Twirl: Equating Quantum and Thermodynamic Entropy Productions

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We search for a "graceful" non-unitary map $\mathcal{W}$ whose von Neumann entropy gain coincides with the calculated thermodynamic entropy production. Our candidate for $\mathcal{W}$ is the so-called "twirl" introduced to quantum information theory by Bennett, also used in the theory of quantum reference frames. Relevance of frame averaging (twirling) for real world irreversibility is outlined.

1. How to equate S_{Qu} with S_{Th}?
2. Simplest example: mechanical friction
3. Graceful irreversible map in MB gas
4. Dissipation in abstract quantum gas
5. Dissipating the external work
6. Twirl $\mathcal{W}$
7. Outlook: Nature's forgetfulness

How to equate S_{Qu} with S_{Th}?

In thermal equilibrium: $S_{\mathrm{Qu}} = S_{\mathrm{Th}}$.
In non-equilibrium: conflict between macro- and microdynamics.

- Second law of thermodynamics: $\Delta S_{\mathrm{Th}} \geq 0$
- Microscopic reversibility: $\Delta S_{\mathrm{Qu}} = 0$

Resolution: coarse grained microscopic dynamics!
Questions remain:

- How much coarse graining?
- What coarse graining?

Answers:

- As much as to equate ΔS_{Qu} with ΔS_{Th}
- Find 'graceful' irreversible map, universal enough

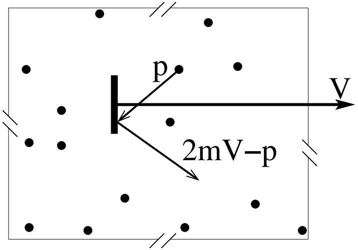
Simplest example: mechanical friction

Irreversible process, double request:

- Calculable ΔS_{Th}
- Transparent microscopic dynamics

Choice: mechanical friction

- $dS_{\mathrm{Th}}/dt = (\eta \times V) \times V/k_B T = \eta\beta V^2$
- Enjoy simplicity of Maxwell-Boltzmann gas:



Dragging the disk at velocity V
needs force 2νmV.
ν=collision rate
η=2νm=friction const.

$dS_{\mathrm{Th}}/dt = 2\nu m\beta V^2$

$dS_{\mathrm{MB}}/dt = 0$ since collisions are reversible

Graceful irreversible map in MB gas

We need $2m\beta V^2$ MB-entropy production per collision!

$$S_{\mathrm{MB}} = S(\rho) = -\int \rho \log \rho dp_1 dp_2 \dots dp_N \qquad (k_B = 1)$$

Pre-collision microscopic state:

$$\rho_\beta(p_1, p_2, \dots, p_r, \dots, p_N) = \mathcal{N} \exp\left(-\frac{\beta}{2m}\sum_{r=1}^{N} p_r^2\right)$$

Post-collision microscopic state (the r'th molecule collided):

$$\rho_\beta^r(p_1, p_2, \dots, p_r, \dots, p_N) = \rho_\beta(p_1, p_2, \dots, p_r - 2mV, \dots, p_N)$$

Zero entropy production per collision: $\Delta S_{\mathrm{MB}} = S(\rho_\beta^r) - S(\rho_\beta) = 0$
Coarse-graining $\mathcal{W}$ of ρ_β^r: erase collider's identity r!

$$\rho_\beta^r \Rightarrow \mathcal{W}\rho_\beta^r = \frac{1}{N}\sum_{r=1}^{N} \rho_\beta^r$$

D. 2002: $S(\mathcal{W}\rho_\beta^r) - S(\rho_\beta^r) = 2m\beta V^2$ (in leading order of V)

Dissipation in abstract qubit gas

Initial state: $\hat{\rho}_\beta = \hat{\rho}^{\otimes N}$ with N equilibrium qubits $\hat{\rho} = \mathcal{N} \mathrm{e}^{-\beta \hat{H}}$
Random unitary 'collisions' on external 'disk': $\hat{\rho} \Rightarrow \hat{\sigma} = \hat{U} \hat{\rho} \hat{U}^\dagger$

$$\hat{\rho}_\beta = \hat{\rho}^{\otimes N} \Rightarrow \hat{\rho}_\beta^r = \hat{\rho}^{\otimes (r-1)} \otimes \hat{\sigma} \otimes \hat{\rho}^{\otimes (N-r)}$$

