Deflection and splitting of atomic beams with counterpropagating, short, chirped laser pulses

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Received June 9, 1997; revised manuscript received September 18, 1997

The application of counterpropagating, short, chirped laser pulses for the deflection and splitting of an atomic beam is investigated. A simple model is proposed to describe the case in which the pulses induce population transfer in the adiabatic-passage regime and in which the effects of spontaneous emission are negligible during the action of a single laser pulse. Spontaneous emission between the pulses is not neglected, however, and it is shown that it has a significant effect on the evolution of the average transversal velocity of the atoms, as well as the dispersion of the transversal velocity. © 1998 Optical Society of America [S0740-3224(98)04101-0]

OCIS codes: 050.1590, 140.3320.

1. INTRODUCTION

The mechanical effects of resonant laser radiation on free atoms have been studied extensively for more than two decades (see Refs. 1–5 and references therein). During this time, most of the attention has been paid to the effects of stationary-state laser fields, i.e., fields whose amplitudes (and, in most cases, frequencies) do not change during the interaction (e.g., various standing-wave configurations). Several interesting effects have been discussed, however, where changing laser fields play an important role. Deflection of atomic beams by means of absorption and stimulated emission from chirped laser pulses,[6] deflection of atomic beams with intense laser pulses retroreflected from a mirror to form standing waves,[7] Doppler cooling in pulsed fields,[8] adiabatic cooling in slowly decaying standing waves,[9,10] coherent beam splitting in multilevel systems, using a counterintuitive pulse sequence in the adiabatic-passage regime,[11,12] and manipulation and cooling by means of chirped laser pulses[13,14] are some examples.

Most of the effects mentioned above (with Refs. 7 and 8 being the only exceptions) involve slowly changing fields that induce changes in the adiabatic-passage regime. These effects are also coherent processes, which might be hampered by spontaneous emission. Some of these methods (e.g., Refs. 11 and 12) are affected very little by spontaneous processes because they involve only ground-state atoms and are thus largely immune to incoherent effects. Other methods for deflection and manipulation of atoms (see Refs. 6, 13, and 14) do not share this property. Because they involve atoms in their excited states, they may be greatly influenced by spontaneous emission. While the changing of the fields should be slow in these processes compared with the Rabi frequency, and therefore with sufficiently strong fields the adiabatic passage regime may in theory be preserved even if the processes last only a fraction of the spontaneous lifetime of the atom, realistic experimental situations may still prevent the usage of these methods of manipulation for most atoms. The reason for this is that it may not be possible to conclude the processes with existing short-pulse lasers in a fraction of a spontaneous lifetime. It is therefore advisable to investigate how spontaneous emission might affect the application of these proposed methods.

In our work we are concerned with the proposals of Refs. 6, 13, and 14 for deflection and manipulation of atoms. The principal idea in these proposals is to use counterpropagating laser pulses, whose frequency is chirped to excite and de-excite the atomic ensemble. These pulses excite and de-excite the whole of the atomic ensemble in the adiabatic-passage regime, and the atoms receive a mechanical momentum of $2\hbar k$ (two-photon momenta) with each absorption–stimulated-emission process. To carry the treatment one step further, we relax the requirement that the whole process take place during a time when spontaneous emission is negligible. The assumption we make is that spontaneous emission is negligible during the action of a single laser pulse but not between the pulses. We thus get valuable predictions about what happens if the action of the series of laser pulses continues for longer than the spontaneous lifetime of the atom. These proposals are similar to that realized in Ref. 15, in which atoms are subject to a series of Π pulses from alternating directions. The greatest handicap of that scheme is that producing an exact inversion of the populations throughout the atomic ensemble is difficult. The use of chirped pulses in the adiabatic-passage regime is much more robust and largely immune to the handicaps of the Π-pulse method. Our analysis, however, is valid also for this case if all the atoms can be assumed to experience a perfect Π pulse.

2. MODEL

We consider the motion of a beam of two-level atoms through a region where the atoms interact with counter-

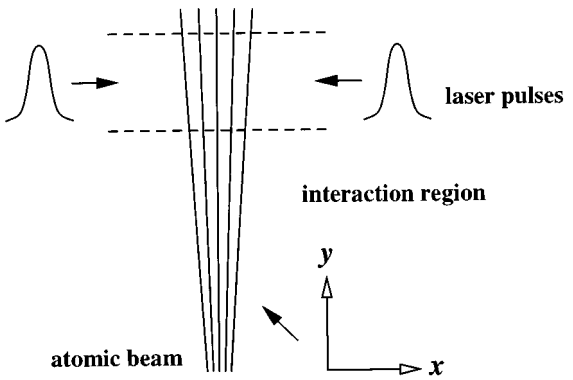

Fig. 1. Schematic arrangement of the situation considered; a collimated atomic beam crosses a region where it interacts with short, chirped laser pulses from alternating directions.

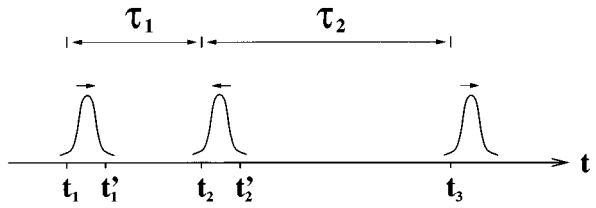

Fig. 2. Time sequence of the laser pulses; one elementary cycle lasts from the beginning of one pulse at t_1 to the beginning of the third pulse at t_3. The arrows on top depict the direction of propagation of the pulses.

propagating, short, frequency-chirped laser pulses. The direction of propagation of the laser pulses is perpendicular to the direction of propagation of the atomic beam (Fig. 1), and the frequency of the pulses is assumed to pass through the resonance frequency of the two-level atoms. Furthermore, the pulses are assumed not to overlap each other in the region of interaction, i.e., atoms in this region experience a series of separate pulses from alternating directions. One pulse propagates toward the positive x direction, then a time τ_1 later one propagates in the opposite direction, and after a time τ_2 this elementary cycle is repeated (Fig. 2). Atoms enter the interaction region at random times, (i.e., the beam of atoms is assumed to be continuous), and the time they interact with the laser pulses is simply the time needed to cross the interaction region, which is determined by the longitudinal velocity. It is assumed that this interaction time is long compared with the cycle time $u = \tau_1 + \tau_2$. We are concerned only with the description of the transverse motion of the atoms; their longitudinal velocity along the beam is not treated in the equations, because it only determines the interaction time with the laser pulses.

