# Quantum control and semiclassical quantumgravity

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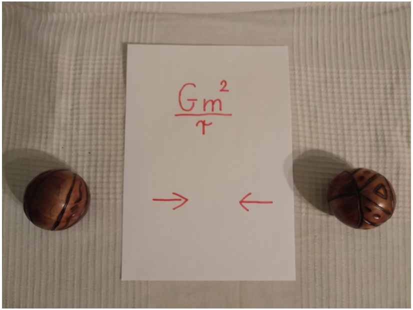
# Abstract

Quantum gravity has not yet obtained a usable theory. We apply the semiclassical theory instead, where the space-time remains classical (i.e.: unquantized). However, the hybrid quantum-classical coupling is acausal, violates both the linearity of quantum theory and the Born rule as well. Such anomalies can go away if we modify the standard mean-field coupling, building on the mechanism of quantum measurement and feed-back well-known in, e.g., quantum optics. The newtonian limit can fully be worked out, it leads to the gravity-related spontaneous wave function collapse theory of Penrose and the speaker.

# Gravity




MATTER IS QUANTIZED

$\psi(x_1)$ $\psi(x_2)$

NO EXPERIMEMENTS!

THEORY?

# Three theories in Newtonian limit

$$\hat{V}_G = -Gm^2 \times$$

$$\begin{cases} \dfrac{1}{|\hat{x}_1 - \hat{x}_2|} & \begin{array}{l}\text{Pair Potential}\\ \text{no relativistic extension}\end{array} \\ \dfrac{1}{|\hat{x}_1 - \langle\hat{x}_2\rangle|} + \dfrac{1}{|\hat{x}_2 - \langle\hat{x}_1\rangle|} & \begin{array}{l}\text{Semiclassical Gravity}\\ \text{relativistic (MøllerRosenfeld1962-63}\\ \text{superluminal, Born rule fails}\end{array} \\ \dfrac{1}{|\hat{x}_1 - \langle\hat{x}_2\rangle - \underset{\substack{\uparrow\\ \text{stochastic}}}{\delta x_2}|} + \dfrac{1}{|\hat{x}_2 - \langle\hat{x}_1\rangle - \underset{\substack{\uparrow\\ \text{stochastic}}}{\delta x_1}|} & \begin{array}{l}\text{Spontaneous Collapse}\\ \text{no relativistic extension}\\ \text{causal, Born rule works}\end{array} \end{cases}$$

# Gravity $\hat{V}^G$: local measurement plus communication

SPONTANEOUS COLLAPSE ("measurement")

$+$

CLASSICAL COMMUNICATION

$\Rightarrow$

DP THEORY OF SPONTANEOUS WAVEFUNCTION COLLAPSE

# DP: Spontaneos collapse of massive superpositions

D. 1986, Penrose 1996

$$|CAT\rangle = \frac{|\mathrm{LEFT}\rangle + |\mathrm{RIGHT}\rangle}{\sqrt{2}} \rightarrow \begin{cases} |\mathrm{LEFT}\rangle \\ \text{or} \\ |\mathrm{RIGHT}\rangle \end{cases}$$

COLLAPSE RATE: $(V_G^i - V_G^f)/\hbar$

Negligible for atomic masses

Extreme fast for large masses

$\sim 10^9/s$ for 1femtogram, $\sim 10^{19}/s$ for 1gram

EXPLAINS EMERGENCE OF CLASSICALITY IN MACROWORLD

# [Semiclassical thought-experiment 1986]

1. electromagnetism
2. nuclear forces
3. weak forces

} quantized, confirmed by tests
unified by the Standard Model

4. gravity (space-time) either quantized or not, no tests so far

Is it sharp or uncertain (fluctuating)?
If it is fluctuating, what is the spectrum of fluctuations $\delta g_{ab}$?
1986: Newtonian limit $\delta g_{00} = \delta\Phi$, thought-experiment with quantized probe + classical $\Phi$, order of magnitude estimate:

$$\langle \delta\Phi_t(\mathbf{r})\delta\Phi_s(\mathbf{y})\rangle_{\text{stoch}} = \text{const} \times \frac{\hbar G}{|\mathbf{x}-\mathbf{y}|}\delta(t-s)$$

$\Rightarrow$ DP positional decoherence for massive objects, testable in the lab?
Can't we get rid of the order-of-magnitude estimate 1986 ?
2016-17: We can (Tilloy & D).

