Coexistence of Classical Continuum and Quantum Theory

Lajos Diósi

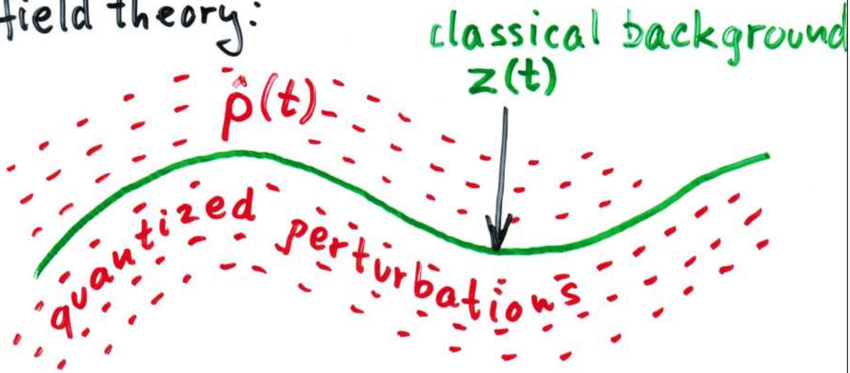
Acknowledgements go to:
Hungarian Scientific Research Fund under Grant No. 75129
EU COST Action MP1006 'Fundamental Problems in Quantum Physics'
Int. Conf. 'Emergent Quantum Mechanics' (Vienna, 2011)

February 11, 2012

The physics issue

Cambridge, 1999 3

Q-field theory:



no backreaction $\hat{\rho}(t) \rightarrow z(t)$

Study the coexistence of $\hat{\rho}(t)$ & $z(t)$!

Vocabulary

- **Classical continuum:** a smooth real function $z(t)$ of time
- **Quantum theory:** dynamics of the density matrix $\rho(t)$ plus its statistical interpretation
- **Coexistence:** $\rho(t)$ and $z(t)$ coexist and depend on each other
- **'Free Will':** my freedom to, conditioned on z, perturbe the dynamics of ρ. I call z **tangible** then.
- **Causality:** perturbation at t has no effect at times $< t$ prior to t
- **Measurement:** math procedure (selective stochastic map) on ρ

Sensitively interrelated: 'Free Will" perturbation vs statistical interpretation, smoothness vs causality.

The quantum-classical coexistence issue

There must be mutual classical⇔quantum influences.

Classical on quantum is trivial:

$$\frac{d\rho}{dt} = \frac{-\mathrm{i}}{\hbar}[H(z), \rho]$$

Quantum on classical (back-reaction) is problematic:

- Mean-Field
  *Moller*1962,*Rosenfeld*1963
- de Broglie-Bohm$^{1927-1952}$
- Decoherence
  *Zeh*1970,*Zurek*1982
- Decoherent Histories
  *Griffith*1984,*Gell−MannHartle*1993
- Measurement
  *vonNeumann*1932
- Continuous Measurement
  *Belavkin*1988,*Diosi*1988
- Hybrid Dynamics
  *SherrySudarshan*1979,...,*Elze*2011

Influence of quantum on classical: Mean-Field?

Classical continuum variable = quantum expectation value:

$$z = \mathrm{tr}[q\rho]$$

Most succesful approximation in optics, cosmology, e.t.c.
Mean-Field $\mathbf{z}(\mathbf{t})$ is smooth and causal.

Free Will test: make $H(t)$ depend on $z(t)$.
Recall influence of classical on quantum:

$$\frac{d\rho}{dt} = \frac{-\mathrm{i}}{\hbar}[H(z), \rho]$$

Nonlinear evolution for ρ denies statistical interpretation. **Free Will doesn't work, Mean-Field $\mathbf{z}$ is not tangible.**

Influence of quantum on classical: Bohm theory?

Restricted for pure states $\rho=\rho^2$ and for coordinate $q \Rightarrow z$.
Amazing: Born probability density is preserved for $z(t)$.

Classical continuum variable senses the quantum potential $V_\rho(z)$:

$$m\frac{d^2z}{dt^2}=-V'(z)-V'_\rho(z)$$

Oldest non-standard theory to generate classical from quantum.
Bohm's $\mathbf{z}(\mathbf{t})$ is smooth and causal.

