On Gao's model of wavefunction collapse

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Based on his RDM interpretation of the wavefunction, Gao has constructed a simple discrete-time energy-conserving model of spontaneous (i.e.: objective) wavefunction collapse. I recast, equivalently, the stochastic equations of this model and I discuss it in the context of alternative collapse models like, e.g., Pearle's gambler's ruin process, previous energy-driven collapse models and the gravity-related model of Penrose and myself.

Background concepts

What I read from Gao:

- Unitary evolution of Ψ and its Born's probability densities are underlied by ergodic random discontinuous motion (RDM) of the particles.
- Also non-unitary collapse dynamics of Ψ is discontinuous.
- The chosen collapse model is discrete in time.

I take these for granted, as a possible alternative to other *spontaneous collapse theories*.

Collapse as Gambler's Ruin

Collapse + Born's rule:
$$|\Psi\rangle = \sum_{k=1}^{N} c_k |k\rangle$$
$$|c_k|^2(\text{final}) = \delta_{kn}, \text{ with probability } |c_n|^2(\text{initial}).$$
Pearle's hint of a stochastic model: N gamblers with initial moneys $p_1 = |c_1|^2, p_2 = |c_2|^2, etc.$ play a fair game which ends when all gamblers go to ruin except for the winner who takes everything.

A possible fair game (there are many more):

1. *They put $\lambda p_1, \lambda p_2, etc.$ into the bank ($\lambda \leq 1$)*
2. *Bank money λ is given to player n with probability p_n*
$$p_k \to (1-\lambda)p_k + \delta_{kn}\lambda$$
3. *Go to 1. until one player wins everything.*

Then
$$p_k(\text{final}) = \delta_{kn}, \text{ with probability } p_n(\text{initial}).$$
That's collapse + Born rule.

Gao's Collapse Model

$$|\Psi(0)\rangle = \sum_{k=1}^{N} c_k(0)|E_k\rangle, \quad \hat{\varrho}(0) = |\Psi(0)\rangle\langle\Psi(0)|$$

$$\hat{\varrho}(t) = \sum_k p_k(t)|E_k\rangle\langle E_k| + \sum_{j\neq k} \varrho_{jk}(t)|E_j\rangle\langle E_k|$$

Discrete stochastic dynamics $\hat{\varrho}(t) \to \hat{\varrho}(t + t_{\mathrm{Pl}})$:

$$\hat{\varrho}(t + t_{\mathrm{Pl}}) = (1-\lambda)\hat{\varrho}(t) + \lambda|E_n\rangle\langle E_n|,$$

with probability $p_n(t) = \langle E_n|\hat{\varrho}(t)|E_n\rangle$.

$\Rightarrow$

$$\underbrace{p_k(t + t_{\mathrm{Pl}}) = (1-\lambda)p_k(t) + \lambda\delta_{kn},}_{\text{like in gambler's ruin}} \quad \underbrace{\varrho_{jk}(t + t_{\mathrm{Pl}}) = (1-\lambda)\varrho_{jk}(t)}_{\text{damping (decoherence)}^*}$$

$$p_k(\infty) = \delta_{kn} \text{ with probability } p_n(0)$$

$$\varrho_{jk}(\infty) = 0 \quad (j \neq k)$$

Diffusive Limit

During $\Delta t = t_{\mathrm{Pl}}$:

discrete change $\Delta p_k \equiv \Delta p_{k|n} = -\lambda(p_k - \delta_{kn})$ with prob. p_n.

Let's calculate 1st & 2nd moments of $\Delta p_{k|n}$:

$$\mathbb{E}\Delta p_{k|n} = \sum_n p_n \Delta p_{k|n} = 0$$

$$\mathbb{E}\Delta p_{j|n}\Delta p_{k|n} = \sum_n p_n \Delta p_{j|n}\Delta p_{k|n} = \lambda^2(p_j\delta_{jk} - p_j p_k)$$

On scales $t >> t_{\mathrm{Pl}}$:

inhomogeneous diffusion, with diff. matrix $t_{\mathrm{Pl}}^{-1}\lambda^2(p_j\delta_{jk} - p_j p_k)$.