External work is $\mathrm{tr}(\hat{H}\hat{\sigma}) - \mathrm{tr}(\hat{H}\hat{\rho})$, *postulate* it has gone *dissipated*.

$$\Delta S_{\mathrm{Th}} = \beta[\mathrm{tr}(\hat{H}\hat{\sigma}) - \mathrm{tr}(\hat{H}\hat{\rho})] = S(\hat{\sigma}|\hat{\rho}) \geq 0$$

'Collision' is reversible, yields $\Delta S_{\mathrm{Qu}} = 0$.
Impose irreversible map $\mathcal{W}$, re-symmetrize the post-collision state:

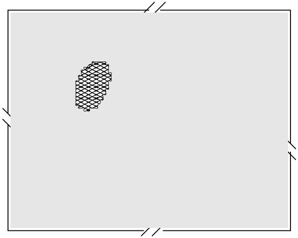
$$\Delta S_{\mathrm{Qu}} = S(\mathcal{W}\hat{\rho}_\beta^r) - S(\hat{\rho}_\beta) \qquad \text{D. et al 2007}$$

To equate ΔS_{Qu} with ΔS_{Th}, the following must be true for $N \to \infty$:

$$S(\mathcal{W}\hat{\sigma} \otimes \hat{\rho}^{\otimes (N-1)}) - S(\hat{\sigma} \otimes \hat{\rho}^{\otimes (N-1)}) \to S(\hat{\sigma}|\hat{\rho})$$

Proof: Csiszár et al. 2007

Dissipating the external work



Homogeneous equilibrium state: $\hat{\rho}_\beta = \mathcal{N} e^{-\beta \hat{H}}$

External field perturbs locally:

$$\hat{\rho}_\beta \longrightarrow \hat{\rho}'_\beta = \hat{U}\,\hat{\rho}_\beta \hat{U}^\dagger$$

Postulate the work is dissipated to heat, then:

$$\Delta S_{Th} = S(\hat{\rho}'_\beta | \hat{\rho}_\beta) > 0$$

Still: $\Delta S_{Qu} = S(\hat{\rho}'_\beta) - S(\hat{\rho}_\beta) = 0$

What irreversible map $\mathcal{W}$ can equate ΔS_{Qu} with ΔS_{Th} ?

Make $\hat{\rho}'_\beta$ forget location of perturbation:

$$\mathcal{W}\hat{\rho}'_\beta = \lim_{V\rightarrow\infty} \frac{1}{V} \int_V e^{-ix\hat{P}} \hat{\rho}'_\beta e^{ix\hat{P}} dx$$

To equate ΔS_{Qu} with ΔS_{Th}, this must hold:

$$S(\mathcal{W}\hat{\rho}'_\beta) - S(\hat{\rho}'_\beta) = S(\hat{\rho}'_\beta | \hat{\rho}_\beta)$$

Proof: D. 2012

Twirl $\mathcal{W}$

is a non-unitary (irreversible) map (Bennett et al. 1996):

$$\hat{\rho} \to \mathcal{W}\hat{\rho} \equiv \int \hat{U}(g)\hat{\rho}\hat{U}^\dagger(g)dg$$

$\hat{U}(g)$: unitary representation of a group; dg: Haar measure

- Friction in ideal gas: permutation group of molecules/qubits
- Dissipation of local work: translational group

$\hat{U}(g)\hat{H}\hat{U}^\dagger(g)=\hat{H}$: no harm to dynamics, 'graceful' entropy production
Postulating $\Delta S_{\mathrm{Qu}} = \Delta S_{\mathrm{Th}}$ yields non-trivial conjecture:

$$\lim_{V\to\infty}\left[S(\mathcal{W}\hat{U}\hat{\rho}_\beta\hat{U}^\dagger) - S(\hat{\rho}_\beta)\right] = \lim_{V\to\infty} S(\hat{U}\hat{\rho}_\beta\hat{U}^\dagger|\hat{\rho}_\beta)\ ,$$

Fenomenology led us to exact math.

Outlook: Nature's forgetfulness

What is the mechanism of microscopic irreversibility?

- Is there a 'graceful' way Nature is forgetting microscopic data?
- Is there a universal non-unitary mechanism?
- Can it produce as much entropy as expected thermodynamically?

Answer: 3xYES

If Nature is twirling (shaking?) reference frames, She is producing just the observed thermodynamic entropy. Whether this is the real and ultimate way for Nature to 'forget' microscopic data remains an open question.