Autonomy of coupled quantum thermodynamic systems

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Quantum thermalization

TLS in heat bath $\beta = 1/k_B T$

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] + \mathcal{L}\hat{\rho}$$

$$\hat{H} = \omega\,\hat{n}, \qquad \hat{n} = \hat{a}^\dagger\,\hat{a}, \qquad \{\hat{a}, \hat{a}^\dagger\} = 1.$$

Thermalizer:

$$\mathcal{L}\hat{\rho} = \Gamma(\hat{a}\hat{\rho}\,\hat{a}^\dagger - \tfrac{1}{2}\{\hat{n}, \hat{\rho}\}) + \Gamma e^{-\beta\omega}(\hat{a}^\dagger\hat{\rho}\,\hat{a} - \tfrac{1}{2}\{1 - \hat{n}, \hat{\rho}\})$$

Stationary state:

$$\hat{\rho}^s = \frac{1}{1 + e^{-\beta\omega}} e^{-\beta\,\hat{n}}$$

Current operator:

$$\hat{J} = \mathcal{L}^\star\,\hat{H} = -\Gamma\omega\,\hat{n} + \Gamma\omega e^{\beta\omega}(1 - \hat{n})$$

Current vanishes in steady state:

$$J = \mathrm{Tr}(\hat{J}\hat{\rho}^s) = 0$$

Coupling of local quantum thermalized systems

TLS A in hot bath β_A, TLS B in cold bath $\beta_B > \beta_A$

$$\frac{d\hat{\rho}_A}{dt} = -i[\hat{H}_A, \hat{\rho}_A] + \mathcal{L}_A\hat{\rho}_A, \qquad \frac{d\hat{\rho}_B}{dt} = -i[\hat{H}_B, \hat{\rho}_B] + \mathcal{L}_B\hat{\rho}_B$$

Joint stationary state:

$$\hat{\rho}^s_{AB} = \hat{\rho}^s_A\hat{\rho}^s_B$$

Coupling: $\hat{K}_{AB} = \epsilon(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a})$

$$\frac{d\hat{\rho}_{AB}}{dt} = -i[\hat{H}_A + \hat{H}_B + \hat{K}_{AB}, \hat{\rho}_{AB}] + (\mathcal{L}_A + \mathcal{L}_B)\hat{\rho}_{AB}$$

Levy-Kosloff arXiv:1402.3825:

- This equation is incorrect, even for weak coupling
- Its stationary state may violate 2nd Law
- Correct equation: joint thermalizer $\mathcal{L}_{AB}$ instead of $\mathcal{L}_A + \mathcal{L}_B$
- $\mathcal{L}_{AB}$ is intricate: autonomy of A and B is lost

Incoherent resonant coupling

My points:

- Revive autonomy of A and B
- It would persist for weak stochastic coupling
- Let's smash coherence of coupling
- ... and restrict to resonant coupling

Coupling: $\hat{K}_{AB} = \sqrt{\epsilon}\left(\xi(t)\,\hat{a}^\dagger\,\hat{b} + \xi^\star(t)\,\hat{b}^\dagger\,\hat{a}\right)$

$\xi = (w_1 + iw_2)/\sqrt{2}$ with standard white-noises w_1, w_2
In resonance $\omega_A = \omega_B$: $[\hat{K}_{AB}, \hat{H}_A + \hat{H}_B] = 0$

$$\frac{d\hat{\rho}_{AB}}{dt} = -i[\hat{H}_A + \hat{H}_B, \hat{\rho}_{AB}] + (\mathcal{L}_A + \mathcal{L}_B + \mathcal{L}_{AB}^K)\hat{\rho}_{AB}$$

$$\mathcal{L}_{AB}^K\hat{\rho} = \epsilon\left(\hat{a}^\dagger\,\hat{b}\hat{\rho}_{AB}\,\hat{b}^\dagger\,\hat{a} + \hat{a}\,\hat{b}^\dagger\hat{\rho}_{AB}\,\hat{b}\,\hat{a}^\dagger - \tfrac{1}{2}\{\hat{n}_A + \hat{n}_B - 2\,\hat{n}_A\,\hat{n}_B, \hat{\rho}_{AB}\}\right)$$

Incoherent coupling $\mathcal{L}_{AB}^K$ preserves autonomy of A and B.

