Pauli-GKLS Hybrid Master Equations

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Prelude: before I learned about CP & GKLS

Master Eq. for positive dynamical semigroup: $d\hat{\rho}/dt = \mathcal{L}\hat{\rho}$.
For pure initial state $\hat{\rho} = |\Psi\rangle\langle\Psi| \equiv \hat{P}$:

$$d\hat{P}/dt = \mathcal{L}\hat{P} = -i\hat{H}_\Psi \hat{P} + i\hat{P}\hat{H}_\Psi^\dagger + \hat{W}_\Psi$$

Effective Hamiltonian: $-i\hat{H}_\Psi = \mathcal{L}\hat{P} - \langle\mathcal{L}\hat{P}\rangle_\Psi$
Transition rate operator: $\hat{W}_\Psi = \mathcal{L}\hat{P} - \{\mathcal{L}\hat{P}, \hat{P}\} + \langle\mathcal{L}\hat{P}\rangle_\psi \hat{P} \;\geq\; 0$
Ito-Stochastic Schrodinger Eq. for all P-preserving ME's (D. 1986):

$$d\Psi = -i\hat{H}_\Psi |\Psi\rangle\, dt + d|\Phi\rangle \qquad d|\Phi\rangle d\langle\Phi| = \hat{W}_\Psi dt$$

Example of non-CP but P ME: $d\hat{\rho}/dt = \mathcal{L}\hat{\rho} = \sum_k c_k(\hat{\sigma}_k\hat{\rho}\hat{\sigma}_k - \hat{\rho})$

$c_1 = 1,\; c_2 = 1,\; c_3 = -1$ (Benatti & al. 2002)

$\hat{H}_\Psi = -\frac{1}{2}\sum c_k(\hat{\sigma}_k - \langle\hat{\sigma}_k\rangle)^2; \qquad \hat{W}_\Psi = \sum c_k(\hat{\sigma}_k - \langle\hat{\sigma}_k\rangle)\hat{P}(\hat{\sigma}_k - \langle\hat{\sigma}_k\rangle)$

Hybrid

- Dynamical coupling between Classical and Quantum (Aleksandrov 1981)
- Coupling between measurement device and Quantum (Sherry & Sudarshan 1978)
- Fundational coexistence between Classical and Quantum (D. 1998)

Mathematics:

	Classical	Quantum	Hybrid
State:	$\rho(x)$	$\hat{\rho}$	$\hat{\rho}(x)$
Dynamics:	Pauli	GKLS	?

Hybrid density

Simplest construction: $\{\rho(x) \textit{ and } \hat{\rho}\} \Rightarrow \rho(x)\hat{\rho} \equiv \hat{\rho}(x)$

Generically:

$$\text{any } \hat{\rho}(x) \geq 0 \quad \forall x; \quad \mathrm{tr} \sum_x \hat{\rho}(x) = 1.$$

Reduced Q	Reduced C	Conditional Q	Conditional C
$\hat{\rho} = \sum_x \hat{\rho}(x)$	$\rho(x) = \mathrm{tr}\hat{\rho}(x)$	$\hat{\rho}_x = \hat{\rho}(x)/\rho(x)$	$\nexists$

E.g.:

- $\hat{\rho}(r,p)$ where $\hat{\rho}$: electrons, (r,p) : nuclei
- $\hat{\rho}[A]$ where $\hat{\rho}$: e^+e^-, A : e.m.field
- $\hat{\rho}(n)$ where $\hat{\rho}$: Q-dot, n : charge count
- $\hat{\rho}(k)$ where $\hat{\rho}$: Q-system, k : measurement outcome

Measurement

What happens to $\hat{\rho}$ under measurement of c.s.o.p. $\{\hat{P}_x\}$?