To treat the transverse motion of atoms along the x axis (the direction of the propagation of the laser pulses), we should try to solve the Schrödinger equation for the density-matrix elements $\varrho_{i,j}(p_1, p_2)$, which determine the state of the atoms in the x direction. Here the indices $i, j \in \{g, e\}$ represent the internal states of the two-level atom (ground and excited states), and the variables p_1, p_2 are the momentum variables along the x axis. The solution of these equations for the case of a large number of consecutive, counterpropagating pulses presents the same problems as for the case of standing waves,[1,14] namely, there are large numbers of coupled equations. To complicate matters further, spontaneous emission couples an infinite number of density-matrix elements, and the combined effects of the laser pulses and spontaneous emission produce a set of equations that are very hard to solve for realistic situations.

It has been shown[13,14,16] that if spontaneous emission can be neglected, and the variation of the envelope and the frequency of the laser pulses is slow enough for the conditions of adiabatic following to be fulfilled, the problem may be simplified. In this case the solutions for the time evolution of the momentum–space distribution functions in the absence of spontaneous emission will be such that at the end of the laser pulse the distributions of the ground and excited states will be exchanged and shifted by $\hbar k$ in momentum space [i.e., $\varrho_{g,g}(p, p, t_1') = \varrho_{e,e}(p + \hbar k, p + \hbar k, t_1)$ and $\varrho_{e,e}(p + \hbar k, p + \hbar k, t_1') = \varrho_{g,g}(p, p, t_1)$; see Fig. 2 for the definitions of the times t_1, t_1']. In other words, all atoms in their ground states are left in their excited states by the laser pulse and receive a momentum $\hbar k$, while those in the excited states before the pulse are all de-excited and receive a momentum $-\hbar k$. Using these results, we may simplify the problem by considering laser pulses whose duration is much shorter than the natural lifetime of the atoms and neglect the effects of spontaneous emission during the action of the pulses. We shall retain the effects of spontaneous emission between the pulses. This means that our model is valid only if $\tau_1, \tau_2, T_1 \gg \tau_p$, where τ_p is the duration of the laser pulses and $T_1 = \Gamma^{-1}$ is the lifetime of the excited state. (Γ is the spontaneous emission rate of the atom.) All these simplifications allow the writing of a simple master equation for the effect of an elementary cycle of two counterpropagating laser pulses on the momentum–space distributions of the atoms.

Let us introduce the notations $a(z) = \varrho_{g,g}(z, z)$ and $b(z) = \varrho_{e,e}(z, z)$ for the distributions of the ground and excited states of the atom, respectively. We have used the dimensionless momentum parameter $z = p/\hbar k$, i.e., the unit of momentum is the photon momentum. Since the first pulse in the elementary cycle propagates toward the positive x direction, the distribution functions at a time t_1' immediately after the laser pulse are related to those at the time t_1 (see Fig. 2) just before the pulse by

$$a(t_1', z) = b(t_1, z + 1),$$
$$b(t_1', z) = a(t_1, z - 1). \tag{2.1}$$

The evolution of the distributions from t_1' to t_2 just before the second pulse, which is due to spontaneous emission, may be written as

$$a(t_2, z) = a(t_1', z) + [1 - \exp(-\Gamma\tau_1)] \times \int_{-1}^{+1} f(z')b(t_1', z + z')\mathrm{d}z',$$
$$b(t_2, z) = \exp(-\Gamma\tau_1)b(t_1', z). \tag{2.2}$$

Here we have denoted the probability of the spontaneously emitted photon to have a momentum z' along the x axis by $f(z')$. Since the probability of spontaneous emis-

sion is the same in all directions, this function is simply the projection of the surface of the three-dimensional unit sphere to the x-axis. The action of the second pulse counterpropagating to the first one has the effect

$$a(t_2', z) = b(t_2, z - 1),$$
$$b(t_2', z) = a(t_2, z + 1), \tag{2.3}$$

while the time τ_2 elapsing from t'_2 after the second pulse till t_3, just before the start of the new cycle, has the effect

$$a(t_3, z) = a(t_2', z) + [1 - \exp(-\Gamma\tau_2)] \times \int_{-1}^{+1} f(z')b(t_2', z + z')\mathrm{d}z',$$
$$b(t_3, z) = \exp(-\Gamma\tau_2)b(t_2', z). \tag{2.4}$$

To simplify matters a little further, we write $f(z') = \delta(z')$, i.e., we neglect the recoil that is due to the spontaneously emitted photons. Combining Eqs. (2.1)–(2.4) and introducing the notations $P_1 = \exp(-\Gamma\tau_1)$ and $P_2 = \exp(-\Gamma\tau_2)$ for the probabilities of the nonoccurrence of spontaneous emission during the time intervals τ_1 and τ_2, we may write the cumulative effect of a full cycle of laser pulses on the distributions as

$$a(t_3, z) = P_1a(t_1, z - 2) + (1 - P_1)(1 - P_2)a(t_1, z) + (1 - P_2)b(t_1, z + 2),$$
$$b(t_3, z) = P_2b(t_1, z + 2) + P_2(1 - P_1)a(t_1, z). \tag{2.5}$$