1. Ito formalism, with $\{\xi_k\}$ white noises:

$$dp_k = \lambda(p_k \sum_n d\xi_n - d\xi_k)$$

$$\mathbb{E}d\xi_k = 0, \quad \mathbb{E}d\xi_j d\xi_k = t_{\mathrm{Pl}}^{-1} p_j \delta_{jk} dt$$

2. Fokker-Planck formalism, for density $\varrho(p_1, p_2, \ldots; t)$:

$$\varrho(p_1, p_2, \ldots; 0) = \prod_{k=1}^N \delta(p_k - p_k(0))$$

$$\frac{\partial \varrho}{\partial t} = \frac{\lambda^2}{t_{\mathrm{Pl}}} \sum_{jk} \frac{\partial^2}{\partial p_j \partial p_k}(p_k\delta_{jk} - p_j p_k)\varrho$$

Two-State Example

Single variable $q = p_1 - p_2$, $q \in [-1, +1]$:

$$p_1 = (1+q)/2, \quad p_2 = (1-q)/2$$

Take initial density $\varrho(q, 0) = \delta(q - p_1(0) + p_2(0))$.
Fokker-Planck eq. reduces to

$$\frac{\partial \varrho(q,t)}{\partial t} = \frac{\lambda^2}{t_{\mathrm{Pl}}} \frac{\partial^2}{\partial q^2}(1-q^2)\varrho(q,t).$$

$\Rightarrow$

$$\varrho(q, \infty) = p_1(0)\delta(q-1) + p_2(0)\delta(q+1)$$

That's collapse + Born rule.

Pearle-Gisin version:

$$\frac{\partial \varrho(q,t)}{\partial t} = \frac{\lambda^2}{2t_{\mathrm{Pl}}} \frac{\partial^2}{\partial q^2}(1-q^2)^2\varrho(q,t)$$

Diffusive Limit for Full Density Matrix

Decoherence time, collapse time

$$\begin{bmatrix} p_1 & \varrho_{12} \\ \varrho_{21} & p_2 \end{bmatrix} \xrightarrow[\tau_D]{\textit{decoherence}} \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} \xrightarrow[\tau_C]{\textit{collapse}} \begin{cases} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{cases}$$

Decoherence is mandatory for collapse.

Decoherence process is falsifiable, collapse process is not. (D)
in known spontaneous collapse theories

Decoherence can be much faster than collapse ($\tau_D \ll \tau_C$):
if $\lambda \ll 1$

$$\left[\tau_D = \frac{1}{\lambda} t_{\mathrm{Pl}},\right] \qquad \tau_C = \frac{1}{\lambda^2} t_{\mathrm{Pl}} \qquad (\lambda < 1).$$

With Gao's choice $\lambda = \Delta E / E_{\mathrm{Pl}}$ (valid for $\Delta E \leq E_{\mathrm{Pl}}$):

$$\tau_C = \frac{\hbar E_{\mathrm{Pl}}}{(\Delta E)^2}, \qquad \Delta E = \text{energy spread.}$$

Coincides with collapse time in old energy-driven models (Percival, Hughston, Milburn).

Physics of Collapse

Relevance of

$$\tau_C = \frac{\hbar E_{\rm Pl}}{(\Delta E)^2}$$

$$\begin{array}{ll}
\Delta E = 1\text{eV (atomic superposition)} & \tau_C = 10^{13}\text{s (irrelevant)} \\
\Delta E = 1\text{GeV (high energy superposition)} & \tau_C = 10^{-5}\text{s (irrelevant)} \\
\Delta E = 1\text{J (macroscopic superposition)} & \tau_C = 10^{-25}\text{s (killing)}
\end{array}$$

Gao: *ΔE is not defined as the uncertainty of the total energy of all sub-systems. [...] each sub-system has its own energy uncertainty that drives its collapse*
... provided system splits into non-interacting subsysems.
If they interact, collapse's energy-conservation will be gone.
CSL and D-Penrose theories prescribe collapses to local mass densities, hence they can not preserve energy.

Summary

I showed that Gao's model:

- is a simple gambler's ruin process
- has a diffusive limit, similar to (but different from) the Pearle-Gisin collapse
- yields collapse time formally equal to old energy driven/conserving models
- and improves them by the statement of subsystem-wise collapses

In the future, it

- can be further discussed in the *zoo of spontaneous collapse theories*
- might lead to similar kind of testable predictions
- will face similar difficulties