# [Fragments from history]

Bronstein (1935): A sharp space-time structure is unobservable (because of Schwartzschild radii of test bodies). Quantization of gravity can not copy quantization of electromagnetism. We may be enforced to reject our ordinary concept of space-time.

1906-1938

Jánossy (1952): Quantum mechanics should be more classical. Expansion of the wave packet might be limited by

$$\dot{\psi}(x) = \frac{i\hbar}{2M}\psi''(x) - \gamma(x - \langle x \rangle)^2\psi(x) + \frac{1}{2}\gamma(\Delta x)^2\psi(x)$$

if we accept superluminality caused by the non-linear term.

1912-1978

Károlyházy (1966): The ultimate unsharpness of space-time structure limits coherent expansion of massive objects' position (while individual particles can expand coherently with no practical limitations).

1929-2012

# [Semiclassical Gravity 1962-63: sharp metric]

Sharp classical space-time metric (Møller, Rosenfeld 1962-63):

$$\mathrm{G}_{ab} = \frac{8\pi G}{c^4}\langle\Psi|\hat{\mathrm{T}}_{ab}|\Psi\rangle$$

Schrödinger equation on background metric $\mathrm{g}$:

$$|\dot{\Psi}\rangle = -\frac{i}{\hbar}\hat{H}[\mathrm{g}]|\Psi\rangle$$

That's our powerful effective hybrid dynamics for $(\mathrm{g}_{ab}, |\Psi\rangle)$, but

- with fundamental anomalies (superluminality, no Born rule, ...)
- that are unrelated to relativity and even gravitation
- just related to quantum-classical coupling
- that makes Schrödinger eq. nonlinear

No deterministic hybrid dynamics is correct fundamentally!
Way out: metric cannot be sharp, must have fluctuations $\delta\mathrm{g}_{ab}$.

# [Sharp metric Newtonian limit]

$$\mathrm{G}_{00} = 8\pi G c^{-4}\langle\Psi|\hat{\mathrm{T}}_{00}|\Psi\rangle \quad \Rightarrow \quad \Delta\Phi = 4\pi G\langle\Psi|\hat{\varrho}|\Psi\rangle$$

$$|\dot{\Psi}\rangle = -(i/\hbar)\hat{H}[\mathrm{g}]|\Psi\rangle \quad \Rightarrow \quad |\dot{\Psi}\rangle =$$

$$-(i/\hbar)\left(\hat{H}_0 + \textstyle\int\hat{\varrho}\Phi dV\right)|\Psi\rangle \quad \Rightarrow$$

$\Rightarrow$ Schrödinger-Newton Equation with self-attraction:

$$|\dot{\Psi}\rangle = -\frac{i}{\hbar}\left(\hat{H}_0 - G\iint\frac{\hat{\varrho}(\mathbf{x})\,\langle\Psi|\hat{\varrho}(\mathbf{y})|\Psi\rangle}{|\mathbf{x}-\mathbf{y}|}d\mathbf{x}d\mathbf{y}\right)|\Psi\rangle$$

Single "pointlike" body c.o.m. motion:

$$\dot{\psi}(\mathbf{x}) = \frac{i\hbar}{2M}\nabla^2\psi(\mathbf{x}) + \underbrace{\frac{i}{\hbar}GM^2\int\frac{|\psi(\mathbf{y})|^2 d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\psi(\mathbf{x})}_{\text{self-attraction}}$$

Solitonic solutions: $\Delta x \sim \hbar^2/GM^3$.

Irrelevant for atomic $M$, grow relevant for nano-$M$:

$$M \sim 10^{-15} g, \ \Delta x \sim 10^{-5} cm$$

That's quantumgravity in the lab [D. 1984].