- *Text-book formalism:* $\hat{\rho}$ jumps randomly to the conditional $\hat{\rho}_x$,

$$\hat{\rho} \longrightarrow \hat{\rho}_x = \frac{1}{\rho(x)}\hat{P}_x\hat{\rho}\hat{P}_x \text{ with probability } \rho(x) = \mathrm{tr}(\hat{P}_x\hat{\rho}\hat{P}_x).$$

- *Hybrid formalism:* $\hat{\rho}$ jumps to the hybrid density $\hat{\rho}(x)$,

$$\hat{\rho} \longrightarrow \hat{\rho}(x) = \hat{P}_x\hat{\rho}\hat{P}_x.$$

$$\text{Generically:} \quad \hat{\rho} \longrightarrow \hat{\rho}(x) = \hat{M}_x\hat{\rho}\hat{M}_x^\dagger \text{ where } \sum_x \hat{M}_x^\dagger\hat{M}_x = \hat{I}.$$

E.g.: unsharp measurement of $\hat{q}$

$$\hat{M}_x = \hat{M}_x^\dagger = (2\pi\sigma^2)^{-1/4}\exp[-(\hat{q}-x)^2/4\sigma^2]$$

$$\hat{\rho} \longrightarrow \hat{\rho}(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{[-(\hat{q}-x)^2/4\sigma^2]}\hat{\rho}e^{[-(\hat{q}-x)^2/4\sigma^2]}$$

Canonical hybrid dynamics?

- *Hybrid canonical bracket*, with hybrid Hamiltonian $\hat{H}(q,p)$:

  $$\dot{\hat{\rho}}(q,p) = -i[\hat{H}(q,p),\hat{\rho}(q,p)] + \textbf{Herm}\{\hat{H}(q,p),\hat{\rho}(q,p)\}_{\mathrm{Poisson}}$$

  Fatally wrong: $\hat{\rho}(q,p) \geq 0$ is not preserved!
  Positivity is preserved if we add noise to the r.h.s.

- *Partial Husimi projection*, from unitary dynamics:
  Q-system + Q-harmonic-oscillator: $\dot{\widehat{\rho}} = -i[\widehat{H},\widehat{\rho}]$.
  Generate hybrid state and ME by partial Husimi projection:

  $$\hat{\rho}(q,p) = \mathrm{tr}_{\mathrm{osc}}\left((\hat{1} \otimes |q,p\rangle\langle q,p|)\widehat{\rho}\right)$$

  $$\dot{\hat{\rho}} = -i\mathrm{tr}_{\mathrm{osc}}\left((\hat{1} \otimes |q,p\rangle\langle q,p|)[\widehat{H},\widehat{\rho}]\right)$$

  Yields canonical hybrid bracket plus "noisy" terms.
  $\hat{\rho}(q,p) \geq 0$ preserved by construction.

Hybrid dynamics

- Markovian Classical ME (Pauli):
  $$\dot{\rho}(x) = -\partial_x v(x)\rho(x) + \sum_y \Big(T(x,y)\rho(y) - T(y,x)\rho(x)\Big), \quad T(x,y) \geq 0$$
  diffusion: $\dot{\rho}(x) = D\partial_x^2 \rho(x), \qquad T(x,y) = \lim_{\tau\to 0} \frac{1/\tau}{\sqrt{4\pi D\tau}} e^{-[(x-y)^2/4D\tau]}$
- Markovian Quantum ME (GKLS):
  $$\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] + \sum_\alpha \Big(\hat{F}_\alpha \hat{\rho} \hat{F}_\alpha^\dagger - \tfrac{1}{2}\{\hat{F}_\alpha^\dagger \hat{F}_\alpha, \hat{\rho}\}\Big)$$
  decoherence: $\dot{\hat{\rho}} = -i[\hat{H},\hat{\rho}] - D[\hat{q},[\hat{q},\hat{\rho}]], \qquad \hat{F} = \hat{F}^\dagger = \sqrt{2D}\hat{q}$
- Generic Markovian Hybrid ME ('Pauli-GKLS'):
  $$\dot{\hat{\rho}}(x) = -i[\hat{H}(x),\hat{\rho}(x)] +$$
  $$+ \sum_{y,\alpha} \Big(\hat{F}_\alpha(x,y)\hat{\rho}(y)\hat{F}_\alpha^\dagger(x,y) - \tfrac{1}{2}\{\hat{F}_\alpha^\dagger(y,x)\hat{F}_\alpha(y,x), \hat{\rho}(x)\}\Big)$$

(Alicki & Kryszewski 2004, D. 2014)