By adding these equations, for the total velocity–space distribution $g(z) = a(z) + b(z)$ we get

$$g(t_3, z) = b(t_1, z + 2) + a(t_1, z)(1 - P_1) + P_1a(t_1, z - 2). \tag{2.6}$$

Equations (2.5) describe the evolution of the velocity-distribution functions of the atoms at discrete times. They describe the effect of one elementary cycle of two counterpropagating laser pulses and the effect of spontaneous emission between the pulses. To analyze them further, we shall consider the evolution of various moments of the distributions under the action of the laser pulses. Introducing the notations $P_a = \int a(z)\mathrm{d}z$ and $P_b = \int b(z)\mathrm{d}z$, the probabilities for the atoms to be in the ground state or the excited state (subject to the constraint $P_a + P_b = 1$), we may integrate Eqs. (2.5) with respect to z to get

$$P_a(t_3) = 1 - P_2 + P_1P_2P_a(t_1), \tag{2.7a}$$
$$P_b(t_3) = P_2 - P_1P_2P_a(t_1). \tag{2.7b}$$

It is easy to prove that these equations describe the relaxation of the ground- and excited-state probabilities to stationary values, which are left unchanged by the action of a full cycle: $P_a^{st} = P_a(t_3) = P_a(t_1)$, $P_b^{st} = P_b(t_3) = P_b(t_1)$. From Eqs. (2.7) we get

$$P_a^{st} = \frac{1 - P_2}{1 - P_1P_2}, \tag{2.8a}$$
$$P_b^{st} = \frac{P_2(1 - P_1)}{1 - P_1P_2}. \tag{2.8b}$$

It can be seen that Eq. (2.7a) is of the form

$$P_a(t_3) = P_a^{st} + [P_a(t_1) - P_a^{st}]P_1P_2, \tag{2.9}$$

and thus the value of the ground-state probability at a time $t = n(\tau_1 + \tau_2)$, where n is the number of full cycles after the arrival of the first pulse of the first cycle at $t_0 = 0$, is

$$P_a(t) = P_a^{st} + [P_a(t_0) - P_a^{st}]\exp(-\Gamma t), \tag{2.10}$$

so that $P_a(t) \to P_a^{st}$ when $t \to \infty$.

Turning to the first-order moments of the distributions, we may multiply Eqs. (2.5) by z and integrate to obtain equations for the average velocities:

$$\bar{z}_a = \int za(z)\mathrm{d}z, \quad \bar{z}_b = \int zb(z)\mathrm{d}z. \tag{2.11}$$

The relation $\bar{z} = \int zg(z)\mathrm{d}z = \bar{z}_a + \bar{z}_b$ is obviously fulfilled. The evolution of $\bar{z}_a$ and $\bar{z}_b$ is now

$$\bar{z}_a(t_3) = \bar{z}_a(t_1)(1 - P_2 + P_1P_2) + (1 - P_2) \times [\bar{z}_b(t_1) - 2P_b(t_1)] + 2P_1P_a(t_1), \tag{2.12a}$$
$$\bar{z}_b(t_3) = P_2[\bar{z}_b(t_1) - 2P_b(t_1)] + P_2(1 - P_1)\bar{z}_a(t_1). \tag{2.12b}$$

By adding these two equations [or by multiplying Eq. (2.6) by z and integrating], we get

$$\bar{z}(t_3) = \bar{z}_a(t_3) + \bar{z}_b(t_3) = \bar{z}(t_1) + 2[P_1P_a(t_1) - P_b(t_1)], \tag{2.13}$$

for the evolution of the average momentum of the atom under the action of a full cycle of laser pulses. This equation has a simple physical meaning; and it could have been derived by the following semiclassical argument: The atoms that are in the ground state just before the first pulse and do not spontaneously fluoresce before the second pulse (the probability of which is P_1P_a) receive a momentum $2\hbar k$ during the interaction. The atoms that are in ground state just before the first pulse but decay before the second pulse do not gain any momentum during the cycle. The atoms that are in an excited state before the first pulse receive a momentum $-2\hbar k$. The average gain in momentum is just $2(P_1P_a - P_b)$ in normalized units.

Remembering that the probabilities P_a and P_b evolve toward steady-state values, it is also evident that the average momentum Δz gained in a cycle also has a steady limit. This value is

$$\Delta z^{st} = 2\frac{P_1 - P_2}{1 - P_1P_2}, \tag{2.14}$$

and is practically reached after a few times Γ^{-1} have elapsed. This means that the average force acting on the atoms relaxes to a steady-state value. It can be seen that the value of the momentum received during a cycle is reversed if τ_1 and τ_2 are exchanged, which reflects the fact that exchanging these two intervals is the same as revers-

ing the directions of propagation of the pulses. The value of Δz^{st} increases with the difference between τ_1 and τ_2. If τ_1 approaches zero with τ_2 being constant, the gained momentum approaches its maximum value of two-laser-photon momenta. This behavior may be explained by noting that this change of momentum depends on P_a^{st}; the larger this value, the more likely that an atom will be pushed in the direction of the first pulse, and the less likely that it will be pushed in the opposite direction [see Eq. (2.13)]. The behavior of P_a^{st} is such that it increases at a given value of τ_2 if τ_1 decreases or at a given τ_1 if τ_2 increases [Eq. (2.8a)]. This is because an atom that is in its ground state before the first pulse will be in its excited state after the first pulse and may decay during τ_1. The shorter τ_1 is, the smaller the probability is that it will decay. On the other hand, if an atom is in its excited state after the second pulse (either because it was in its excited state before the first pulse or because it was in its ground state before the first pulse but decayed during τ_1), it has a chance to decay during τ_2 and to be in the ground state before the start of the next cycle. The longer τ_2 is, the greater is the probability that it does so. It is thus understandable that decreasing τ_1 or increasing τ_2 both increase P_a^{st}. If $\tau_1 = \tau_2$, the force approaches zero in the steady-state limit.

