Pulse propagation in a dressed, degenerate system

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We study the propagation of coherent light pulses in a medium of three-level atoms with degenerate ground- and excited-state sublevels in an electromagnetically induced transparency (EIT)-type configuration. Both the strong control field and the weak probe pulse have elliptical polarization, which gives rise to concurrent multipath couplings between the ground-state sublevels and the auxiliary stable state. We derive the probe field susceptibility and show that in general, the probe field propagates in two separate polarization modes, one of which is attenuated, the other of which displays EIT. This generic result is valid provided the atomic medium is prepared to be in a pure quantum state over the ground-state sublevels initially. We also investigate the case when the initial state of the medium is described by an incoherent mixture of ground-state sublevels and show how EIT-like pulse propagation degrades. The possibility of controlling the probe susceptibility matrix with control field polarization provides a convenient tool for probing the quantum state of the medium on the degenerate ground-state sublevels.

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1. INTRODUCTION

It has been known for nearly two decades that coherence between atomic states changes dramatically the susceptibility of a medium: an absorbing medium becomes transparent for a weak probe field in the process of electromagnetically induced transparency (EIT) [1–5]. Based on EIT a number of interesting and important applications have been developed: matched pulse generation in a lambda system [6–10], enhanced nonlinear frequency conversion and field generation [11–20], creation and recall of spatial excitation distribution in dielectric media [21,22], and coherent quantum memory for photons [23,24]. For recent review on EIT-based applications see [25].

The control over the susceptibility of the medium can be further enhanced by extending the EIT concept to multistate systems, such as the double-lambda configuration: two ground states are coupled to two excited states with four coherent laser fields. It has been shown that in a double-lambda one can implement amplification without inversion [26], EIT and matched pulse generation [27,28], resonantly enhanced parametric nonlinear optical processes [29], and phase-dependent resonant nonlinear optics [30].

Another multistate system, which is still simple enough to describe as a combination of lambda systems, is the tripod configuration in which two lower states are coupled to a common excited state with weak probe fields, which is coupled further to an auxiliary state with a strong control field. The tripod system has been analyzed from the point of view of adiabatic state preparation [31], where dark states [32] emerge naturally. Pulse propagation in the tripod system has been studied extensively: the nonlinear coupling between the weak fields has been considered in [33,34]. The propagation of adiabatically varying pulsed fields has been discussed in [35], while propagation of quantized fields under EIT condition has also been studied [36]. The application of the tripod configuration for quantum storage of few photon pulses has been proposed as well [37].

EIT has also been investigated in a number of cases when the states of the lambda atom have Zeeman degenerate substates. A lot of these studies (e.g., [38]) were motivated by the true level structure of alkali atoms, which are used extensively in EIT and related experiments. In [39], it was assumed that the probe and control fields are of identical circular polarization. In this case, the degenerate multilevel system reduces to a set of concurrent simple EIT systems because of selection rules, but the coupling strength of the fields are different for each one due to the difference in Clebsch–Gordan coefficients. It was shown that the multilevel system can be transformed into a simple three-level system under adiabatic conditions. In [40], the effects of Zeeman degeneracy was investigated on the storage of light pulses that can be achieved in lambda systems under EIT conditions. In [41,42], the propagation of adiabatons was investigated for the case of Zeeman degenerate substates of the lambda system, while light propagation in degenerate two-level systems showing EIT and electromagnetically induced absorption phenomena was investigated in [43,44].

In the previous paragraph we have listed several works in which EIT and related phenomena have been studied in systems with degenerate energy levels. However, none of these works considered the situation in which the populated ground-state space is degenerate, and its sublevels are coupled to the auxiliary states(s) through multiple, concurrent pathways. Hence, in this paper we study an EIT-like scheme, in which the ground and excited states are formed of degenerate atomic Zeeman sublevels, and there is a single auxiliary state. Initially the ground-state space is populated. The ground-state and excited-state sublevels are coupled by an elliptically polarized weak probe field, whereas the excited-state sublevels are coupled to the auxiliary state by a strong, elliptically polarized control (dressing) field. We have studied various coherent population transfer methods in this level configuration in some of our previous works [45–48], however, field propagation effects have not been considered. The consequence of such a complex scheme is increased controllability: the susceptibility for the probe field is controlled by not only the strong coupling field, but by the initially prepared ground state of the system as well, which can be a pure or a mixed quantum state. In Section 2, we define the model system. We provide a qualitative analysis of the probe field propagation in Section 3. We derive analytically the probe field susceptibility matrix in Section 4. We show the behavior of the susceptibility through specific examples in Section 5. Finally, the results are summarized in Section 6.

