Wavefunction collapse in hybrid formalism: GRW hybrid master equation

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Abstract

Hybrid quantum-classical formalism is a simple alternative to textbook formalism of the wavefunction's random collapse. This alternative applies when the simple von Neumann collapse is upgraded to time-continuous collapse. The usual stochastic equations of the conditional quantum state and the measured signal can be rewritten in a single hybrid master equation. As a specific example, we consider the GRW stochastic equations of the wavefunction's conjectured spontaneous dynamical collapse and construct the theory's hybrid master equation.

Hybrid (QC) framework

- Dynamics: coupling between Classical and Quantum systems
- Measurement: coupling between Device and Quantum system
- Foundations: whatever coexistence between Classical and Quantum

Mathematics:

	Classical	Quantum	Hybrid
Density:	$\rho(x)$	$\hat{\rho}$	$\hat{\rho}(x)$
Master Eq.:	Pauli	Lindblad	?

Hybrid density

Simplest construction: $\{\rho(x) \textit{ and } \hat{\rho}\} \Rightarrow \rho(x)\hat{\rho} \equiv \hat{\rho}(x)$
Generically:

$$\text{any } \hat{\rho}(x) \geq 0 \quad \forall x; \qquad \mathrm{Tr} \sum\!\!\!\!\!\!\int_x \hat{\rho}(x) = 1.$$

Reduced Q	Reduced C	Conditional Q	Conditional C
$\hat{\rho} = \sum\!\!\!\!\!\!\int_x \hat{\rho}(x)$	$\rho(x) = \mathrm{Tr}\hat{\rho}(x)$	$\hat{\rho}_x = \hat{\rho}(x)/\rho(x)$	$\nexists$

E.g.:

- $\hat{\rho}(r,p)$ where $\hat{\rho}$: electrons, (r,p) : nuclei
- $\hat{\rho}[A]$ where $\hat{\rho}$: e^+e^-, A : e.m.field
- $\hat{\rho}(n)$ where $\hat{\rho}$: Q-dot, n : charge count
- $\hat{\rho}(x)$ where $\hat{\rho}$: Q-system, x : measurement outcome

Measurement in alternative formalisms

What happens to $\hat{\rho}$ under measurement of c.s.o.p. $\{\hat{P}_x\}$?

- Text-book: $\hat{\rho}$ jumps randomly to the conditional $\hat{\rho}_x$,

$$\hat{\rho} \longrightarrow \hat{\rho}_x = \frac{1}{\rho(x)}\hat{P}_x\hat{\rho}\hat{P}_x \text{ with probability } \rho(x) = \mathrm{Tr}(\hat{P}_x\hat{\rho}\hat{P}_x).$$

- Hybrid QC: $\hat{\rho}$ jumps to the hybrid density $\hat{\rho}(x)$,

$$\hat{\rho} \longrightarrow \hat{\rho}(x) = \hat{P}_x\hat{\rho}\hat{P}_x.$$

Generically: $\hat{\rho} \longrightarrow \hat{\rho}(x) = \hat{M}_x\hat{\rho}\hat{M}_x^\dagger$ where $\sum\!\!\!\!\!\!\int_x \hat{M}_x^\dagger\hat{M}_x = \hat{I}$.

E.g.: $\hat{M}_x = \hat{M}_x^\dagger = (2\pi\sigma^2)^{-1/4}\exp[-(\hat{q}-x)^2/4\sigma^2]$,

$$\hat{\rho} \longrightarrow \hat{\rho}(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{[-(\hat{q}-x)^2/4\sigma^2]}\hat{\rho}e^{[-(\hat{q}-x)^2/4\sigma^2]}$$

Hybrid dynamics

- All Markovian Classical ME (Pauli):

$$\dot{\rho}(x) = -\partial_x v(x)\rho(x) + \sum\!\!\!\!\!\!\int_y \big(T(x,y)\rho(y) - T(y,x)\rho(x) \big)$$

transition rates $T(x,y) \geq 0$

- All Markovian Quantum ME (Lindblad):