# Interaction generated "on phone line"

Alice $\qquad\qquad$ Bob

$$M_A \; ////////// \; \overset{\text{spring}}{V(x_A - x_B)} \; ////////// \; M_B$$

at $x_A$ $\qquad\qquad$ at $x_B$

We can replace the spring by a phone line + two local springs

under local control of Alice and Bob, resp.:

$$\text{Alice} - - - \overset{\text{phone line}}{\text{to communicate}} - - - \text{Bob}$$

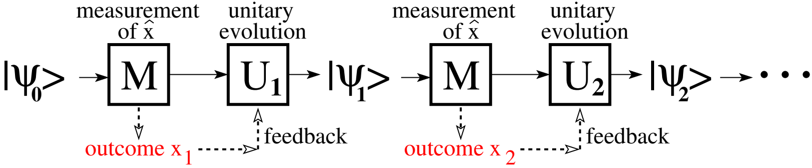
$$\searrow \qquad x_A \to \mathrm{B}, \mathrm{A} \leftarrow x_B \qquad \swarrow$$

$$M_A \; ///| \qquad\qquad |/// \; M_B$$

Trivial classically, non-trivial in quantum.
(Kafri, Taylor & Milburn 2014)

# Quantum control to generate potential (tutorial)

Sequential measurements of $\hat{x}$ plus feedback:




At $\infty$ repetition frequency:

standard theory of time-continuous monitoring+feedback.

$$\underbrace{x_t}_{signal} = \underbrace{\langle\Psi_t|\hat{x}|\Psi_t\rangle}_{mean} + \underbrace{\delta x_t}_{noise} \qquad \underbrace{\mathbb{E}\delta x_t \delta x_s}_{correlation} = \underbrace{\gamma^{-1}}_{\gamma=precision}\delta(t-s)$$

To generate a potential, take

$\hat{H}_{fb}(t) = R x_t \hat{x} = R(\langle\Psi_t|\hat{x}|\Psi_t\rangle + \delta x_t)\hat{x}$.

$$|\dot{\Psi}\rangle = \frac{-i}{\hbar}(\hat{H}_0 + \underbrace{\tfrac{1}{2}R\hat{x}^2}_{fb-generated})|\Psi\rangle - \tfrac{1}{8}\underbrace{[\gamma + 4\gamma^{-1}(R/\hbar)^2]}_{to\ be\ minimized}\underbrace{(\hat{x} - \langle\hat{x}\rangle)^2}_{localisation}|\Psi\rangle + \underbrace{\ldots\delta x}_{stochastic}|\Psi\rangle$$

$$\dot{\hat{\rho}} = \frac{-i}{\hbar}[\hat{H}_0 + \tfrac{1}{2}R\hat{x}^2, \hat{\rho}] - \frac{1}{2\hbar}R[\hat{x},[\hat{x},\hat{\rho}]]$$

# Quantum control to generate potential (summary)

Assume $\hat{x}$ is being monitored, yielding signal
$x_t = \langle\Psi_t|\hat{x}|\Psi_t\rangle + \delta x_t$.
Apply feedback via the hybrid Hamiltonian

$$\hat{H}_{fb}(t) = R x_t \hat{x} = \underbrace{R\langle\hat{x}\rangle_t\hat{x}}_{sharp\ \ semiclassical\ \ coupling} + \underbrace{R\delta x_t\hat{x}}_{(white)noise\ \ part\ \ of\ \ coupling}$$

Sharp+noisy terms together cancel nonlinearity (and related anomalies) from the quantum dynamics:

$$\dot{\hat{\rho}} = \frac{-i}{\hbar}[\hat{H}_0 + \tfrac{1}{2}R\hat{x}^2, \hat{\rho}] - \frac{1}{2\hbar}R[\hat{x}, [\hat{x}, \hat{\rho}]]$$

New potential has been generated 'semiclassically' and consistently with quantum mechanics, but at the price of decoherence.