Finally, we deduce the evolution of the width $\sigma_z^2 = \bar{z}^{-2} - \bar{z}^{-2}$ of the velocity distribution. Multiplying Eq. (2.6) by z^2, integrating it, and using Eq. (2.13), after some straightforward calculations, we obtain

$$\sigma_z{}^2(t_3) = \sigma_z{}^2(t_1) + 4(P_1 + 1)Q(t_1) + W(t_1), \tag{2.15}$$

where we have introduced the notations

$$Q = P_b\bar{z}_a - P_a\bar{z}_b, \tag{2.16a}$$

$$W = 4(P_b + P_1P_a) - 4(P_1P_a - P_b)^2. \tag{2.16b}$$

It can be seen from these equations that the changing of the width of the distribution during a cycle may easily be negative, i.e., it is possible to imagine situations where the distribution becomes narrower after the action of two laser pulses.

As for the long-term evolution, it is obvious that $W(t)$, being composed only of the probabilities of the ground and the excited states, has a steady-state value to which it relaxes. The quantity $Q(t)$ is proportional to the difference of the average momentum of the ground-state distribution and that of the excited-state distribution $Q = P_aP_b(\bar{z}_a/P_a - \bar{z}_b/P_b)$. Using Eqs. (2.12), one may deduce that the asymptotic form of the equation for this quantity (i.e., the form taken by the equation when enough time has elapsed so that $P_{a,b} = P_{a,b}^{st}$) is similar to that of the probabilities

$$Q(t_3) = Q^{st} + P_1P_2[Q(t_1) - Q^{st}], \tag{2.17}$$

with

$$Q^{st} = \frac{2P_1P_2(1 - P_1)(1 - P_2{}^2)}{(1 - P_1P_2)^3}. \tag{2.18}$$

All this means that the long-term evolution of the standard deviation will be similar to a diffusion:

$$\sigma_z{}^2(t_3) = \sigma_z{}^2(t_1) + 4(P_1 + 1)Q^{st} + W^{st} = \sigma_z{}^2(t_1) + K, \tag{2.19}$$

i.e., the square of the standard deviation will increase by the same amount in each cycle. By substitution of Eqs. (2.8), (2.16b), and (2.18) into Eq. (2.19) one may verify that K is positive for arbitrary values of τ_1, τ_2 so that the long-term evolution of the width of the distribution will be a spreading.

Finally, with the notations introduced above, we may write the equation giving the quantity $\bar{z}(n)$ after n cycles in terms of the initial values in a relatively simple form:

$$\begin{aligned}\bar{z}(n) &= \bar{z}(0) + \sum_{i=0}^{n-1} 2[P_a(i)P_1 - P_b(i)] \\ &= \bar{z}(0) + n\Delta z^{st} + 2(P_1 + 1) \\ &\quad \times [P_a(0) - P_a^{st}]\,\frac{(1 - P_1P_2)^n}{1 - P_1P_2}.\end{aligned} \tag{2.20}$$

The evolution of the average momentum thus consists of two separate terms. The second term describes a change that is constant with each cycle, the cumulative effect of which is simply proportional to the number of cycles applied. This constant depends solely on the times τ_1, τ_2 that elapse between the pulses. This term describes the behavior of an atom that has reached the stationary state with respect to the probabilities P_a and P_b. The other term in the equation describes a transient behavior that is due to a difference between the initial state of the atom and the stationary state. This term is constant for times $t \gg \Gamma^{-1}$ and may simply be calculated as the convergent sum of an infinite geometrical series.

The precise time evolution of $\sigma_z{}^2$ is not this easy to express. However, iterating Eqs. (2.9), (2.12), and (2.15), we may calculate the evolution of the width of the distribution for arbitrary initial conditions.

It is also appropriate to investigate the domain of applicability of our model. The predictions of the present model should be compared with a full numerical simulation of the problem [i.e., without the initial assumption of perfect adiabatic passage without spontaneous emission, as in Eq. (2.5)]. Such a comparison can provide valuable information as to precisely how much larger τ_1, τ_2, and Γ^{-1} need to be than τ_p (laser-pulse duration) for the predictions of the model to coincide with the more exact simulations. Although a full quantum simulation has not been performed because of the difficulties, a much simpler simulation of the expectation value of the center-of-mass coordinate (the first moment of the Schrödinger equation) derived from Ehrenfest's theorem[17] has been done. The results were compared with Eqs. (2.9) and (2.20). It has been found that for a pulse duration of $\tau_p \approx 0.006\,\Gamma^{-1}$ and $\tau_1, \tau_2 \geqslant 10\tau_p$ the predictions of the present model were already almost identical to the exact simulations.

3. DISCUSSION

A. Deflection of an Atomic Beam

We are now in a position to consider the interaction of atoms with a series of chirped pulses for the deflection of

the atomic beam. There are two major requirements: to shift the mean value of the transversal momentum of the atoms as much as possible (in other words, we would like the average force to be as large as possible) and to limit the dispersion of the transversal velocity as much as possible. In fact, the spread of the transversal momentum should be kept much smaller than the change of average momentum during the given interaction time if deflection is to have a meaning at all.