2. MODEL SYSTEM

We consider the coupling configuration shown in Fig. 1: three sets of angular momentum states are coupled by elliptically polarized CW light fields in a generalized lambda configuration. A probe light field couples the degenerate angular momentum states of the g-set ($J_g = 2$) to those of the e-set ($J_e = 1$). The states of the e-set are coupled by a strong control field to a single $J_a = 0$ auxiliary state. Initially the system is prepared in the coherent superposition of the states from the g-set with $M_g = -2, 0, +2$.

The coupling fields may be decomposed as

$$\vec{E}_k(t) = \frac{1}{2}([\boldsymbol{e}_- \mathcal{E}_-^k(t) + \boldsymbol{e}_+ \mathcal{E}_+^k(t)]e^{-i\omega_k t} + \text{c.c.}), \tag{1}$$

where k stands for the control (c) or probe (p) fields, respectively. Here, $\boldsymbol{e}_q$ is a unit polarization vector, of helicity $q = \pm 1$ ($\boldsymbol{e}_+ = -(\hat{x} + i\hat{y})/\sqrt{2}$ and $\boldsymbol{e}_- = (\hat{x} - i\hat{y})/\sqrt{2}$), as is appropriate for the expression of elliptical polarization as a combination of circular polarizations σ_- and σ_+ [49,50]. The atomic energy eigenstates are denoted by $|b, J_b, M_b\rangle$, where the label $b = a, g, e$ identifies the atomic energy level and J_b, M_b are the usual angular momentum quantum numbers. Using the Wigner–Eckart theorem [50,51], the Rabi frequencies $\Omega^p_{M_e M_g}$ associated with the electric dipole transitions between the atomic energy eigenstates $|g, J_g, M_g\rangle$ and $|e, J_e, M_e\rangle$ are defined through the relation

$$\hbar\Omega^p_{M_e M_g}(t) = -\mathcal{E}^p_q(t)\langle e, J_e, M_e|d\boldsymbol{e}_q|g, J_g, M_g\rangle, = \hbar\Omega^p_q(t)\xi^{M_e}_{M_g} e^{i\varphi_q} \tag{2}$$

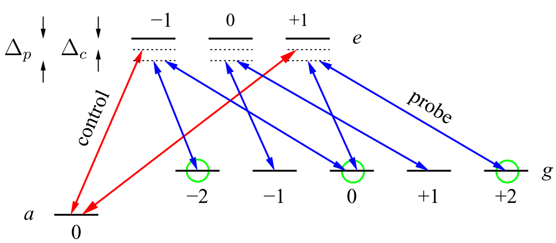

Fig. 1. (Color online) Scheme of the coupling configuration. The ground-state manifold g consists of the sublevels of a five-fold degenerate $J_g = 2$ state. The sublevels are coupled by an elliptically polarized weak probe field to the excited-state manifold e, which consists of the sublevels of a $J_e = 1$ state. The probe field is detuned by Δ_p from exact resonance. The excited-state manifold e is coupled further by an elliptically polarized strong control field to a $J_a = 0$ auxiliary state. The control field is detuned by Δ_c from exact resonance. The circles on the $M_g = -2, 0, +2$ sublevels represent the initial occupations.

with $\xi^{M_e}_{M_g}$ being a Clebsch–Gordan coefficient $\xi^{M_e}_{M_g} = (J_g M_g, 1q|J_e M_e)$ [51], d is the electronic dipole operator. For $J_g = 2$ and $J_e = 1$ we have $\xi^{-1}_{-2} = \xi^{+1}_{+2} \equiv \xi^1_2 = \sqrt{6/10}$ and $\xi^{-1}_0 = \xi^{+1}_0 \equiv \xi^1_0 = \sqrt{1/10}$. The phase φ_q is the sum of the phases associated with the field amplitude $\mathcal{E}^p_q(t)$ *and* the reduced matrix element $d = \langle e, J_e \| d \| g, J_g\rangle$. The Rabi frequencies $\Omega^c_{M_e M_a}$ for the transitions $|a, J_a, M_a\rangle \leftrightarrow |e, J_e, M_e\rangle$ are defined in a similar way:

$$\hbar\Omega^c_{M_e M_a}(t) = -\mathcal{E}^c_q(t)\langle e, J_e, M_e|d\boldsymbol{e}_q|a, J_a, M_a\rangle, = \hbar\Omega^c_q(t)\zeta^{M_e}_{M_a} e^{i\vartheta_q} \tag{3}$$

with $\zeta^{M_e}_{M_a} = (J_a M_a, 1q|J_e M_e)$. For $J_a = 0$ and $J_e = 1$ we have $\zeta^{-1}_0 = \zeta^{+1}_0 = 1$. The definition of ϑ_q is similar to that of φ_q.

The system is described by the Master equation

$$\frac{d}{dt}\varrho = -\frac{i}{\hbar}[H, \varrho] + \mathcal{L}_R(\varrho) + \mathcal{L}_\phi(\varrho). \tag{4}$$

In the Hamiltonian only the relevant states are included from the degenerate angular momentum energy levels. In the ordered basis $\{|a,0\rangle, |e,-1\rangle, |e,+1\rangle, |g,-2\rangle, |g,0\rangle, |g,+2\rangle\}$ (for brevity we introduce the notation $|b, J_b, M_b\rangle \equiv |b, M_b\rangle$) the Hamiltonian is given by

$$H = \hbar \begin{bmatrix} 0 & \frac{1}{2}\Omega^{c\dagger} & 0 \\ \frac{1}{2}\Omega^c & \Delta_c \boldsymbol{I}_2 & \frac{1}{2}\Omega^p \\ 0 & \frac{1}{2}\Omega^{p\dagger} & \delta \boldsymbol{I}_3 \end{bmatrix}, \tag{5}$$

where $\boldsymbol{I}_n$ are identity matrices of dimension n, the detunings are defined as $\Delta_p = \omega_{eg} - \omega_p$, $\Delta_c = \omega_{ea} - \omega_c$, $\delta = \Delta_c - \Delta_p$; $\omega_{ab} = (\varepsilon_a - \varepsilon_b)/\hbar$, ε_n being the energy of the level n. The control field coupling matrix reads

$$\Omega^c = \begin{bmatrix} \Omega^c_- \zeta^1_0 e^{i\vartheta_-} \\ \Omega^c_+ \zeta^1_0 e^{i\vartheta_+} \end{bmatrix}, \tag{6}$$

whereas the probe field coupling matrix is given by

$$\Omega^p = \begin{bmatrix} \Omega^p_+ \xi^1_2 e^{i\varphi_+} & \Omega^p_- \xi^1_0 e^{i\varphi_-} & 0 \\ 0 & \Omega^p_+ \xi^1_0 e^{i\varphi_+} & \Omega^p_- \xi^1_2 e^{i\varphi_-} \end{bmatrix}. \tag{7}$$

The radiative decay term $\mathcal{L}_R(\varrho)$ in the Master equation (4) is defined by

$$\mathcal{L}_R(\varrho) = \frac{\Gamma_{eg}}{2}\left[2\sum_{q=0,\pm} M_q \varrho M_q^\dagger - |e\rangle\langle e|\varrho - \varrho|e\rangle\langle e|\right] + \frac{\Gamma_{ea}}{2}\left[2\sum_{q=0,\pm} N_q \varrho N_q^\dagger - |e\rangle\langle e|\varrho - \varrho|e\rangle\langle e|\right], \tag{8}$$

where the matrices M_q (N_q) describe spontaneous decay from the degenerate manifolds e to g (e to the state a). They have the property $\sum_q M_q^\dagger M_q = |e\rangle\langle e|$ and $\sum_q N_q^\dagger N_q = |e\rangle\langle e|$, where $|e\rangle\langle e| \equiv \sum_{M_e} |e, M_e\rangle\langle e, M_e|$. We do not need their explicit form in the rest of the calculations. The quantities Γ_{eg} and Γ_{ea} are decay rates from the state manifold e to a and g, respectively. The dephasing term $\mathcal{L}_\phi(\varrho)$ of Eq. (4) describes phase decay between the manifolds $a \leftrightarrow g$, $e \leftrightarrow g$, and $e \leftrightarrow a$. It reads