# [Decoherent Semiclassical Gravity: unsharp metric]

- Assume $\hat{\mathrm{T}}_{ab}$ is spontaneously measured (monitored)
- Let $\mathrm{T}_{ab}$ be the measured value (called signal in control theory)
- Replace Møller-Rosenfeld 1962-63 by

$$\mathrm{G}_{ab} = \frac{8\pi G}{c^4}\mathrm{T}_{ab} = \frac{8\pi G}{c^4}(\langle\hat{\mathrm{T}}_{ab}\rangle + \delta\mathrm{T}_{ab})$$

  i.e.: source Einstein eq. by the noisy signal (meanfield+noise)
- For backaction of monitoring, add terms to Schrödinger eq.:

$$\frac{d}{dt}|\Psi\rangle = -\frac{i}{\hbar}\hat{H}[\mathrm{g}]|\Psi\rangle + \mathrm{nonlinear} + \mathrm{stoch.\ terms}$$

- Tune precision of monitoring by Principle of Least Decoherence

D 1990, Kafri, Taylor & Milburn 2014, Tilloy & D 2016-17

# Unsharp metric Newtonian limit

- Assume mass density $\hat{\varrho}(\mathbf{x})$ is spontaneously measured (monitored)
- Let $\varrho_t(\mathbf{x})$ be the measured value (called signal in control theory)
- Source classical Newtonian gravity by the signal:

$$\Phi_t(\mathbf{x}) = -G\int \frac{d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\ \varrho_t(\mathbf{y})$$

- Introduce $\hat{H}_{fb} = \int \hat{\varrho}\Phi dV$ to induce Newton interaction
- For backaction of monitoring, add standard terms to Schrödinger eq.:

$$\frac{d}{dt}|\Psi\rangle = -\frac{i}{\hbar}(\hat{H}_0 + \hat{H}_{fb})|\Psi\rangle + \mathrm{nonlinear} + \mathrm{stoch.\ terms}$$

- Tune precision of monitoring by Principle of Least Decoherence

Such theory of unsharp semiclassical gravity coincides with ...

# ... coincides with DP wavefunction collapse theory

Unique ultimate unsharpness of Newton potential $\Phi$ (metric):

$$\langle\delta\Phi_t(\mathbf{r})\delta\Phi_s(\mathbf{y})\rangle_{\text{stoch}} = \frac{\hbar G}{2|\mathbf{x}-\mathbf{y}|}\delta(t-s)$$

By averaging over the stochastic $\Phi$ (metric), master eq. (D. 1986):

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\Big[\hat{H}_0 \underbrace{-\frac{G}{2}\iint\frac{d\mathbf{x}d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\hat{\varrho}(\mathbf{x})\hat{\varrho}(\mathbf{y})}_{Newton\ \ pair\ \ potential}, \hat{\rho}\Big] \underbrace{-\frac{G}{2\hbar}\iint\frac{d\mathbf{x}d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}[\hat{\varrho}(\mathbf{x}),[\hat{\varrho}(\mathbf{y}),\hat{\rho}]]}_{DP\ \ decoherence}$$

Double merit:

- Semiclassical theory of gravity, a hybrid dynamics of $(\Phi, |\Psi\rangle)$ free of anomalies (no superluminality, valid Born rule).
- Theory of G-related spontaneous collapse (Schrödinger's Cats go collapsed).

# Summary

Møller-Rosenfeld (sharp) Semiclassical Gravity is quantum-nonlinear, with related fundamental anomalies and particular effects:

- superluminality, fall of $\Psi$'s statistical interpretation (anomaly)
- self-attraction (main effect for tests)

These fundamental anomalies and self-attraction are missing in (unsharp) Decoherent Semiclassical Gravity. But new anomalies and effects arise:

- non-conservation of energy, momenta, etc. (anomaly)
- decoherence, c.o.m. Brownian motion, ... (effects for tests)
- submicron cutoff against diverging decoherence (open problem)
- submicron breakdown of Newton force (effect for tests)

# References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