Non-Markovian Open Quantum Systems: Input-Output Theory

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1 System+Bath→System+Memory+Detector

If the memory of B cannot be ignored for S, Markovian tools don't work. In such non-Markovian (NM) case, S is coherently interacting with a finite part of B over a finite time.

How can we divide the environment B into the memory M and detector D?

M is continuously entangled with S, while S+M should be Markovian open system. D contains information on S, can be continuously disentangled (monitored) without changing the dynamics of S.

Answer: Markovian field representation [GarCol85] of B. The local Markov field interacts with S in a finite range (M). Information on S is carried away by the output field (D). Markovian theory [GarCol85] of monitoring apply invariably to the composite system S+M.

2 Markovian bath, non-Markovian coupling

The composite S+B dynamics:

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}$$

$$\hat{H}_B = \int \omega \hat{b}_\omega^\dagger \hat{b}_\omega d\omega \qquad \hat{H}_{SB} = i\hat{s} \int \kappa_\omega \hat{b}_\omega^\dagger d\omega + \text{h.c.}$$

$\hat{s}$ is a S-operator that couples to the B-modes.

$$[\hat{b}_\omega, \hat{b}_{\omega'}^\dagger] = \delta(\omega - \omega'), \qquad \hat{b}_\omega|0\rangle = 0$$

B is Markovian (flat spectrum). Memory is encoded in coupling κ_ω. Markovian limit: $\kappa_\omega = \text{const.}$
Switch for abstract field representation!

3 Markovian local field, non-local coupling

$$\hat{b}(z) = \frac{1}{\sqrt{2\pi}} \int \hat{b}_\omega \mathrm{e}^{-i\omega z} d\omega, \qquad z \in (-\infty, \infty)$$

$$[\hat{b}(z), \hat{b}^\dagger(z')] = \delta(z - z')$$

The field can be measured *independently* at all locations.
Free Heisenberg field: $\hat{b}_t(z) = \hat{b}(z + t)$.
The composite S+B dynamics:

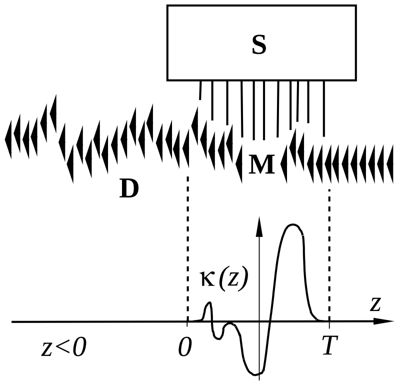
$$\hat{H}_B = \frac{i}{2} \int \hat{b}^\dagger(z) \partial_z \hat{b}(z) dz + \mathrm{h.c.}$$

$$\hat{H}_{SB} = i\hat{s} \int \hat{b}^\dagger(z) \kappa(z) dz + \mathrm{h.c.},$$

$\kappa(z)$=Fourier-tr. of κ_ω. Markovian limit $\kappa(z) \propto \delta(z)$.
Heisenberg field [GarCol85]:

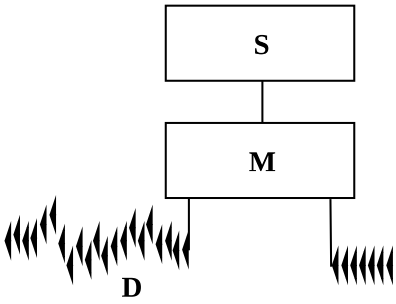
$$\hat{b}(z, t) = \hat{b}(z + t) + \int_0^t \hat{s}(t - \tau) \kappa(z + \tau) d\tau$$

Input-output fields



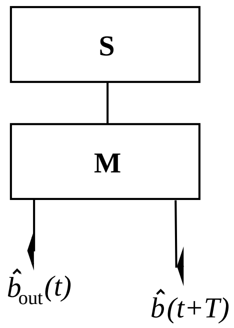
The bath field $\hat{b}(z,t)$, when free, is propagating from *right to left* without dispersion at velocity 1. The unperturbed input field from range $z \geq T$ propagates through the interaction range $z \in [0,T]$ of non-zero coupling $\kappa(z)$, gets modified by, and entangled with the system S, then it leaves to freely propagate away to *left* infinity as the output field. The interaction range makes the memory M and the output range $z \leq 0$ makes the detector D which can continuously be read out (monitored).