$$\mathcal{L}_\phi(\varrho) = \frac{\Gamma_g^\phi}{2}[2|g\rangle\langle g|\varrho|g\rangle\langle g| - |g\rangle\langle g|\varrho - \varrho|g\rangle\langle g|] + \frac{\Gamma_a^\phi}{2}[2|a\rangle\langle a|\varrho|a\rangle\langle a| - |a\rangle\langle a|\varrho - \varrho|a\rangle\langle a|], \quad (9)$$

where $\Gamma_{a,g}^\phi$ are dephasing rates. The projector $|b\rangle\langle b|$ is defined as

$$|b\rangle\langle b| \equiv \sum_{M_b} |b, M_b\rangle\langle b, M_b|. \quad (10)$$

In the following we shall use the notation ϱ_{ab}, which refers to a submatrix of ϱ given by $\varrho_{ab} = |a\rangle\langle a|\varrho|b\rangle\langle b|$.

3. QUALITATIVE DESCRIPTION IN THE COUPLED–UNCOUPLED STATE PICTURE

In this section we present a qualitative picture about the excitation process from the atomic point of view. Our approach is based on the Morris–Shore (MS) transformation [52], which is applied now to the control field transition. The MS transformation renders the complex coupling between two degenerate state manifolds (ground and excited states) to a set of two level coupled systems and uncoupled states. This is achieved by redefining the basis vectors for the ground and excited states, separately. In our system, for the control field MS transformation, the auxiliary state $|a\rangle$ plays the role of the ground state and the excited states are $|e, \pm 1\rangle$. The structure of the MS unitary transformation matrix U is rather simple:

$$U = \begin{bmatrix} 1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_3 \end{bmatrix}, \quad (11)$$

where A is a 2×2 unitary matrix, the precise definition is not needed for the qualitative analysis. In the basis defined by Eq. (11), the atomic Hamiltonian reads

$$H_{\mathrm{MS}} = U^\dagger H U = \hbar \begin{bmatrix} 0 & \frac{1}{2}\Omega^{c\dagger}A & 0 \\ \frac{1}{2}A^\dagger\Omega^c & \Delta_c I_2 & \frac{1}{2}A^\dagger\Omega^p \\ 0 & \frac{1}{2}\Omega^{p\dagger}A & \delta I_3 \end{bmatrix}. \quad (12)$$

The matrix A is chosen such that the control field coupling matrix takes the simple form

$$\begin{bmatrix} \Omega^c \\ 0 \end{bmatrix} = A^\dagger \Omega^c, \quad (13)$$

i.e., we have a coupled and an uncoupled state in the excited-state manifold. Simultaneously, the probe field coupling matrix gets redefined, $\tilde{\Omega}^p = A^\dagger\Omega^p$. In general, all six matrix elements of the 2×3 matrix $\tilde{\Omega}^p$ are nonzero. The result of the so defined MS transformation is depicted in Fig. 2: the auxiliary state is coupled to one of the transformed excited states $|e_1'\rangle$, which is coupled further to all three initially populated ground states with the probe field. The other transformed excited state $|e_2'\rangle$ is coupled only to the ground states. There are two typical coupling schemes present simultaneously: (1) an EIT-like lambda-coupling configuration $|a\rangle \leftrightarrow |e_1'\rangle \leftrightarrow |g\,\text{states}\rangle$, and (2) a coupled two-level system $|g\,\text{states}\rangle \leftrightarrow |e_2'\rangle$.

In this paper we are going to study the propagation of a weak probe pulse while the control field is kept constant.

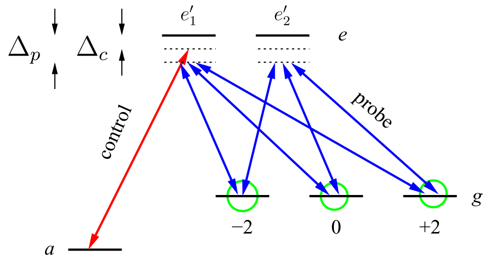

Fig. 2. (Color online) MS transformation for the control field transition results in a pair of coupled–uncoupled excited states to the auxiliary state.

The field components associated with the two-level system get absorbed in the course of propagation, whereas the field components contributing to the lambda system display EIT-type propagation. In the next section we derive explicitly the susceptibility of the dressed system in the weak probe field limit.