Incoherent resonant coupling: steady state

Full master equation:

$$\frac{d\hat{\rho}_{AB}}{dt} = -i\omega[\hat{n}_A + \hat{n}_B, \hat{\rho}_{AB}]$$
$$+\Gamma_A\left(\hat{a}\hat{\rho}_{AB}\hat{a}^\dagger - \tfrac{1}{2}\{\hat{n}_A, \hat{\rho}_{AB}\}\right) + \Gamma_A e^{-\beta_A\omega}\left(\hat{a}^\dagger\hat{\rho}_{AB}\hat{a} - \tfrac{1}{2}\{1-\hat{n}_A, \hat{\rho}_{AB}\}\right)$$
$$+\Gamma_B\left(\hat{b}\hat{\rho}_{AB}\hat{b}^\dagger - \tfrac{1}{2}\{\hat{n}_B, \hat{\rho}_{AB}\}\right) + \Gamma_B e^{-\beta_B\omega}\left(\hat{b}^\dagger\hat{\rho}_{AB}\hat{b} - \tfrac{1}{2}\{1-\hat{n}_B, \hat{\rho}_{AB}\}\right)$$
$$+\epsilon\left(\hat{a}^\dagger\hat{b}\hat{\rho}_{AB}\hat{b}^\dagger\hat{a} + \hat{a}\hat{b}^\dagger\hat{\rho}_{AB}\hat{b}\hat{a}^\dagger - \tfrac{1}{2}\{\hat{n}_A + \hat{n}_B - 2\hat{n}_A\hat{n}_B, \hat{\rho}_{AB}\}\right)$$

Take (for simplicity) $\Gamma_A = \Gamma_B = \Gamma \gg \epsilon$

$$\hat{\rho}^s_{AB} = \hat{\rho}^s_A\hat{\rho}^s_B - \frac{\epsilon}{\Gamma}\frac{D^2\hat{\rho}^s_A\hat{\rho}^s_B + \hat{D}_A\hat{D}_B + 2\hat{D}_A + 2\hat{D}_B}{(2 + e^{-\beta_A\omega} + e^{-\beta_B\omega})(1 + e^{-\beta_A\omega})(1 + e^{-\beta_B\omega})}$$

$$D = e^{-\omega\beta_A} - e^{-\omega\beta_B}, \quad \hat{D}_A = e^{-\omega\beta_A\hat{n}_A} - e^{-\omega\beta_B\hat{n}_A}, \quad \hat{D}_B = e^{-\omega\beta_B\hat{n}_B} - e^{-\omega\beta_A\hat{n}_B}$$

Incoherent resonant coupling: heat flow

Current operators:

$$\hat{J}_A = \mathcal{L}_A^\star \hat{H}_A = -\Gamma_A \hat{n}_A + \Gamma_A e^{-\beta_A \omega_A}(1 - \hat{n}_A)$$
$$\hat{J}_B = \mathcal{L}_B^\star \hat{H}_B = -\Gamma_B \hat{n}_B + \Gamma_B e^{-\beta_B \omega_B}(1 - \hat{n}_B)$$

Currents in steady state: $J_A^s = \mathrm{Tr}(\hat{J}_A \hat{\rho}_{AB}^s)$ and $J_B^s = \mathrm{Tr}(\hat{J}_B \hat{\rho}_{AB}^s)$

$$J_A^s + J_B^s = 0.$$

In our case:

$$J_A^s = \mathrm{Tr}(\hat{J}_A \hat{\rho}_{AB}^s) = \epsilon\omega(e^{-\omega\beta_A} - e^{-\omega\beta_B})$$

Γ cancels from J_A in lowest order in ϵ/Γ.
Heat flows from hot to cold baths, 2nd Law respected*.

Summary

- Autonomy of local Q-thermodynamic systems is lost for coherent coupling
- Autonomy of local Q-thermodynamic systems is preserved for incoherent coupling
- Resonant coupling seems also necessary for consistency
- Heat flows from hot to cold
- Proof is given for weak incoherent resonant coupling in special case $\Gamma_A = \Gamma_B$