Let us first investigate the effects of the laser pulses in the steady-state limit. We may define an average force acting on the atoms by $f^{st} = \Delta z^{st}/(\tau_1 + \tau_2)$ and an average diffusion constant by $D^{st} = K/(\tau_1 + \tau_2)$, noting that the increase of $\bar{z}$ and $\sigma_z{}^2$ may be calculated by multiplying these values by the time elapsed only at discrete moments at the end of each cycle. It is instructive to plot this force and this diffusion constant as functions of two variables composed of τ_1 and τ_2: $u = \tau_1 + \tau_2$ gives the cycle time, while $v = (\tau_2 - \tau_1)/(\tau_1 + \tau_2)$ gives the relation of τ_1 and τ_2 within one cycle. The plots of f^{st} and D^{st} in the region $u \in [0.01,0.1]$ are shown on Fig. 3. The unit of time is Γ^{-1}. It can be seen that for any given v, both the force and the diffusion constant grow with the decrease of u, except for $v = 0$, in which case the force remains zero. In fact it is not hard to deduce from Eqs. (2.14) and (2.16)–(2.19) that both the force and the diffusion constant diverge as the cycle time approaches zero, except for $v = 0$, in which case f^{st} remains zero. This is understandable from the fact that the number of cycles per unit time increases as the cycle time decreases. It can be seen from Fig. 3(a) that for any given u, the force is an odd function of v, which means that exchanging τ_1 and τ_2 reverses the force. The reason is that in the stationary regime, when the initial conditions are no longer of importance, our choice of first pulse within a cycle is arbitrary. Exchanging τ_1 and τ_2 is therefore equivalent to saying that we shall call the pulse propagating toward the negative x direction the first pulse. The value of the force increases with the difference between τ_1 and τ_2. This is a consequence of the dependence of Δz^{st} on τ_1 and τ_2 already discussed. The fact that the diffusion constant is maximum for a given cycle time at $v = 0$ and decreases toward $v = \pm 1$ may be explained by noting that the two parts of the distribution $a(z)$ and $b(z)$ are pushed in opposite directions by the pulses in momentum space. The smaller the fraction of the atoms in an excited state before the first pulse (the smaller P_b^{st}), the smaller is the fraction of the atoms that move opposite to the majority in momentum space, and hence the smaller the dispersive effect on the distribution.

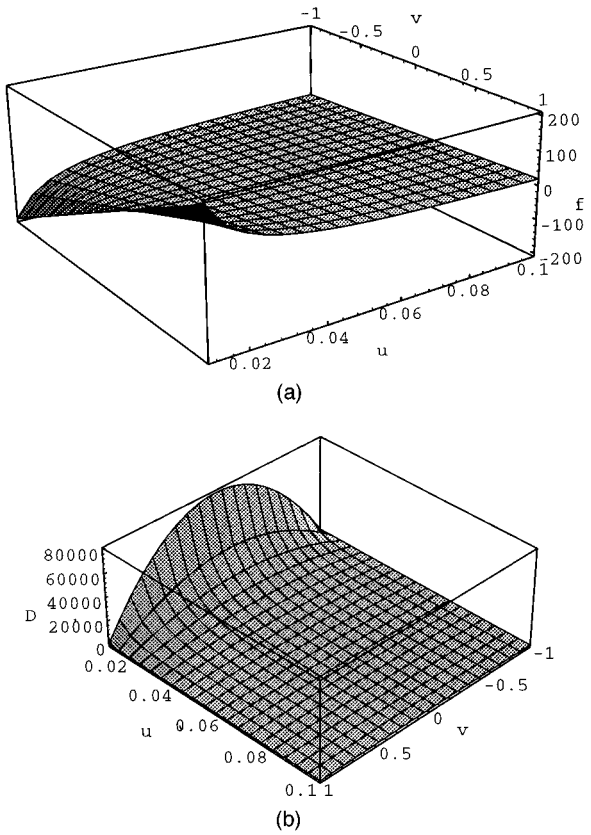

Fig. 3. (a) Stationary-state force f^{st} and (b) diffusion constant D^{st} as functions of $u = \tau_1 + \tau_2$ pulse length and $v = (\tau_2 - \tau_1)/(\tau_1 + \tau_2)$ in the region $u \in [0.01, 0.1]$. The unit of momentum is $\hbar k$, and the unit of time is Γ^{-1}.

Comparison of the numerical values of f^{st} and D^{st} on Fig. 3 is not easy because D^{st} gives the increase of the square of the width of the distribution per unit time. It is obvious, however, that the two major requirements (the force should be as large as possible, while the widening of the distribution should be as small as possible) both require v to be in the vicinity of 1, i.e., τ_1 to be small compared to τ_2.

We shall now consider atoms with a single longitudinal velocity, i.e., the interaction time with the laser pulses is the same for all atoms. The initial distribution of the transversal velocity is assumed to be some smooth distribution $g_0(z)$ (e.g., a Gaussian), located around $z = 0$ and having some initial spread σ_{z0}. The average transversal velocity is thus assumed to be zero. Atoms arrive at the interaction region in their ground states. To determine the average transversal momentum and the width of the distribution, we must first determine the initial values $P_{a,b}(0)$, $\bar{z}_{a,b}(0)$, and $\sigma_z{}^2(0)$. We must remember, however, that atoms arrive at the interaction region at random times, i.e., we have no way of knowing whether they actually interact with the pulse we call first in a cycle initially. Therefore we must say that the initial distribution consists of two terms: One, when the atom arrives during the interval τ_2 with a probability $\tau_2/(\tau_1 + \tau_2)$ and experiences a pulse propagating toward the positive x direction first, i.e., what we refer to as the first pulse of a cycle. The other term arises when the atom arrives during the interval τ_1 and interacts with the second pulse of a cycle first. The first full cycle of laser pulses will therefore find this atom in a different state from the state in which it was prepared before the interaction. These atoms will obviously start moving in the opposite direction from the rest, and this will have a dispersive effect on the distribution. The initial distribution will therefore be of the form

$$a_0(z) = \frac{\tau_2}{\tau_1 + \tau_2} g_0(z) + (1 - P_2) \frac{\tau_1}{\tau_1 + \tau_2} g_0(z + 1), \tag{3.1a}$$