Memory and Detector



If we form a memory subsystem M from the local field oscillators of the interaction range then the system S and the memory M constitutes a Markovian open system. It is pumped by the standard Markovian quantum noise (input field) and it creates the Markovian output field D that can be monitored.

System+Memory=MarkovianOpenSystem



The system-plus-memory is pumped by the standard (external) quantum white-noise $\hat{b}(t+T)$ and monitored through the modified quantum white-noise $\hat{b}_{\text{out}}(t)$ just like Markovian open quantum systems, apart form the delay T of read-out w.r.t. pump.

Mathematical realizations:

I/O relationship [GarCol85] for the measured signal.

Lindblad Master Equation for S+M (formal).

Stoch. Sch-Ito Eq for the conditional state of S+M (?).

NM Stoch. Sch Eq for the conditional state of S.

4 Monitoring

Measurement in coherent state overcomplete basis parametrized by the complex field $\xi(z)$. Bargman coherent states

$$|\xi\rangle = \exp\left(\int \xi(z)\hat{b}^\dagger(z)dz\right)|0\rangle$$

form an overcomplete basis: $\mathbf{M}|\xi\rangle\langle\xi^*| = \hat{1}$.

$$\mathbf{M}\xi(z) = 0,\;\; \mathbf{M}\xi(z)\xi(z') = 0,\;\; \mathbf{M}\xi(z)\xi^*(z') = \delta(z-z').$$

If we perform the measurement, the state of B collapses on $|\xi\rangle$ randomly, the complex field $\xi(z)$ becomes the random read-out. But its statistics depends on the pre-measurement state. In the vacuum state $|0\rangle$, the read-outs $\xi(z)$ follow the $\mathbf{M}$-statistics. It gets modified by the B-S interaction: $\mathbf{M}\xi(z)$ becomes non-vanishing.

5 Stochastic Schrödinger equation

S-statevector under monitoring, conditioned on signal ξ:

$$
\begin{aligned}
\frac{d|\Psi_S[\xi^*;t]\rangle}{dt} &= \hat{s}_t \int_0^T d\tau \kappa(\tau)\xi^*(t+\tau)|\Psi_S[\xi^*;t]\rangle \\
&- \hat{s}_t^\dagger \int_0^T d\tau \kappa^*(\tau) \frac{\delta|\Psi_S[\xi^*;t]\rangle}{\delta\xi^*(t+\tau)}
\end{aligned}
$$

The r.h.s. would contain the measured signal $\xi(t+\tau)$ at later times w.r.t. t, these data are not yet available at time t.
Either we propagate *conditional mixed state* (compromise i) or we propagate the *retrodicted pure state* (compromise ii).
This SSE is equivalent with the Strunz-Diosi SSE (1997).

6 Structured bath→Markovian bath

Strunz-D SSE works in structured bath of spectral density $\alpha_\omega \geq 0$ while coupling is 1. Its interpretation drew debates. No pure state monitoring exists [GamWis03]. Mixed state monitoring is possible [JackCollWall99]. Pure state retrodiction [Dio08]. Causality structure is involved.
Trick:
Structured B ($\alpha_\omega \geq 0, \kappa_\omega = 1$) is equivalent with
Markovian B ($\alpha_\omega = 1, \kappa_\omega \neq 1$) if we solve [Cho24]

$$\alpha(t) = \int \kappa(t+\tau)\kappa^*(\tau)d\tau$$

Strunz-D SSE takes the ξ-driven earlier form of transparent causality structure.

7 Summary

S+M becomes Markovian if you split B into M+D properly.
Markovian (even Ito) technologies must work.
Issue of monitorability is transparent: S+M is monitorable.
Key problem: how to represent (approximate) M.
Compromises: mixed state or retrodicted pure state trajectories.
To MarVacHugBur: Is all S+M asymptotically Markovian?
To MazManPiiSuoGar: Is information flow more transparent in I/O?

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