4. CALCULATION OF THE PROBE FIELD SUSCEPTIBILITY

The atomic response to the probe field is a microscopic polarization. In our case, the polarization induced by the probe field differs significantly from the polarization of the bare atoms due to the interaction with the strong control (dressing) laser. The probe field consists of $\sigma_\pm$ components; therefore, the induced atomic polarization also has these two components. Starting from the definition

$$\vec{P} = \frac{1}{2}[\vec{P}^{(+)} + \vec{P}^{(-)}] = \mathcal{N}\,\mathrm{Tr}\{\varrho d\}, \quad (14)$$

one finds that both components of the atomic polarization consist of the sum of two terms: for the σ_- component, the sum includes the contributions of the coherences between the states $|e, -1\rangle \leftrightarrow |g, 0\rangle$ and $|e, +1\rangle \leftrightarrow |g, +2\rangle$, whereas for the σ_+ component, it includes the coherences between the states $|e, -1\rangle \leftrightarrow |g, -2\rangle$ and $|e, +1\rangle \leftrightarrow |g, 0\rangle$. Therefore, the slowly varying positive frequency components $\mathcal{P}_\pm^{(+)}(t, z)$ of the atomic polarization vector, $P_\pm^{(+)}(t, z) = \mathcal{P}_\pm^{(+)} \exp(i[k_p z - \omega_p t])$ and $\vec{P}^{(+)}(t, z) = P_+^{(+)}(t, z)\boldsymbol{e}_+ + P_-^{(+)}(t, z)\boldsymbol{e}_-$, are given by the equations

$$\mathcal{P}_-^{(+)}(t, z) = 2\mathcal{N} d(\xi_0^1 \varrho_{e_{-1}, g_0} + \xi_2^1 \varrho_{e_{+1}, g_{+2}}), \quad (15a)$$

$$\mathcal{P}_+^{(+)}(t, z) = 2\mathcal{N} d(\xi_2^1 \varrho_{e_{-1}, g_{-2}} + \xi_0^1 \varrho_{e_{+1}, g_0}), \quad (15b)$$

where $\mathcal{N}$ denotes the atomic density, $\varrho_{e_{q1}, g_{q2}}$ are slowly varying density matrix elements. The CG coefficients in front of the atomic coherences multiply the reduced dipole moment matrix element d to yield the dipole moment strength associated with the particular transition. We conclude that in order to obtain the linear susceptibility matrix for the probe field we have to calculate four coherences of the atomic system.

As we have said in the introduction, it is assumed that the probe field is much weaker than the control field. Furthermore, initially only the ground-state manifold ϱ_{gg} is populated. Therefore, in case of CW excitations both for the control and probe fields, in order to obtain the steady-state solution of the Master equation (4), it is sufficient to take into account the effect of the probe field up to first order. The structure of the equations for the matrix elements of the density matrix imply that up to first order in the probe field amplitude, the ground-state manifold ϱ_{gg} is preserved, the coherences ϱ_{eg} and ϱ_{ag} depend linearly on the probe field, finally the matrix elements ϱ_{ee}, ϱ_{ea}, ϱ_{aa} are all equal to zero. Therefore, we can establish a linear relation between the atomic coherences and the probe field strengths. Consequently, we can define a linear susceptibility matrix χ for the probe field through the relation

$$\begin{bmatrix} \mathcal{P}_-^{(+)}(t,z;\Delta_p) \\ \mathcal{P}_+^{(+)}(t,z;\Delta_p) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \chi_{--} & \chi_{-+} \\ \chi_{+-} & \chi_{++} \end{bmatrix} \begin{bmatrix} \mathcal{E}_-^p(t,z;\Delta_p) \\ \mathcal{E}_+^p(t,z;\Delta_p) \end{bmatrix}, \tag{16}$$

where ε_0 denotes the vacuum permittivity. Here, the argument Δ_p signifies that the quantities are evaluated at a certain frequency ω_p, which corresponds to the probe field detuning Δ_p, defined under Eq. (5).