$$b_0(z) = P_2 \frac{\tau_1}{\tau_1 + \tau_2} g_0(z + 1), \tag{3.1b}$$

and so we should determine the initial quantities from these new initial distributions:

$$P_a(0) = 1 - \frac{P_2\tau_1}{\tau_1 + \tau_2}, \tag{3.2a}$$

$$\bar{z}_a(0) = \frac{(P_2 - 1)\tau_1}{\tau_1 + \tau_2}, \qquad \bar{z}_b(0) = -\frac{P_2\tau_1}{\tau_1 + \tau_2}, \tag{3.2b}$$

$$\sigma_z^2(0) = \sigma_{z0}^2 + \frac{\tau_1\tau_2}{(\tau_1 + \tau_2)^2}. \tag{3.2c}$$

For a given time $t > \Gamma^{-1}$ we may now approximate Eq. (2.20) as the convergent sum of an infinite geometrical series from the transient term, plus the stationary force multiplied by the time that has elapsed. By doing this, we have neglected the fact that, because of the discrete nature of the interaction, some atoms may not experience the same number of cycles [and certainly not a fractional number of cycles if $t \neq n(\tau_1 + \tau_2)$], but may exit the interaction region half a cycle or a full cycle sooner depending on the time of arrival. This jitter, however, causes an error that is of the order of one photon momentum, which is at most the same order of magnitude (and usually less) than the error caused by neglecting the recoil of the spontaneously emitted photons. When writing Eqs. (3.1) we have also neglected the fact that some atoms may arrive during a pulse, and interaction with a fraction of a pulse may not completely invert their internal states of excitation but leave them in some superpositional state. This approximation is justified by assuming $\tau_{1,2} \gg \tau_p$.

The total momentum transferred and the width of the distribution, assuming an initial width of $\sigma_{z0} = 10$ after a time $t = 50\Gamma^{-1}$, can be seen in Fig. 4. It can clearly be seen that effective deflection of the atomic beam is possible and that the results are best if the cycle time and the ratio of τ_1 to τ_2 are as small as possible. It may be seen from the figure, that significant momentum transfer (of the order of $10^4\hbar k$) may occur in a relatively short time (by comparison, the momentum transfer from the scattering force exerted by a simple cw laser beam from one side would be at most a few times $10\hbar k$ during this time), and the widening of the distribution may be kept smaller then the deflection.

It is notable that when the goal is to deflect the atomic beam, making τ_1 as small as possible compared with τ_2 is best for three reasons. The average force acting on the atoms at a given cycle time is largest and the diffusion constant is smallest when the proportion of τ_1 is smallest; furthermore, the initial distribution [Eqs. (3.2)] is the most favorable. (Fewer atoms have a chance of arriving during τ_1 to experience a second pulse first and start moving in the wrong direction in velocity space.)

Up to now we have considered only the average velocity of the distribution and its width in velocity space. This information is only a fraction of what there is to know about the distribution function. If one examines the precise evolution of the distribution [by numerical simulation

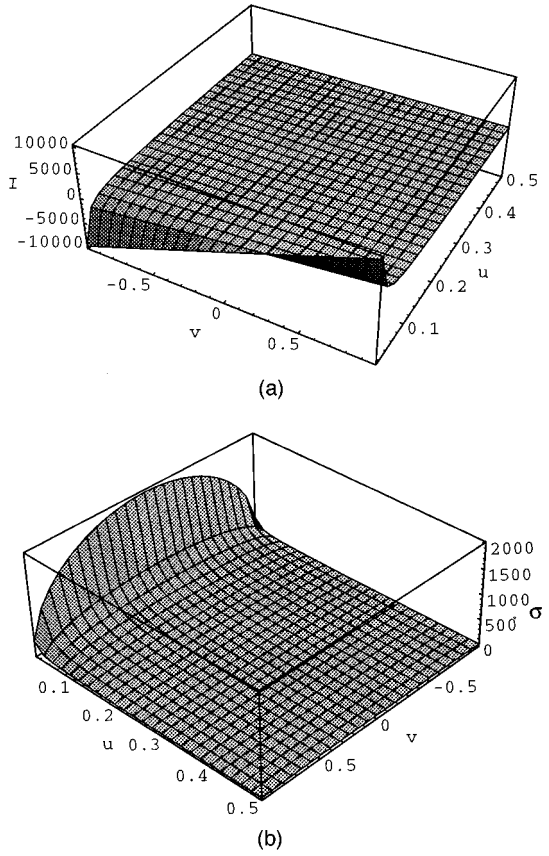

Fig. 4. (a) Total momentum I transferred to the atoms during a time $50\Gamma^{-1}$ and (b) width σ_z of the distribution after this time in the region $u \in [0.01, 0.1]$, assuming an initial width of $\sigma_{z0} = 10$.

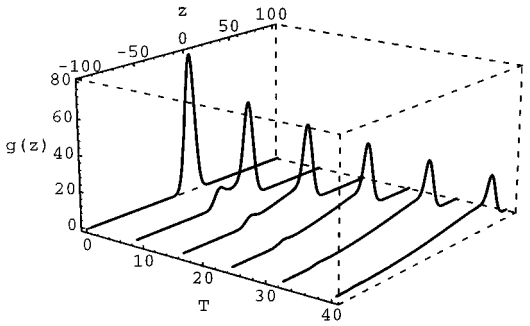

Fig. 5. Evolution of the velocity distribution of atoms $g(z)$ for the first 40 cycles of laser pulses. The intervals between the pulses are $\tau_1 = 0.03$, $\tau_2 = 0.1$, the initial Gaussian distribution has been normalized to $N = 1000$, and the initial width is $\sigma_z = 5\hbar k$. T on the horizontal axis gives time measured by the number of cycles.

of the master equation, Eq. (2.5)], it turns out that there are several other peculiarities that may be easily understood.

