Relativistic Covariant Formalism of Quantum Measurements and Wavefunction Collapse

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Abstract

In quantum measurement theory, the wavefunction collapse is traditionally viewed as an instantaneous event across all of space. This raises concerns about its compatibility with the principles of relativity. We demonstrate that this longstanding issue can be resolved positively. We develop a simple covariant formalism for quantum measurements and the associated wavefunction collapses.

The Problem & Solution

Collapse Relativistic Covariance:

- No problem at the level of physical predictions.
- But wavefunction collapse is superluminal: instantaneous across the whole space.

The covariance of relativistic quantum theories resides exclusively in the experimental probabilities, and not in the underlying quantum states (Aharonov&Albert 1981).

Still, I argue for Covariance of Collapse:

- Collapse knows no reference frame.
- Outcome a is available in the forward lightcone.
- Postmeasurement state $\hat{\rho}|_a$ and probability of outcome a are both encoded in hybrid state

$$\hat{\rho}(a) = p(a)\hat{\rho}|_a \;.$$

Measurement

Kraus operators $\hat{K}(a)$: $\sum_a \hat{K}^\dagger(a)\hat{K}(a) = \hat{I}$

Kraus superoperator: $\mathcal{K}(a)\hat{\rho} = \hat{K}(a)\hat{\rho}\hat{K}^\dagger(a)$

$$\hat{\rho} \Rightarrow \hat{\rho}|_a = \frac{1}{p(a)}\mathcal{K}(a)\hat{\rho}$$

$$p(a) = \mathrm{tr}\left(\mathcal{K}(a)\hat{\rho}\right)$$

Same in hybrid formalism (D. 2014):

$$\hat{\rho} \Rightarrow \mathcal{K}(a)\hat{\rho} \equiv \hat{\rho}(a)$$

$$\hat{\rho}|_a = \frac{\hat{\rho}(a)}{p(a)}$$

$$p(a) = \mathrm{tr}\hat{\rho}(a)$$

Local Measurement

Interaction picture

Initial state $\hat{\rho}$.

Space-time location $x = (t, r)$

Attribute x to Kraus operators and outcomes:

$$\sum_{a_x} \hat{K}^\dagger(a_x; x)\hat{K}(a_x; x) = \hat{I}$$

scalars, for simplicity

$$\mathcal{K}(a_x; x)\hat{\rho} = \hat{K}(a_x; x)\hat{\rho}\hat{K}^\dagger(a_x; x)$$

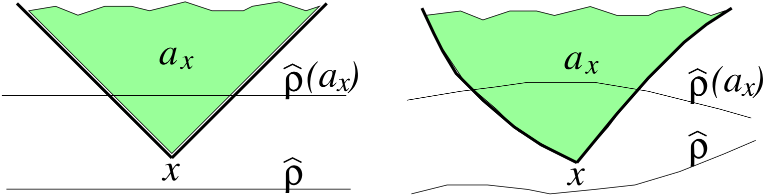
Local measurement in hybrid formalism:

$$\hat{\rho} \Rightarrow \mathcal{K}(a_x; x)\hat{\rho} = \hat{\rho}(a_x)$$

Remember starting observations:

- Collapse knows no reference frame.
- a_x is available in forward lightcone of x.

Local Measurement Covariant Form



$$\hat{\rho}_\Sigma = \begin{cases} \hat{\rho} & x \prec \Sigma \\ \hat{\rho}(a_x) & x \succ \Sigma \end{cases}$$

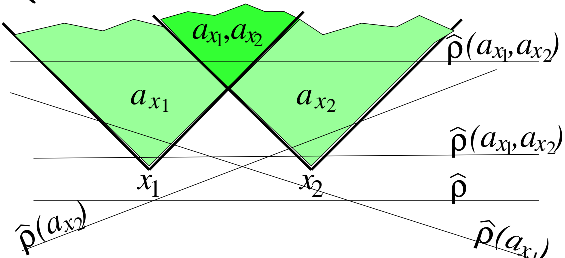
Remember interpretation:

$$\begin{aligned} \hat{\rho}|_{a_x} &= \frac{\hat{\rho}(a_x)}{p(a_x)} \\ p(a_x) &= \mathrm{tr}\hat{\rho}(a_x) \end{aligned}$$

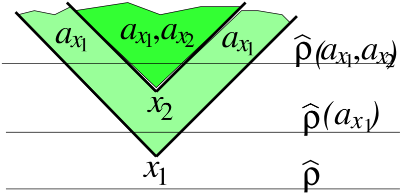
Two Local Measurements

post-meas' states
$$\begin{cases} \mathcal{K}(a_{x_1}; x_1)\hat{\rho} & = \hat{\rho}(a_{x_1}) \\ \mathcal{K}(a_{x_2}; x_2)\hat{\rho} & = \hat{\rho}(a_{x_2}) \\ T\mathcal{K}(a_{x_1}; x_1)\mathcal{K}(a_{x_2}; x_2)\hat{\rho} & = \hat{\rho}(a_{x_1}, a_{x_2}) \end{cases}$$

x_1, x_2 spacelike:



$x_1 \prec x_2$ timelike:



Multiple Local Measurements Cov' Form

State evolution under multiple local measurements at discrete locations $\{x\} = \{x_1, x_2, \dots\}$:

$$\hat{\rho}_{\Sigma_f}(\{a_x; x \prec \Sigma_f\}) = T\left(\prod_{x \prec \Sigma_f} \mathcal{K}(a_x; x)\right)\hat{\rho}$$

$$= T\left(\prod_{\Sigma_f \prec x \prec \Sigma_i} \mathcal{K}(a_x; x)\right)\hat{\rho}_{\Sigma_i}(\{a_x; x \prec \Sigma_i\})$$

Remember:

$$\hat{\rho}|_{\{a_x\}} = \frac{\hat{\rho}(\{a_x\})}{p(\{a_x\})}$$

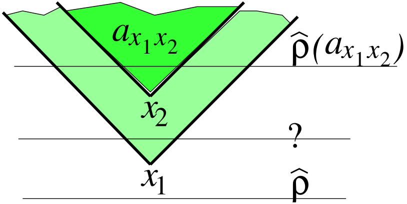
$$p(\{a_x\}) = \mathrm{tr}\hat{\rho}(\{a_x\})$$

Nonlocal Measurement

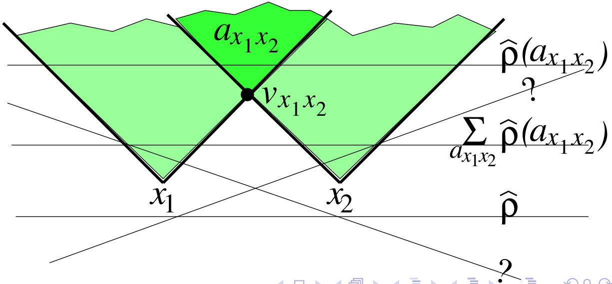
$\hat{K}(a_{x_1 x_2}, x_1, x_2)$: function of op's at both $x_1 \& x_2$

post-meas' state: $\mathcal{K}(a_{x_1 x_2}; x_1, x_2)\hat{\rho} = \hat{\rho}(a_{x_1 x_2})$

$x_1 \prec x_2$ timelike:



x_1, x_2 spacelike:



Nonlocal Measurement Covariant Form

Post-measurement state: $\mathcal{K}(a_{x_1x_2}; x_1, x_2)\hat{\rho} = \hat{\rho}(a_{x_1x_2})$
Vertex $v_{x_1x_2}$: apex of narrowest backward lightcone containing both $x_1 \& x_2$

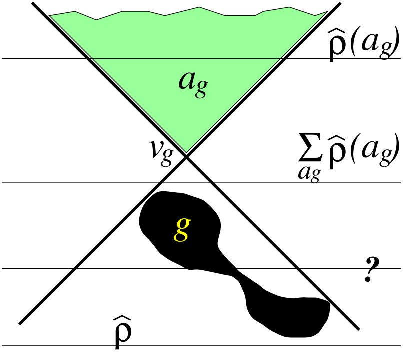
$$\hat{\rho}_\Sigma = \begin{cases} \hat{\rho} & (\Sigma \prec x_1, x_2) \\ \text{depends on } \mathcal{K}\text{-details} & (\Sigma \not\prec x_1, x_2; \Sigma \prec v_{x_1x_2}) \\ \sum_{a_{x_1x_2}} \hat{\rho}(a_{x_1x_2}) & (x_1, x_2 \prec \Sigma \prec v_{x_1x_2}) \\ \hat{\rho}(a_{x_1x_2}) & (v_{x_1x_2} \prec \Sigma) \end{cases}$$

Field Measurement Covariant Form

$x \Rightarrow$ domain g

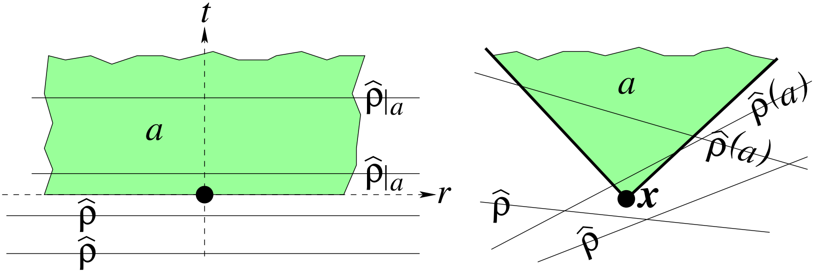
$v_x \Rightarrow v_g$ apex of narrowest backward lightcone containing domain g

$\mathcal{K}(a_g; g)\hat{\rho} = \hat{\rho}(a_g)$ post-meas' state



$$\hat{\rho}_\Sigma = \begin{cases} \hat{\rho} & (\Sigma \prec g) \\ \text{depends on } \mathcal{K}\text{-details} & (\Sigma \cap g \neq \emptyset; \Sigma \prec v_g) \\ \sum_{a_g} \hat{\rho}(a_g) & (g \prec \Sigma \prec v_g) \\ \hat{\rho}(a_g) & (v_g \prec \Sigma) \end{cases}$$

Summary: Key to Covariance



non-relativistic relativistic

Covariance of collapse:

$$\hat{\rho}_\Sigma = \begin{cases} \hat{\rho} & x \prec \Sigma \\ \hat{\rho}(a_x) & x \succ \Sigma \end{cases}$$

- Collapse does not happen in a specific frame.
- Outcome a is available in the forward lightcone.
- Hybrid state $\hat{\rho}(a) = p(a)\hat{\rho}|_a$ encodes both quantum state and outcome's probability