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \sum\!\!\!\!\!\!\int_\alpha \big(\hat{L}_\alpha \hat{\rho} \hat{L}_\alpha^\dagger - \tfrac{1}{2}\{\hat{L}_\alpha^\dagger \hat{L}_\alpha, \hat{\rho}\} \big)$$

Lindblad generators $\hat{L}_\alpha$

- Generic Markovian Hybrid ME ('Pauli-Lindblad'):

$$\dot{\hat{\rho}}(x) = -i[\hat{H}(x), \hat{\rho}(x)] - \partial_x v(x)\hat{\rho}(x) +$$

$$+ \sum\!\!\!\!\!\!\int_{y,\alpha} \big(\hat{L}_\alpha(x,y)\hat{\rho}(y)\hat{L}_\alpha^\dagger(x,y) - \tfrac{1}{2}\{\hat{L}_\alpha^\dagger(y,x)\hat{L}_\alpha(y,x), \hat{\rho}(x)\} \big)$$

hybrid generators $\hat{L}_\alpha(x,y)$

[Case study: Q-monitoring]

$\hat{\rho}$: Q-particle, $\dot{x}$: monitored value of $\hat{q}$

Naive hybrid ME:

$$\dot{\hat{\rho}}(x) = -i[\hat{H}, \hat{\rho}(x)] - \tfrac{1}{2}\partial_x\{\hat{q}, \hat{\rho}(x)\}$$

Check: $d\langle x\rangle/dt = \mathrm{Tr}\int x\dot{\hat{\rho}}(x)dx = \mathrm{Tr}\int \hat{q}\hat{\rho}(x)dx = \mathrm{Tr}(\hat{q}\hat{\rho}) = \langle\hat{q}\rangle$

Problem: term $-\frac{1}{2}\partial_x\{\hat{q}, \hat{\rho}(x)\}$ may violate positivity of $\hat{\rho}(x)$

Try Lindblad ME for $\widehat{\rho}$ in big space, choose

$\widehat{L} = \hat{q}/\sqrt{2D} \otimes \hat{I} + \sqrt{2D}\,\hat{I} \otimes \int (\partial_x|x\rangle)\langle x|\,dx \quad \Rightarrow$

$$\dot{\hat{\rho}}(x) = -i[\hat{H}, \hat{\rho}(x)] - \frac{1}{2}\partial_x\{\hat{q}, \hat{\rho}(x)\} - \frac{1}{8D}[\hat{q}, [\hat{q}, \hat{\rho}(x)]] + D\partial_x^2\hat{\rho}(x)$$

Hybrid ME of Q-monitoring.

In particular: $\dot{\rho}(x) = -\partial_x\langle\hat{q}\rangle_x\rho(x) + D\partial^2\rho(x)$

Equivalent with the Ito-formalism of Q-monitoring.

GRW hybrid ME

Recall generic HME:

$$\dot{\hat{\rho}}(x) = -i[\hat{H}(x), \hat{\rho}(x)] - \partial_x v(x)\rho(x) +$$

$$+ \sum\!\!\!\!\!\!\int_{y,\alpha} \left(\hat{L}_\alpha(x,y)\hat{\rho}(y)\hat{L}^\dagger_\alpha(x,y) - \tfrac{1}{2}\{\hat{L}^\dagger_\alpha(y,x)\hat{L}_\alpha(y,x), \hat{\rho}(x)\} \right)$$

With constituent coordinates $\hat{q}_\alpha$, vN measurement outcomes x_α (aka "flashes"), drift $v(x) = 0$, and hybrid generators

$$\hat{L}_\alpha(x,y) = \lambda^{\frac{1}{2}}(2\pi\sigma^2)^{-1/4}\exp[-(\hat{q}_\alpha - x_\alpha)^2/4\sigma^2]$$