First of all we shall examine the first phase of the evolution, a transient domain, during which the probabilities of the ground and excited states P_a, P_b reach their stationary state values. The time needed for this is a few times Γ^{-1}. Figure 5 depicts the evolution of the distribution function for the first 40 cycles of laser pulses. The

intervals between the pulses are $\tau_1 = 0.03\Gamma^{-1}$ and $\tau_2 = 0.1\Gamma^{-1}$ (so the first 40 cycles amount to $3\Gamma^{-1}$), and the initial distribution is taken to be a Gaussian with a width of $\sigma_{z0} = 5\hbar k$ and normalized to $N = 1000$ for convenience. It can be seen that the distribution develops to be highly asymmetric, because those atoms that enter during the interval τ_1 start moving in the opposite direction as those that enter during τ_2. These two peaks can be clearly seen from the figure. (Because the two intervals are not equal, a smaller number of atoms arrive during τ_1 and the corresponding peak is proportionally smaller.) As the separation of the peaks continues, the smaller peak, whose atoms spend most of their time in the excited state, gradually disperses because the atoms undergo spontaneous emission and reverse their motion in velocity space. After time $3\,\Gamma^{-1}$ has elapsed, it can no longer be distinguished, but the whole distribution is highly asymmetrical with a long tail. Note that if the initial width of the distribution is wider than the separation of the two distinct peaks during this short time, the two peaks will not be resolved at all.

Next we shall examine the evolution of the distribution on a slightly longer scale. After the transient domain discussed earlier, the probabilities P_a, P_b reach their stationary values, and the changing of the average velocity and the width of the distribution becomes constant with each cycle. The shape of the distribution, however, exhibits an asymmetric shape for times much larger than Γ^{-1}. Figure 6 depicts the evolution of the distribution function from the 80th cycle to the 400th cycle of laser pulses. (Note that the orientation of the figure is different from that of Fig. 5 for a clear view.) The intervals between the pulses are $\tau_1 = 0.02\Gamma^{-1}$ and $\tau_2 = 0.1\Gamma^{-1}$ (slightly different from the previous figure). The initial distribution is the same as in Fig. 5. It can be seen that the asymmetry of the distribution persists for times $t \gg \Gamma^{-1}$. This phenomenon may be explained as follows: Atoms enter the interaction region and start accelerating in one direction or the other depending on when they arrived. Occasionally, they make a spontaneous transition when they are in an excited state and reverse their motion in velocity space. This behavior is similar to that of the case of a two-valued fluctuating force as described in Ref. 18. By the time the first transient phase is over (i.e., one peak has already dispersed) the distribution will consist of a peak, which moves at a rate of $2\hbar k$ per cycle in velocity space and a tail, which drags behind. Initially, this tail will consist mainly of atoms that have entered the interaction region during τ_1, but as time progresses, more and more atoms will break off from the large peak whose atoms entered during τ_2 as they, too, spend some of their time in the excited state and undergo spontaneous emission. Thus the leading peak, which bears the characteristics of the original distribution and is made up of atoms that have not yet undergone spontaneous emission (and the corresponding reversal of movement in velocity space), shrinks and the distribution smoothens. This, however, can be expected to happen only if all the atoms have undergone spontaneous emission at least several times. The average number of spontaneous transitions is proportional to Γ and to the average time that an atom spends in its excited state during the interaction. If we assume the atoms to have reached the stationary state with respect to the internal state, this average probability turns out to be

$$\langle P_b \rangle = \frac{2(1 - P_1)P_a^{st}}{\Gamma(\tau_1 + \tau_2)}, \tag{3.3}$$

which can be very small if $\tau_1 \ll \tau_2$. This means that the average time during which an atom continues gathering speed in a certain direction, $(\langle P_b \rangle \Gamma)^{-1}$, may be much larger than Γ^{-1}. As in this particular case, $\langle P_b \rangle \approx 1/3.6$, this peak can be seen to move at a rate of $2\hbar k$ per cycle for times much greater than the spontaneous lifetime of the atoms (see Fig. 6). As more and more atoms fall behind from the original peak, the maximum of the distribution shifts, but the characteristics of the initial distribution are preserved in the form of a steep leading edge for a long time. If the initial distribution is not this narrow, the effects are much less dramatic.

Equation (3.3) also helps in understanding why the broadening of the velocity distribution caused by the recoil of spontaneously emitted photons is negligible compared with the widening caused by the occasional reversal of the force acting on the atom after an event of spontaneous fluorescence. Since the atom spends most of its time in its ground state, the average rate of spontaneous emission is much smaller than Γ. For example, for the distribution of Fig. 6, we would have to include the widening effects of approximately 13 photons per atom emitted in random directions during the whole 400 cycles depicted in the figure, which is negligible.

