# CONTINUOUS WAVE FUNCTION COLLAPSE IN QUANTUM-ELECTRODYNAMICS?

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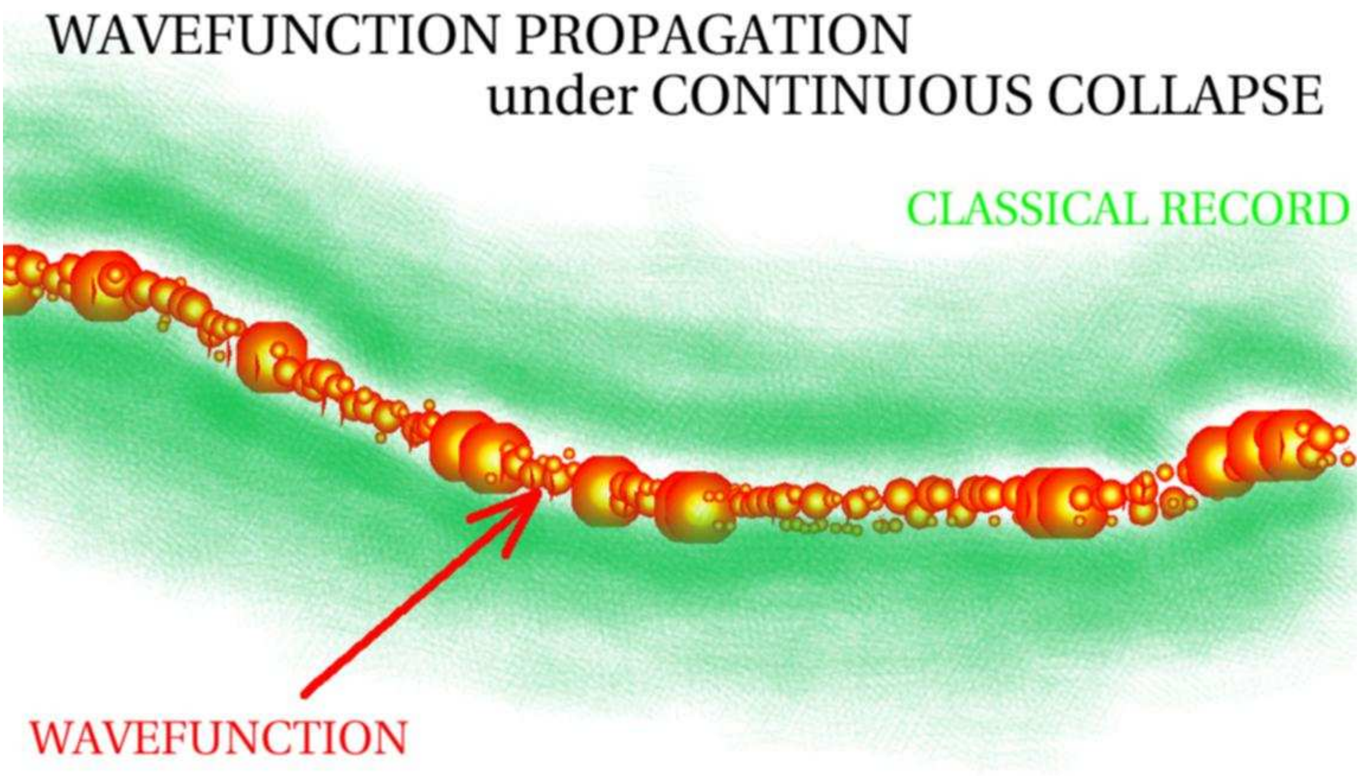
## CONTENT:



## PEOPLE:

- Real Continuous Collapse: Mott, Castin-Dalibard-Molmer, Carmichael, Milburn-Wiseman, ...
- Fictitious Continuous Collapse: Bohm, Károlyházi, Pearle, Gisin, GRW, Diósi, Penrose, Percival, Adler, ... .
- Furthermore: Barchielli-Lanz-Prosperi, Blanchard-Jadczyk, Diósi-Wiseman, ...
- Non-linear Markov SSE: Gisin, Diósi, Belavkin, Pearle, Carmichael, Milburn-Wiseman, ...
- Non-Markov SSE: Strunz, -Diósi, -Gisin-Yu, Budini, Stockburger-Grabert, Bassi, -Ghirardi, Gambetta-Wiseman, ...
- Lorentz Invariance: Pearle, Diósi, Breuer-Petruccione, Percival-Strunz, Rimini, Ghirardi, -Bassi, Tumulka, ...
- Coexistence of classical and quantum: Kent, Diósi, Dowker-Herbaut, ...