The GRW HME:
$$\dot{\hat{\rho}}(x) = -i[\hat{H}, \hat{\rho}(x)] +$$

$$+\lambda \sum_\alpha \left(\frac{1}{2\pi\sigma^2} \exp\left[-\frac{(\hat{q}_\alpha - x_\alpha)^2}{4\sigma^2} \right] \hat{\rho} \exp\left[-\frac{(\hat{q}_\alpha - x_\alpha)^2}{4\sigma^2} \right] - \hat{\rho}(x) \right)$$

GRW: quantum and classical reduced dynamics

The GRW HME: $\dot{\hat\rho}(x) = -i[\hat H, \hat\rho(x)] +$

$$+\frac{\lambda}{\sqrt{2\pi\sigma^2}}\sum_\alpha\left(\exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{4\sigma^2}\right]\hat\rho\exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{4\sigma^2}\right] - \hat\rho(x)\right)$$

Quantum reduced dynamics for $\hat\rho = \int_x \hat\rho(x)$:

$$\dot{\hat\rho} = -i[\hat H, \hat\rho] + \frac{\lambda}{\sqrt{2\pi\sigma^2}}\sum_\alpha\left(\int_x \exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{4\sigma^2}\right]\hat\rho\exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{4\sigma^2}\right] - \hat\rho(x)\right)$$

Classical reduced dynamics for $\rho(x) = \mathrm{Tr}\,\hat\rho(x)$:

$$\dot\rho(x) = \frac{\lambda}{\sqrt{2\pi\sigma^2}}\sum_\alpha\left(\mathrm{Tr}\left(\exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{2\sigma^2}\right]\hat\rho\right) - \rho(x)\right)$$

That's a jump process, with jump rates:

$$T_\alpha(x,y) = \frac{\lambda}{\sqrt{2\pi\sigma^2}}\sum_\alpha \mathrm{Tr}\left(\exp\left[-\frac{(\hat q_\alpha - x_\alpha)^2}{2\sigma^2}\right]\hat\rho(y)\right)$$

GRW stochastic equations from GRW hybrid ME

Stochastic interpretations (aka "unravellings")
- Classical $\rho(x,t)$ in terms of stochastic trajectories x_t
- Quantum $\hat{\rho}(t)$ in terms of stochastic trajectories ψ_t
- Hybrid $\hat{\rho}(x,t)$ in terms of stochastic trajectories (ψ_t, x_t):

$$\begin{aligned} \dot{x}_t &= 0 \\ \dot{\psi}_t &= -i\hat{H}\psi_t \end{aligned}$$

apart from random jumps

$$\begin{aligned} (x_t)_\alpha &\to (x_{t+0})_\alpha \\ \psi_t &\to \psi_{t+0} = \mathcal{N}\exp\left[-(\hat{q}_\alpha - (x_\alpha)_{t+0})^2/(4\sigma^2)\right]\psi_t \end{aligned}$$

at rates $T_\alpha(x_{t+0}, x_t) = \frac{\lambda}{\sqrt{2\pi\sigma^2}}\langle\exp\left[-(\hat{q}_\alpha - (x_\alpha)_{t+0})^2/(2\sigma^2)\right]\rangle_{\psi_t}$.

That's GRW theory standard stochastic form (Bell).

Fundamental message

Impact of classical variable x on quantum state $\hat{\rho}$:

parametric through $\hat{H}(x)$.

Impact of quantum state $\hat{\rho}$ on classical variable x:

von Neumann collapse, nothing else!!!

Formalisms of single collapse:

stochastic	hybrid
$\hat{\rho} \to \hat{P}_x \hat{\rho} \hat{P}_x$	$\hat{\rho} \to \hat{P}_x \hat{\rho} \hat{P}_x \equiv \hat{\rho}(x)$
prob $x : \mathrm{Tr}(\hat{P}_x \hat{\rho})$	

Formalisms of dynamical collapse (laboratory, spontaneous):

stochastic	hybrid
ψ_t obeys SSE	$\hat{\rho}(x,t)$ obeys HME
signal x_t obeys SE	

Collapse theories GRW,DP,CSL:
rooted in vN collapse, SSEs are equivalent with HMEs.