Note that the physical mechanisms causing the deflection and spreading of the atomic beam in our scheme are very similar to those of Ref. 19, in which bichromatic standing waves were used to slow an atomic beam. The effect of bichromatic standing waves under certain circumstances may be interpreted as the action of counterpropagating Π pulses. Our method has the advantage that, if $\tau_1 \gg \tau_2$, the force acting on the atoms will be large and the dispersive effect of the pulses on the velocity distribution will be small for arbitrarily long times. As the situation corresponding to $\tau_1 \gg \tau_2$ may not be achieved

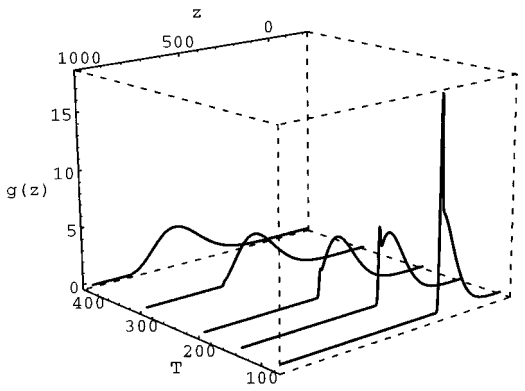

Fig. 6. Evolution of the velocity distribution of atoms $g(z)$ from after the 80th cycle to the 400th cycle. The intervals between the pulses are $\tau_1 = 0.02$, $\tau_2 = 0.1$, the initial Gaussian distribution has been normalized to $N = 1000$, and the initial width is $\sigma_z = 5\hbar k$. Note that the orientation of this figure is different from that of Fig. 5, for a clear view.

with bichromatic standing waves, the usefulness of bichromatic standing waves is clear only for times $t < \Gamma^{-1}$.

B. Splitting of an Atomic Beam

In the previous subsection, we have considered the long-term evolution of the whole ensemble of atoms. The fact that some atoms arrive during the interval τ_1 and thus start moving in the wrong direction was taken into account in the initial distributions [Eqs. (3.1)]. After a long time the difference between these two cases is obviously smeared out by the dispersion of the beam. It has been mentioned, however, that for short times ($t < \Gamma^{-1}$) these two parts of the atomic beam start moving in opposite directions initially. This may cause a clear splitting of the atomic beam.

When the splitting of an atomic beam is desired, the usual requirements are that the beam should be split in two equal parts. They should be separated clearly, as much as possible in velocity space, and they should remain collimated as far as possible. If $\tau_1 = \tau_2$, the number of atoms arriving during these two intervals will be equal. It is also clear from Eqs. (2.20) and (2.14) that in this case the steady-state force is zero, and the change of average momentum is described by the transient term in Eq. (2.20).

Let us now consider the same situation as in Subsection 3.A, i.e., a monochromatic beam of atoms with some initial transversal-velocity spread crossing the interaction region. We shall assume $\tau_1 = \tau_2$ and consider the atoms arriving at the interaction region during τ_1 and τ_2 separately. Those that arrive during the interval τ_2 will see a pulse propagating toward the positive x direction first and will thus begin acquiring momentum in this direction as described by Eq. (2.20). On the other hand, for the atoms that arrive during τ_1, we may consider the pulse propagating toward the negative x direction to be the first in the cycle and still apply Eq. (2.20) with only its sign reversed (i.e., we are now considering two separate cases when applying the equations, as opposed to the approach of Subsection 3.A, in which we combined these two cases in the initial distributions, Eqs. 3.1). The equation describing the dispersion of the distribution is the same in both cases. Thus the two parts of the beam will be deflected in opposite directions, and if the spreading of these two beams can be kept much smaller than this deflection, we have realized a beam splitter. It is for times $t < \Gamma^{-1}$ that the effects of dispersion can be expected to be negligible compared with the separation of these two packets. It is also evident that the initial distribution must be narrower than the separation for the two peaks to be resolved at all.

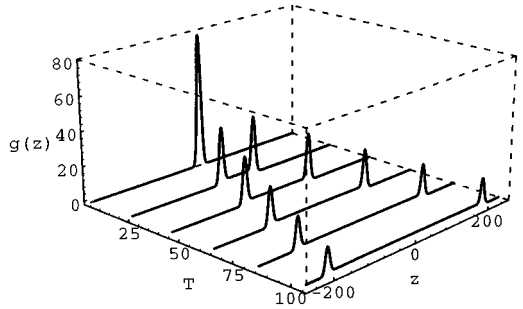

Fig. 7. Evolution of the velocity distribution of atoms for the first 100 cycles of laser pulses when $\tau_1 = \tau_2 = 0.02$. Initial distribution is the same as for Figs. 5 and 6.

Figure 7 shows the evolution of the distribution function of the atoms when $\tau_1 = \tau_2 = 0.02$. For times much shorter than the spontaneous lifetime, the two peaks are clearly resolved. As time progresses, more and more atoms are lost from these peaks owing to spontaneous emission. Note that the time of evolution depicted on the figure is $t = 4\Gamma^{-1}$ (100 cycles) to show how atoms are lost from the peaks as time progresses. For interaction times $t < \Gamma^{-1}$, the separated peaks at the end would not be so much smaller than half the one at the beginning. Making $\tau_1 = \tau_2$ as small as possible increases the efficiency of the beam splitter because the atoms acquire more momentum during unit time as the cycle time decreases. There is no optimum cycle time for efficiency within this model. Decreasing cycle time increases the efficiency without limits, but for very small cycle times the model will cease to be valid because the duration of the pulses will not be negligible compared with the cycle time for a realistic situation, or the two-level atom approximation may not be sufficient for pulses of very short durations and hence very large bandwidths. Also note that this scheme of beam splitting is analogous to the one using bichromatic standing waves.[20]

4. SUMMARY

The effect of a series of counterpropagating, chirped laser pulses on the transversal velocity distribution of an atomic beam has been investigated. The duration of the pulses has been assumed to be very short compared with the spontaneous lifetime of the atoms. For this reason, the effects of spontaneous emission were assumed to be nonnegligible only between the laser pulses. It has been shown that depending on the ratio of the times that elapse between the pulses and interaction time, the atomic beam may be effectively deflected or split into two halves. A strong dispersive effect on the transversal-velocity distribution of the atoms has also been predicted, wherein the atoms interact with the laser pulses for longer than the spontaneous lifetime. It is the conclusion of this paper that such a series of laser pulses may be useful for the manipulation of atomic beams.

ACKNOWLEDGMENT

This research was supported by the Hungarian Scientific Research Fund of the Hungarian Academy of Sciences under contract T 019683.

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