SMALLEST REFRIGERATOR WITHOUT MOVING PARTS

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CONTENTS

Linden, Popescu, Skrzypczyk: How small thermal machines can be?
LSP: The smallest possible refrigerator, PRL **105**, 130401 (2010)
SBrunnerLP: On the efficiency of very small refrigerators, arXiv:1009.0865
P: Maximally efficient quantum thermal machines: The basic principles, arXiv:1010.2536
LSP: The smallest possible heat engines, arXiv:1010.6029
Also L.D.: Short Course on Quantun Information Theory (Springer, 2nd, to appear)

SMALLEST REFRIGERATOR: 3-LEVEL-SYSTEM

Hot and cold reservoirs: $T_h > T_c$. Refrigerator will yield $T_0 < T_c$.
Transition $|0\rangle \to |1\rangle$ is heated by T_h, $|0\rangle \to |2\rangle$ is cooled by T_c.
Let $\epsilon_c > \epsilon_h$!

$-- \epsilon_c -- \; exp(-\epsilon_c/k_B T_c) \quad -- \epsilon_c -- \; exp[-(\epsilon_c/k_B T_c) + (\epsilon_h/k_B T_h)]$

$-- \epsilon_h -- \; exp(-\epsilon_h/k_B T_h) \quad -- \epsilon_h -- \; 1$

$-- 0 --- \; 1$

Make $exp[-(\epsilon_c/k_B T_c) + (\epsilon_h/k_B T_h)] \; = \; exp[-(\epsilon_c - \epsilon_h)/k_B T_0] \quad \Rightarrow$

Effective temperature of the TLS $|1_h\rangle \, , |1_c\rangle$:

$$T_0 = \frac{1 - \frac{\epsilon_h}{\epsilon_c}}{1 - \frac{\epsilon_h}{\epsilon_c}\frac{T_c}{T_h}} T_c \qquad (< T_c).$$

2nd SMALLEST REFRIGERATOR: 2xTLS

Hot and cold reservoirs: $T_h > T_c$. Refrigerator will yield $T_0 < T_c$.
Transition $|0\rangle \to |1\rangle$ is heated by T_h, $|0\rangle \to |2\rangle$ is cooled by T_c.
Let $\epsilon_c > \epsilon_h$!

$|1_c\rangle$ $--\epsilon_c--$ $exp(-\epsilon_c/k_B T_c)$

$|1_h\rangle$ $--\epsilon_h--$ $exp(-\epsilon_h/k_B T_h)$

$|0_c\rangle$ $--0_c--$ 1

$|0_h\rangle$ $--0_h--$ 1

Make $exp[-(\epsilon_c/k_B T_h) + (\epsilon_h/k_B T_c)] \ = \ exp[-(\epsilon_c - \epsilon_h)/k_B T_0] \quad \Rightarrow$

Effective temperature of the TLS $|1_h\rangle, |1_c\rangle$:

$$T_0 = \frac{1 - \frac{\epsilon_h}{\epsilon_c}}{1 - \frac{\epsilon_h}{\epsilon_c}\frac{T_c}{T_h}} T_c \qquad (< T_c).$$

TLS THERMALIZATION DYNAMICS

TLS: $\hat{a} = |0\rangle\langle 1|$, $\hat{a}^\dagger = |1\rangle\langle 0|$, $\hat{H} = \epsilon\hat{a}^\dagger\hat{a}$; Heat bath: $\beta = 1/k_B T$.
Thermalization master equation:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\epsilon[\hat{a}^\dagger\hat{a},\hat{\rho}] + \Gamma\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\{\hat{a}^\dagger\hat{a},\hat{\rho}\}\right) + \mathrm{e}^{-\beta\epsilon}\Gamma\left(\hat{a}^\dagger\hat{\rho}\hat{a} - \frac{1}{2}\{\hat{a}\hat{a}^\dagger,\hat{\rho}\}\right).$$

2nd term: spontaneous decay $|1\rangle \to |0\rangle$ at rate Γ.
3rd term: thermal excitation $|0\rangle \to |1\rangle$ at rate $\Gamma\times$Boltzmann factor.
Competition $\Rightarrow$ Gibbs stationary state at (inverse) temperature β:

$$\rho \longrightarrow |0\rangle\langle 0| + exp(-\beta\epsilon)|1\rangle\langle 1| \quad (t \gg \mathrm{e}^{\beta\epsilon}/\Gamma).$$

MLS: Any TL subspace may likewise be thermalized.
$\hat{a} = |n\rangle\langle m|$, $\epsilon = \epsilon_n - \epsilon_m > 0, \Gamma = \Gamma_{nm}$
Each TL subspace may have different temperatures T_{nm}. If some are equilibrated by reservoirs, the rest obtains calculable ‘effective temperatures’.

2nd SMALLEST Q-REFRIGERATOR DYNAMICS

No external resources of energy just heat flow $T_h \to T_c$.
Refrigerator: 2xTLS, in contact with T_h, T_c where $T_h > T_c$ and $\epsilon_c > \epsilon_h$.
Develops a temperature $T_0 < T_c$ for the TLS subspace $|1_h\rangle, |1_c\rangle$.
Can cool a 'thermometer' to temperature $T_0 < T_c$.
Thermometer: third TLS $\hat{a}_3, \hat{a}_3^\dagger, \quad \epsilon_3 = \epsilon_0 = \epsilon_c - \epsilon_h$.
Coupled to $\hat{a}_0 = |1_h\rangle\langle 1_c|, \; \hat{a}_0^\dagger = |1_c\rangle\langle 1_h|$ of the refrigerated subspace.
Master eq. in interaction picture:

$$
\begin{aligned}
\frac{d\hat{\rho}}{dt} = \;& \Gamma_c\left(\hat{a}_c\hat{\rho}\hat{a}_c^\dagger - \frac{1}{2}\{\hat{a}_c^\dagger\hat{a}_c, \hat{\rho}\}\right) + \mathrm{e}^{-\beta_c\epsilon_c}\Gamma_c\left(\hat{a}_c^\dagger\hat{\rho}\hat{a}_c - \frac{1}{2}\{\hat{a}_c\hat{a}_c^\dagger, \hat{\rho}\}\right) + \\
& + \Gamma_h\left(\hat{a}_h\hat{\rho}\hat{a}_h^\dagger - \frac{1}{2}\{\hat{a}_h^\dagger\hat{a}_h, \hat{\rho}\}\right) + \mathrm{e}^{-\beta_h\epsilon_h}\Gamma_h\left(\hat{a}_h^\dagger\hat{\rho}\hat{a}_h - \frac{1}{2}\{\hat{a}_h\hat{a}_h^\dagger, \hat{\rho}\}\right) + \\
& - i\frac{g}{\hbar}\left[\hat{a}_3^\dagger\hat{a}_0 + \hat{a}_0^\dagger\hat{a}_3, \hat{\rho}\right]
\end{aligned}
$$

If coupling $g \ll \Gamma_c, \Gamma_h$ then $\hat{\rho}_3 \to |0_3\rangle\langle 0_3| + \exp(-\epsilon_3/k_B T_0)\,|1_3\rangle\langle 1_3|$.

2nd SMALLEST HEAT ENGINE: 2xTLS

We want a negative T_0 (population inversion).
Change the role of T_h and T_c in refrigerator: $\Rightarrow$ T_0 may be negative!
Reorganized refrigerator becomes heat engine.
Transition $|0_h\rangle \to |1_h\rangle$ is heated by T_h, $|0_c\rangle \to |1_c\rangle$ is cooled by T_c.
Let $\epsilon_h > \epsilon_c$ now (opposite than for refrigerator)!

$|1_h\rangle$ $--\epsilon_h--$ $exp(-\epsilon_h/k_B T_h)$

$|1_c\rangle$ $--\epsilon_c--$ $exp(-\epsilon_c/k_B T_c)$

$|0_h\rangle$ $--0_h--$ 1

$|0_c\rangle$ $--0_c--$ 1

Make $exp[-(\epsilon_h/k_B T_h) + (\epsilon_c/k_B T_c)] \;=\; exp[-(\epsilon_h - \epsilon_c)/k_B T_0] \quad \Rightarrow$

Negative effective temperature of the TLS $|1_h\rangle, |1_c\rangle$:

$$T_0 = \frac{1 - \frac{\epsilon_c}{\epsilon_h}}{1 - \frac{\epsilon_c}{\epsilon_h}\frac{T_h}{T_c}} T_h \qquad < \; 0 \text{ if } \frac{T_h}{T_c} > \frac{\epsilon_h}{\epsilon_c} > 1.$$

Negative T_0 means population inversion between $|1_c\rangle$ and $|1_h\rangle$. It can 'lift a weight' at constant speed!

2nd SMALLEST HEAT ENGINE DYNAMICS

Resource: heat flow $T_h \to T_c$.
Engine: 2xTLS, in contact with T_h, T_c where $T_h/T_c > \epsilon_h/\epsilon_c > 1$.
Develops population inversion $T_0 < 0$ for the TLS subspace $|1_c\rangle, |1_h\rangle$.
Can 'lift a weight' at stationary power.
Weight: harmonic oscillator $\hat{a}_3, \hat{a}_3^\dagger$, $\epsilon_3 = \epsilon_0 = \epsilon_h - \epsilon_c$.
Coupled to $\hat{a}_0 = |1_c\rangle\langle 1_h|$, $\hat{a}_0^\dagger = |1_h\rangle\langle 1_c|$ of the population inverted TLS.
Master eq. in interaction picture (formally same as refrigerator's, just $[\hat{a}_3, \hat{a}_3^\dagger] = 1$):

$$\begin{aligned}\frac{d\hat{\rho}}{dt} = \quad & \Gamma_c \left(\hat{a}_c\hat{\rho}\hat{a}_c^\dagger - \frac{1}{2}\{\hat{a}_c^\dagger\hat{a}_c, \hat{\rho}\}\right) + \mathrm{e}^{-\beta_c\epsilon_c}\Gamma_c \left(\hat{a}_c^\dagger\hat{\rho}\hat{a}_c - \frac{1}{2}\{\hat{a}_c\hat{a}_c^\dagger, \hat{\rho}\}\right) + \\ & + \Gamma_h \left(\hat{a}_h\hat{\rho}\hat{a}_h^\dagger - \frac{1}{2}\{\hat{a}_h^\dagger\hat{a}_h, \hat{\rho}\}\right) + \mathrm{e}^{-\beta_h\epsilon_h}\Gamma_h \left(\hat{a}_h^\dagger\hat{\rho}\hat{a}_h - \frac{1}{2}\{\hat{a}_h\hat{a}_h^\dagger, \hat{\rho}\}\right) + \\ & - i\frac{g}{\hbar}\left[\hat{a}_3^\dagger\hat{a}_0 + \hat{a}_0^\dagger\hat{a}_3, \hat{\rho}\right]\end{aligned}$$

If coupling $g \ll \Gamma_c, \Gamma_h$ then, for $T_h/T_c > \epsilon_h/\epsilon_c$, the oscillator energy $\langle \epsilon_3 \hat{a}_3^\dagger \hat{a}_3 \rangle$ grows like $\sim g^2 t$. Carnot-efficiency is reached at $g \to 0$ and $T_h/T_c \to \epsilon_h/\epsilon_c$.