Hybrid ME: Derivation

- Embed Hybrid into a bigger Quantum:
$$\hat\rho(x)\rightarrow\widehat\rho=\sum_x\hat\rho(x)\otimes|x\rangle\langle x|\,,\quad \hat H(x)\rightarrow\widehat H=\sum_x\hat H(x)\otimes|x\rangle\langle x|$$
- Assume GKLS ME:
$$\dot{\widehat\rho}=-i[\widehat H,\widehat\rho]+\widehat F\widehat\rho\widehat F^\dagger-\frac12\{\widehat F^\dagger\widehat F,\widehat\rho\}]$$
- Project back by $\hat I\otimes|x\rangle\langle x|$, introduce $\hat F(x,y)=\mathrm{tr}'[(\hat I\otimes|y\rangle\langle x|)\widehat F]$
$$\dot{\hat\rho}(x)=-i[\hat H(x),\hat\rho(x)]+$$
$$+\sum_y\left[\hat F(x,y)\hat\rho(y)\hat F^\dagger(x,y)-\frac12\{\hat F^\dagger(y,x)\hat F(y,x),\hat\rho(x)\}\right]$$

That's general (Markovian) hybrid ME if $\hat F(x,y)$ is regular.
But, e.g., $\hat F(x,y)\sim\delta'(x-y)$, yields different forms.
Coming example: $\widehat F$ contains $(\partial_x|x\rangle)\langle x|$.

Case study: Q-monitoring

$\hat{\rho}$: Q-particle, $\dot{x}$: monitored value of $\hat{q}$

$$d\langle x\rangle/dt = \langle\hat{q}\rangle$$

- Naive hybrid ME:
  $$\dot{\hat{\rho}}(x) = -i[\hat{H},\hat{\rho}(x)] - \tfrac{1}{2}\partial_x\{\hat{q},\hat{\rho}(x)\}$$
  Problem: term $-\frac{1}{2}\partial_x\{\hat{q},\hat{\rho}(x)\}$ violates positivity of $\hat{\rho}(x)$

- Try GKLS ME for $\widehat{\rho}$ in 'big' space, choose
  $$\widehat{F} = \frac{\hat{q}}{\sqrt{2D}}\otimes\hat{I} + \sqrt{2D}\,\hat{I}\otimes\int\frac{\partial|x\rangle}{\partial x}\langle x|dx \quad \Rightarrow$$
  $$\dot{\hat{\rho}}(x) = -i[\hat{H},\hat{\rho}(x)] - \tfrac{1}{2}\partial_x\{\hat{q},\hat{\rho}(x)\} - \frac{1}{16D}[\hat{q},[\hat{q},\hat{\rho}(x)]] + D\partial_x^2\hat{\rho}(x)$$
  That's Hybrid ME of Q-monitoring $\hat{q}$ (D. 2014).
  Equivalent with the Ito-formalism (Belavkin 1988, D. 1988).
  (Similar HMEs: Bauer & Bernard & Tilloy 2014)

Summary

- Q-measurements $\Rightarrow$ fundational hybrids:

$$\hat{\rho} \longrightarrow \hat{P}_x \hat{\rho} \hat{P}_x \equiv \hat{\rho}(x)$$

- Structural theorem is missing for HME. What we do instead:
  1) Dilate from hybrid to full quantum: $\hat{\rho}(x) \Rightarrow \widehat{\rho}$
  2) Impose GKLS ME: $\dot{\widehat{\rho}} = \mathcal{L}_{GKLS}\widehat{\rho}$
  3) Project back to hybrid ME: $\dot{\hat{\rho}}(x) = \mathcal{L}_{hybr}\hat{\rho}(x)$
- Time-continuous Q-measurement $\Rightarrow$ fundational HME:

$$\dot{\hat{\rho}}(x) = -i[\hat{H}, \hat{\rho}(x)] - \tfrac{1}{2}\partial_x\{\hat{q}, \hat{\rho}(x)\} - \frac{1}{16D}[\hat{q}, [\hat{q}, \hat{\rho}(x)]] + D\partial_x^2\hat{\rho}(x)$$

$$\frac{d\langle x\rangle}{dt} = \langle\hat{q}\rangle$$

- Outlook: Classical gravity decoheres (D. 2011); Is gravity LOCC? (Kafri,Taylor,Milburn 2014)