The gravity-related decoherence master equation from hybrid dynamics

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Naive hybrid: Dirac+Poisson

Liouville equation for $\rho(q,p)$ with Hamilton func. $H(q,p)$:

$$\dot{\rho} = \{H,\rho\}_P \equiv \sum_n \Big(\frac{\partial H}{\partial q_n} \frac{\partial \rho}{\partial p_n} - \frac{\partial \rho}{\partial q_n} \frac{\partial H}{\partial p_n} \Big)$$

von Neumann equation for $\hat{\rho}$ with Hamiltonian $\hat{H}$:

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] \equiv -\frac{i}{\hbar}\big(\hat{H}\hat{\rho} - \hat{\rho}\hat{H}\big).$$

Naiv hybrid equation for $\hat{\rho}(q,p)$ with $\hat{H}(q,p) = \hat{H}_Q + H_C(q,p) + \hat{H}_{QC}(q,p)$:

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathrm{Herm}\{\hat{H},\hat{\rho}\}_P \quad \text{(Aleksandrov 1981)}$$

It may not preserve $\hat{\rho}(q,p) \geq 0$. (Boucher, Traschen 1988)

Blurring Dirac+Poisson

In total Hamiltonian $\hat{H} = \hat{H}_Q + H_C + \hat{H}_{QC}$, the interacting part:

$$\hat{H}_{QC} = \sum_r \hat{f}^r \phi^r$$

Blurring the 'interacting' currents by classical noises $\delta f, \delta\phi$:

$$\hat{H}^{noise}_{QC} = \sum_r \left(\hat{f}^r + \delta f^r(t)\right)\left(\phi^r + \delta\phi^r(t)\right)$$

$$\left\langle \delta f^r(t')\delta f^s(t) \right\rangle_{noise} = D_Q^{rs}\delta(t'-t)$$

$$\left\langle \delta \phi^r(t')\delta \phi^s(t) \right\rangle_{noise} = D_C^{rs}\delta(t'-t)$$

Total Hamilton becomes noisy: $\hat{H}^{noise} = \hat{H}_Q + H_C + \hat{H}^{noise}_{QC}$.

Hybrid master equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \mathrm{Herm}\{\hat{H}, \hat{\rho}\}_P$$

Recall, this naive hybrid dynamics may not preserve $\hat{\rho}(q,p) \geq 0$. We replace $\hat{H}$ by $\hat{H}^{noise}$ and take the average of the naive dynamics.

$$\hat{H}^{noise} = \hat{H} + \sum_r (\hat{f}^r \delta\phi^r + \phi^r \delta f^r)$$

Noise terms add $-[\hat{f}, [\hat{f}, \hat{\rho}]]$ and $\{\phi, \{\phi, \hat{\rho}\}_P\}_P$ to the naive hybrid eq.:

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \mathrm{Herm}\{\hat{H}, \hat{\rho}\}_P -$$

$$-\frac{1}{2\hbar^2}\sum_{r,s} D_C^{rs}[\hat{f}^r, [\hat{f}^s, \hat{\rho}]] + \frac{1}{2}\sum_{r,s} D_Q^{rs}\{\phi^r, \{\phi^s, \hat{\rho}\}_P\}_P$$

This *hybrid master eq.* preserves $\hat{\rho}(q,p) \geq 0$ if $D_Q D_C \geq \hbar^2/4$.[(D.1995)].

Hybrid master equation for matter plus gravity

Quantized matter Hamiltonian $\hat{H}_Q$, its mass density $\hat{f}(r)$, coupled to weak classical gravitational field $\phi \equiv \frac{1}{2}c^2(g_{00}-1)$. Conjugate variables $q_n \Rightarrow \phi(r)$ and $p_n \rightarrow \xi(r)$. Hybrid state: $\hat{\rho}(\phi,\xi)$. The total Hamiltonian:

$$\hat{H}(\phi,\xi) = \hat{H}_Q + H_C(\phi,\xi) + \hat{H}_{QC}(\phi)$$

$$H_C(\phi,\xi) = \int_r \left(2\pi G c^2\ \xi^2 + \frac{|\nabla\phi|^2}{8\pi G}\right); \qquad \hat{H}_{QC}(\phi) = \int_r \hat{f}(r)\phi(r)$$

As $\hat{f}^r \Rightarrow \hat{f}(r)$ and $\phi^r \Rightarrow \phi(r)$, and $\sum_r \Rightarrow \int_r$, the hybrid master eq. reads:

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathrm{Herm}\{\hat{H},\hat{\rho}\}_P -$$

$$-\frac{1}{2\hbar^2}\int_{r,s} D_C(r,s)[\hat{f}(r),[\hat{f}(s),\hat{\rho}]] + \frac{1}{2}\int_{r,s} D_Q(r,s)\{\phi(r),\{\phi(s),\hat{\rho}\}_P\}_P$$