**Real or Fictitious Continuous Collapse**

**Classicality emerges from Quantum via real or hypothetic, often time-continuous measurement [detection, observation, monitoring, ...] of the wavefunction $\psi$.**

- **Real: particle track detection, photon-counter detection of decaying atom, homodyne detection of quantum-optical oscillator, ...**
- **Fictitious: theories of spontaneous [universal, intrinsic, primary, ...] localization [collapse, reduction, ...].**

**To date, the mathematics is the same for both classes above! We know almost everything about the mathematical and physical structures if markovian approximation applies. We know much less beyond that approximation.**

What Equation describes the wavefunction under time-continuous collapse?

## The Markovian Stochastic Schrödinger Equation

$$\begin{aligned} \frac{\mathrm{d}\psi(t,z)}{\mathrm{d}t} = & -i\widehat{H}\psi(t,z) && \text{hermitian hamiltonian} \\ & -i\hat{q}z\psi(t,z) && \text{non-hermitian noisy hamiltonian} \\ & -\tfrac{1}{2}\gamma\hat{q}^2\psi(t,z) && \text{non-hermitian dissipative hamiltonian} \end{aligned}$$

where $z$ is complex Gaussian hermitian white-noise: $\mathrm{M}[z^\star(t)z(s)] = \gamma\delta(t-s)$. The equation is not norm-preserving. We define the physical state by $\psi/\|\psi\|$ and its statistical weigth is multiplied by $\|\psi\|^2$:

$$\begin{aligned} \psi(t,z) &\longrightarrow \frac{\psi(t,z)}{\|\psi(t,z)\|} \equiv |t,z\rangle \\ \mathrm{M}[\ldots] &\longrightarrow \mathrm{M}[\,\|\psi(t,z)\|^2\ldots] \equiv \widetilde{\mathrm{M}}_t[\ldots] \end{aligned}$$

There exists a closed non-linear SSE for $|t,z\rangle$.

The markovian SSE describes perfectly the time-continuous collapse of the wavefunction in the given observable(s) $\hat{q}$. The state $|t,z\rangle$ is conditioned on $\{\, z(s); s \le t \,\}$ causally. The individual solutions $|t,z\rangle$ can, in principle, be realized by time-continuous monitoring of $\hat{q}$. Then $z(t)$ becomes the classical record explicitly related to the monitored value of $\hat{q}$.

Our key-problems will be: causality, realizability, and Lorentz-invariance. So far, for markovian SSE: causality OK, realizability OK, Lorentz-invariance NOK.

Why do we need non-markovian SSE?

## The non-Markovian Stochastic Schrödinger Equation

**Driving noise is non-white-noise:**

$$\mathrm{M}[z^\star(t)z(s)] = \alpha(t-s)$$

**SSE contains memory-term:**

$$\frac{\mathrm{d}\psi(t,z)}{\mathrm{d}t} = -i\widehat{H}\psi(t,z) - i\hat{q}z\psi(t,z) + i\hat{q}\int_0^t \alpha(t-s)\frac{\delta\psi(t,z)}{\delta z(s)}\mathrm{d}s$$

**The equation is not norm-preserving. We define the state by $\psi/\|\psi\|$ and its statistical weight is multiplied by $\|\psi\|^2$:**

$$\psi(t,z) \longrightarrow \frac{\psi(t,z)}{\|\psi(t,z)\|} \equiv |t,z\rangle$$
$$\mathrm{M}[\ldots] \longrightarrow \mathrm{M}[\,\|\psi(t,z)\|^2 \ldots] \equiv \widetilde{\mathrm{M}}_t[\ldots]$$

**There exists a closed non-linear non-markovian SSE for $|t,z\rangle$.**

**The non-markovian SSE describes the t e n d e n c y of time-continuous collapse of the wavefunction in the given observable(s) $\hat{q}$. The state $|t,z\rangle$ is conditioned on $\{\, z(s); s \leq t \,\}$ causally. The individual solutions $|t,z\rangle$ can n o t be realized by any known way of monitoring. The non-markovian SSE corresponds mathematically to the influence of a real or fictitious oscillatory reservoir whose Husimi-function is sampled stochastically. Disappointedly, $z(t)$ can n o t be interpreted as classical record, it only corresponds to mathematical paths in the parameter-space of the reservoir's coherent states.**

**Status of key-problems for non-markovian SSE: causality OK, realizability NOK, Lorentz-invariance NOK.**

Can we enforce Lorentz-invariance?

