Trans-Planckian non-relativistic motion is not an oxymoron

Lajos Diósi

Wigner Center, Budapest

11 June 2019, Vaxjø

Supported by
Foundational Questions Institute & Fetzer Franklin Fund, National Research Development and Innovation Office of Hungary
Grant number K12435

No QG effects below 10^{19}GeV?

length	$\ell_{\rm Pl} = \sqrt{\frac{\hbar G}{c^3}}$	$= (1.6\times)$	10^{-33}	cm
time	$\tau_{\rm Pl} = \sqrt{\frac{\hbar G}{c^5}}$	$= (5.4\times)$	10^{-44}	s
mass	$m_{\rm Pl} = \sqrt{\frac{\hbar c}{G}}$	$= (2.2\times)$	10^{-5}	g
temperature	$T_{\rm Pl} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	$= (1.4\times)$	10^{32}	K
energy	$E_{\rm Pl} = \sqrt{\frac{\hbar c^5}{G}}$	$= (1.4\times)$	10^{19}	GeV
		$= (3.0\times)$	10^{16}	erg

High Energy Physics wisdom:

- QG is beleived to dominate on the trans-Planckian scale
- QG effects are totally ingorable below the Planck-scale
- Non-relativistic (NR) QM can not overlap with QG
- Planckian or trans-Planckian slow $\ll c$ motion is an oxymoron
- ... like “QG in the Lab”

Still there is a loophole to QG in the lab

Google's wisdom ("quantum gravity in the lab"):

- 10 results before 2000
- 12.600 results until 2019 (7 June)

How can this deal exist?

My ~~wisdom~~ hint:

- HEP requests $10^{19}GeV$
- ... per elementary particle — unavailable after Big Bang
- 10^{19}GeV$\sim 10^{16}$erg per massive object — available in the lab

$$E = 10\text{kg}\times(10\text{km/s})^2 = 10^{16}\text{erg}\sim E_{\rm Pl}$$

Loophole from high energies toward the lab: massive d.o.f. to test Planck-scale!

Next: Planck lenght $\ell_{\rm Pl}$ can be hit earlier than Planck energy $E_{\rm Pl}$.

Trans-Planckian de Broglie wavelength

$$\underbrace{\lambda_{\mathrm{deBroglie}}}_{\frac{2\pi\hbar}{mv}\sqrt{1-v^2/c^2}} \sim \underbrace{\ell_{\mathrm{Planck}}}_{\sqrt{\frac{\hbar G}{c^3}}}.$$

λ_{dB} sinks to and below ℓ_{Pl} in two ways:

- Relativistic way: elementary particles velocity v closes c, requests 10^{19}GeV/particle — forget it!
- NR way: velocity v remains $\ll c$ but *mass grows macroscopic* — might be viable in the lab

$$\lambda_{\mathrm{dB}} = \frac{2\pi\hbar}{\mathbf{10g}\times\mathbf{10km/s}} \sim \mathbf{10^{-33}cm} \sim \ell_{\mathrm{Pl}}$$

1) NR (trans-)Planckian $m = 10g$ Schrödinger cat:

$$|v=-10km/s\rangle + |v=+10km/s\rangle$$

2) NR (trans-)Planckian vibration of $m{=}10$kg body:

$$\lambda_{\mathrm{dB}} = \frac{2\pi\hbar}{m\omega a} = \frac{2\pi\hbar}{10kg\times 100kHz \times 10^{-2}cm} \sim 10^{-33}cm \sim \ell_{\mathrm{Pl}}$$

What happens at the Planck scale?

- Bronstein 1935: Notion of space-time continuum may be lost.
- Wheeler 1962: Foamy space-time
- Hawking 1983: Non-unitary (decohering) dynamics

No accepted dynamics for scales ℓ_{Pl} (Garay 1995, Hossenfelder 2013)
Phenomenology: ℓ_{Pl}-scale metric fluctuations
Simplest choice: stochastic conformal fluctuations

$$\{g_{ab}\} = (1+h)\mathrm{diag}(1,-1-1-1), \qquad |h| \ll 1$$

Lorentz-invariant correlation (D 2019):

$$\mathbb{E}h(x)h(y) = \ell_{\mathrm{Pl}}^2 \int \frac{d^4k}{(2\pi)^4} \frac{\theta(-k^2)}{-k^2} e^{-ik(x-y)}, \tag{1}$$

Write h as $h = 2\Phi/c^2$, take $c \to \infty$, then c cancels!

$$\mathbb{E}\Phi(t,\mathbf{x})\Phi(s,\mathbf{y}) = \frac{\hbar G}{|\mathbf{x}-\mathbf{y}|}\delta(t-s).$$

Trans-Planckian NR Schrödinger cats decohere

... and die.

NR QM becomes modified by the stochastic Newton-potential:

$$\frac{d\Psi}{dt} = -\frac{i}{\hbar}\left(\hat{H} + \sum m_n \Phi(\hat{x}_n, t)\right)\Psi$$

$$\mathbb{E}\Phi(t, \mathbf{x})\Phi(s, \mathbf{y}) = \frac{\hbar G}{|\mathbf{x} - \mathbf{y}|}\delta(t - s),$$

causing G-related decoherence in NR QM (D 1987, Penrose 1996).

Trans-Planckian NR Schrödinger cat decays

$$\frac{|-10km/s\rangle + |10km/s\rangle}{\sqrt{2}} \Rightarrow \begin{cases} |-10km/s\rangle & 50\% \\ |\ \ 10km/s\rangle & 50\% \end{cases}$$

at the DP-decay time

$$10^{-15} - 10^{-9}s.$$

Summary

- NR wavefunction of (isolated) massive d.o.f. acquires trans-Planckian structure.
- Hence NR QM should get some corrections for large masses.
  - We have no theory there — since we have no safe QG theory.
  - Yet, QG-phenomenology says: metric fluctuates heavily there.
  - Fluctuations are ignorable for any NR atomic object.
  - Cumulative effect of fluctuations does matter for massive d.o.f.
  - Their primary effect is decoherence of massive d.o.f.
  - DP version seems to clear NR states of their trans-Planckianity.
- **Trans-Planckian & non-relativistic? Yes, in massive d.o.f.**, theorists should do something about QM there.
- **QG & lab? Yes! Quantum control of "big-enough" d.o.f.** would come soon (10yy? — ask e.g. Aspelmayer).