Recall $D_Q D_C \geq \hbar^2/4$.
We concentrate on the reduced dynamics of $\hat{\rho}_Q = \int \hat{\rho}(\phi,\xi)\mathcal{D}\phi\mathcal{D}\xi$. Most terms on the r.h.s. cancel but we are left with:

Reduced quantum master equation

$$\dot{\hat\rho}_Q = -\frac{i}{\hbar}[\hat H_Q, \hat\rho_Q] - \frac{i}{\hbar}\int_r \phi(r)[\hat f(r), \hat\rho(\phi)]\mathcal{D}\phi - \frac{1}{2\hbar^2}\int_{r,s} D_C(r,s)[\hat f(r), [\hat f(s), \hat\rho_Q]]$$

where $\hat\rho(\phi) = \int \hat\rho(\phi,\xi)\mathcal{D}\xi$. Suppose the *post-mean-field* Ansatz:

$$\int \phi(r)\hat\rho(\phi)\mathcal{D}\phi = \mathrm{Herm}\hat\phi(r)\hat\rho_Q.$$

$$\hat\phi(r) =: -G\int_s \frac{\hat f(s)}{|r-s|}$$

We obtain the following result:

$$\dot{\hat\rho}_Q = -\frac{i}{\hbar}[\hat H_Q + \hat H_G, \hat\rho_Q] - \frac{1}{2\hbar^2}\int_r\int_s D_C(r,s)[\hat f(r), [\hat f(s), \hat\rho_Q]$$

where $\hat H_G$ is the well-known Newtonian potential energy:

$$\hat H_G = -\frac{G}{2}\int_r\int_s \frac{\hat f(r)\hat f(s)}{|r-s|}.$$

The gravity-related decoherence matrix

We determine the decoherence matrix $D_C(r,s)$. Recall:

$$\left\langle \delta f(r',t')\delta f(r,t)\right\rangle_{noise} = D_Q(r',r)\delta(t'-t)$$

$$\left\langle \delta\phi(r',t')\delta\phi(r,t)\right\rangle_{noise} = D_C(r',r)\delta(t'-t)$$

The mean-fields satisfy: $\Delta\langle\phi(r)\rangle = 4\pi G\langle\hat{f}(r)\rangle$. If we requested the same equation for the fluctuations, the above two correlations would lead to:

$$\Delta\Delta' D_C(r,r') = (4\pi G)^2 D_Q(r,r')$$

With minimum blurring $D_C D_Q = \hbar^2/4$, the unique translation invariant solution:$^{(cf. D., Lukacs 1987)}$

$$D_C(r,r') = (G\hbar/2)|r-r'|^{-1}$$

The reduced master eq. of quantized matter follows:$^{(D. 1987; cf. Penrose 1994)}$

$$\dot{\hat{\rho}}_Q = -\frac{i}{\hbar}[\hat{H}_Q + \hat{H}_G, \hat{\rho}_Q] - \frac{1}{4}\int_r\int_{r'}\frac{G/\hbar}{|r-r'|}[\hat{f}(r),[\hat{f}(r'),\hat{\rho}_Q]].$$

Appendix: Full expansion of the hybrid master equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}_Q, \hat{\rho}] - \frac{i}{\hbar}\int_r \phi(r)[\hat{f}(r), \hat{\rho}]$$

$$-\int_r\Big(4\pi Gc^2\xi(r)\frac{\delta\hat{\rho}}{\delta\phi(r)} + \frac{1}{4\pi G}\Delta\phi(r)\frac{\delta\hat{\rho}}{\delta\xi(r)}\Big) + \mathrm{Herm}\int_r \hat{f}(r)\frac{\delta\hat{\rho}}{\delta\xi(r)}$$

$$-\frac{1}{2\hbar^2}\int_r\int_{r'} D_C(r,r')[\hat{f}(r),[\hat{f}(r'),\hat{\rho}]\,] + \frac{1}{2}\int_r\int_{r'} D_Q(r,r')\frac{\delta^2\hat{\rho}}{\delta\xi(r)\delta\xi(r')}.$$

The status of the *post-mean-field* Ansatz

$$\int \phi(r)\hat{\rho}(\phi)\mathcal{D}\phi = \mathrm{Herm}\hat{\phi}(r)\hat{\rho}_Q \ ,$$

$$\hat{\phi}(r) =: -G\int_s \frac{\hat{f}(s)}{|r-s|}$$

is yet to be clarified. It may follow from the $c \to \infty$ non-relativistic limit, or may at least be consistent with it. But it may, in the worst case, contradict to the hybrid master equation.

References

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