Time-local Free Will test passes, $H(t)$ depends on $z(t)$:

$$\frac{d\rho}{dt}=\frac{-\mathrm{i}}{\hbar}[H(z),\rho]$$

Does Bohm remain consistent when $H(t)$ depends on $z(t'<t)$?
If causal Free Will fails: Bohm's $\mathbf{z}$ is not tangible.

Influence of quantum on classical: Measurement?

Classical variable = outcome of quantum measurement:

$$\rho \longrightarrow \frac{P(z)\rho P(z)}{p(z)} \equiv \rho_z \text{ with prob. } p(z)$$

Standard theory to generate classical from quantum.
Measurement z(t) is not continuous (though causal).

Free Will test, make H depend on z and average the dynamics over z:

$$\rho(t) = \sum_z p(z) e^{-(i/\hbar)H(z)t} \rho_z e^{(i/\hbar)H(z)t} = \sum_z U(z,t)P(z)\rho_z P(z)U^\dagger(z,t)$$

This is linear for ρ. **Free Will works, Measurement z is tangible.**

Influence of quantum on classical: Continuous Measurement?

Classical variable = outcome of time-continuous quantum measurement:

$$z = \mathrm{tr}[q\rho] + \text{white-noise}$$

Now standard theory to generate classical from quantum in Markovian approximation. **Continuous Measurement $\mathbf{z(t)}$ is not smooth (though continuous and causal).**

Free Will test: make $H(t)$ depend on $z(<t)$ and average over z. We get linear equation for ρ at the end. **Free Will works, the Continuous Measurement $\mathbf{z(t)}$ is tangible.**

The quantum-classical coexistence issue (re-shown)

There must be mutual classical⇔quantum influences.

Classical on quantum is trivial:

$$\frac{d\rho}{dt} = \frac{-\mathrm{i}}{\hbar}[H(z), \rho]$$

Quantum on classical (back-reaction) is problematic:

- Mean-Field
  *Moller*1962,*Rosenfeld*1963
- de Broglie-Bohm$^{1927-1952}$
- Decoherence
  *Zeh*1970,*Zurek*1982
- Decoherent Histories
  *Griffith*1984,*Gell*−*MannHartle*1993
- Measurement
  *vonNeumann*1932
- Continuous Measurement
  *Belavkin*1988,*Diosi*1988
- Hybrid Dynamics
  *SherrySudarshan*1979,...,*Elze*2011

Summary: Coexistence (co-influence) of quantum and classical

Classical on quantum is trivial:

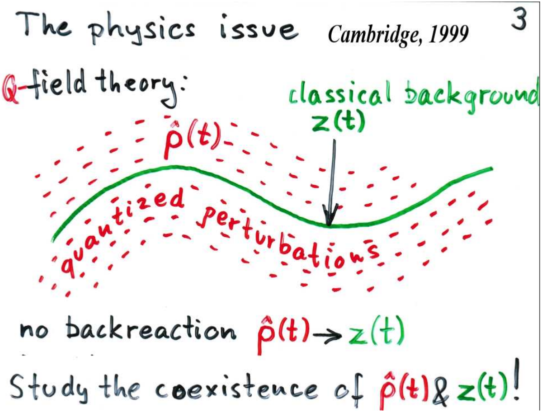
$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H(z), \rho]$$

Quantum on classical (back reaction): The only **tangible (cf. Free Will)** and **smooth** classical 'field' $\mathbf{z(t)}$:

Classical variable = outcome of time-continuous non-Markovian quantum measurement:

$$z = \mathrm{tr}[q\rho] + \text{colored-noise}$$

Causality structure of Non-Markovian Continuous Measurement is tricky. Progress after Cambridge 1999, with recent debates$^{JackColletWalls, WisemanGambetta, Diosi (1999-2011)}$.



On coexistence of classical continuum and quantum theory
Cambridge, *July* 1999 *www.rmki.kfki.hu/~diosi/slides/cambridge.pdf*

Remark
Bielefeld, *Febr* 2004 *www.rmki.kfki.hu/~diosi/slides/bielefeld.pdf*

Continuous wave function collapse in quantum-electrodynamics?
AIP Conf.Proc. 844,133(2006); *arXiv:quant−ph*/0603164