## Case study: quantum-electrodynamics

$x = (x_0, \vec{x})$: 4-vector of space-time coordinates
$\widehat{A}(x)$: 4-vector of second-quantized electromagnetic potential
$\widehat{\chi}(x)$: Dirac-spinor of second-quantized electron-positron-field
$\widehat{J}(x) = e\overline{\widehat{\chi}}(x)\gamma\widehat{\chi}(x)$: 4-vector of fermionic current
$D(x) = i\langle \text{e.m.vac}|\widehat{A}(x)\widehat{A}(0)|\text{e.m.vac}\rangle$: electromagnetic correlation

Schrödinger equation in interaction picture:

$$\frac{\mathrm{d}\Psi(t)}{\mathrm{d}t} = -i\int_{x_0=t}\mathrm{d}x\,\widehat{J}(x)\widehat{A}(x)\,\Psi(t)$$

Restrict for $\Psi(-\infty) = \psi(-\infty)\otimes|\text{e.m.vac}\rangle$ and seek SSE for the electron-positron wavefunction $\psi(t)$ continuously localized by the electromagnetic field.

Driving noise is the negative-frequency part $A^-(x)$ of the e.m. "vacuum-field" $A^+ + A^-$, satisfying

$$\mathrm{M}[A^-(x)A^+(y)] = \langle \text{e.m.vac}|\widehat{A}(x)\widehat{A}(0)|\text{e.m.vac}\rangle = -iD(x-y)$$

SSE contains memory-term:

$$\frac{\mathrm{d}\psi(t,A^-)}{\mathrm{d}t} = -i\int_{x_0=t}\mathrm{d}x\widehat{J}(x)A^-(x)\psi(t,A^-) - \int_{x_0=t}\mathrm{d}x\int_{y_0<t}\mathrm{d}y\,\widehat{J}(x)D(x-y)\frac{\delta\psi(t,A^-)}{\delta A^-(y)}$$

There exists a closed non-markovian SSE for the normalized state $|t,A^-\rangle$ as well.

The solutions of this "relativistic" SSE, when averaged over $A^-$, describe the exact QED fermionic reduced state:

$$\mathrm{M}[\psi(t,A^-)\psi^\dagger(t,A^+)] = \mathrm{tr}_{\text{e.m.}}[\Psi(t)\Psi^\dagger(t)]$$

The "relativistic" SSE describes the t e n d e n c y of time-continuous collapse of the fermionic wavefunction in the current $\widehat{J}$ although the collapse happens in (certain) Fourier-components rather than the local values $\widehat{J}(x)$. The wavefunction $\psi(t,A^-)$ is conditioned on the classical field $\{\ A^-(x); x_0 \le t\ \}$ causally. The individual solutions $|t,A^-\rangle$ can n o t be realized by any known way of monitoring. Therefore the classical field $A^-$ can n o t be interpreted as classical record. It carries certain information on the collapsing components of the current $\widehat{J}$ but, first of all, $A^-$ carries information on the quantized e.m. field $\widehat{A}(x)$.

## Lorentz invariance?

Solution of "relativistic" SSE emerging from the initial state $\psi(-\infty)$:

$$\psi(t, A^-) = \mathrm{T}\exp\left\{-i\int_{x_0<t} \mathrm{d}x \widehat{J}(x) A^-(x) - \iint_{y_0<x_0<t} \mathrm{d}x\mathrm{d}y \widehat{J}(x) D(x-y) \widehat{J}(y)\right\} \psi(-\infty)$$

Consider the expectation value of the local e.m. current at some $t$:

$$J(t, \vec{x}, A^-) = \frac{\psi^\dagger(t, A^+) \widehat{J}(t, \vec{x}) \psi(t, A^-)}{\psi^\dagger(t, A^+) \psi(t, A^-)}$$

Trouble: $J(x, A^-)$ may depend on $A^-(y)$ for $y_0 \langle x_0$ which is causality in the given frame while it may violate causality in other Lorentz frames. If $J(x, A^-)$ depends not only on $A^-$ inside but also outside the backward light-cone of $x$ then "relativistic" SSE is not Lorentz-invariant.

Status of key-problems for "relativistic" SSE: causality NOK, realizability NOK, Lorentz-invariance NOK.

## Summary

"Classicality emerges from Quantum via real or hypothetic, often time-continuous measurement [detection, observation, monitoring, ...] of the wavefunction $\psi$."

- Markovian models of continuous collapse turn out to be mathematically equivalent with standard (though sophisticated) quantum measurements.
- Non-markov models are still equivalent with standard quantum reservoir dymamics, i.e., with its formal stochastic decomposition (unravelling).
- Lorentz invariance of individual continuously localized quantum trajectories is likely to remain a problem.

Can we construct more general models that are more likely to liberate us from the mathematical structure of standard quantum theory?

Replace, please, "Emergence of Classicality from Quantum" by "Coexistence of Classical and Quantum".

- Classical fields $C(x)$ and quantum fields $\widehat{Q}(x)$
- Causal and Lorentz invariant relationship