It follows from Eq. (4) that the equations for the relevant density matrix elements are given by

$$i\frac{\partial}{\partial t}\varrho_{a_0,g_{-2}} = \frac{1}{2}\Omega_-^{c*}\varrho_{e_{-1},g_{-2}} + \frac{1}{2}\Omega_+^{c*}\varrho_{e_{+1},g_{-2}} - (\delta + i\gamma_{ag}^{\phi})\varrho_{a_0,g_{-2}}, \tag{17a}$$

$$i\frac{\partial}{\partial t}\varrho_{a_0,g_0} = \frac{1}{2}\Omega_-^{c*}\varrho_{e_{-1},g_0} + \frac{1}{2}\Omega_+^{c*}\varrho_{e_{+1},g_0} - (\delta + i\gamma_{ag}^{\phi})\varrho_{a_0,g_0}, \tag{17b}$$

$$i\frac{\partial}{\partial t}\varrho_{a_0,g_{+2}} = \frac{1}{2}\Omega_-^{c*}\varrho_{e_{-1},g_{+2}} + \frac{1}{2}\Omega_+^{c*}\varrho_{e_{+1},g_{+2}} - (\delta + i\gamma_{ag}^{\phi})\varrho_{a_0,g_{+2}}, \tag{17c}$$

$$i\frac{\partial}{\partial t}\varrho_{e_{-1},g_{-2}} = \frac{1}{2}\Omega_-^{c}\varrho_{a_0,g_{-2}} + \frac{1}{2}\Omega_-^{p}\xi_0^1\varrho_{g_0,g_{-2}} + \frac{1}{2}\Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_{-2}} + [\Delta_p - i\gamma_{eg}^{\phi}]\varrho_{e_{-1},g_{-2}}, \tag{17d}$$

$$i\frac{\partial}{\partial t}\varrho_{e_{-1},g_0} = \frac{1}{2}\Omega_-^{c}\varrho_{a_0,g_0} + \frac{1}{2}\Omega_-^{p}\xi_0^1\varrho_{g_0,g_0} + \frac{1}{2}\Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_0} + [\Delta_p - i\gamma_{eg}^{\phi}]\varrho_{e_{-1},g_0}, \tag{17e}$$

$$i\frac{\partial}{\partial t}\varrho_{e_{+1},g_0} = \frac{1}{2}\Omega_+^{c}\varrho_{a_0,g_0} + \frac{1}{2}\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_0} + \frac{1}{2}\Omega_+^{p}\xi_0^1\varrho_{g_0,g_0} + [\Delta_p - i\gamma_{eg}^{\phi}]\varrho_{e_{+1},g_0}, \tag{17f}$$

$$i\frac{\partial}{\partial t}\varrho_{e_{+1},g_{+2}} = \frac{1}{2}\Omega_+^{c}\varrho_{a_0,g_{+2}} + \frac{1}{2}\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_{+2}} + \frac{1}{2}\Omega_+^{p}\xi_0^1\varrho_{g_0,g_{+2}} + [\Delta_p - i\gamma_{eg}^{\phi}]\varrho_{e_{+1},g_{+2}}, \tag{17g}$$

where $\gamma_{eg}^{\phi} = (\Gamma_{eg} + \Gamma_g^{\phi})/2$, $\gamma_{ag}^{\phi} = (\Gamma_a^{\phi} + \Gamma_g^{\phi})/2$, and we have taken into account the vanishing of ϱ_{ee}, ϱ_{ae}, and ϱ_{aa} to first order in $\Omega_{\pm}^{p}$. We are looking for the steady-state solution of this set of equations for fixed parameters, hence we set the left-hand sides to zero. Then one can easily solve Eqs. (17a)–(17c). Substituting these results to the remaining four equations for ϱ_{e_m,g_n}, we arrive at the steady-state solutions

$$\varrho_{e_{-1},g_{-2}} = \frac{1}{D}\{-Y\cdot(\Omega_-^{p}\xi_0^1\varrho_{g_0,g_{-2}} + \Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_{-2}}) + Z\cdot(\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_{-2}} + \Omega_+^{p}\xi_0^1\varrho_{g_0,g_{-2}})\}, \tag{18a}$$

$$\varrho_{e_{-1},g_0} = \frac{1}{D}\{-Y\cdot(\Omega_-^{p}\xi_0^1\varrho_{g_0,g_0} + \Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_0}) + Z\cdot(\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_0} + \Omega_+^{p}\xi_0^1\varrho_{g_0,g_0})\}, \tag{18b}$$

$$\varrho_{e_{+1},g_0} = \frac{1}{D}\{-X\cdot(\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_0} + \Omega_+^{p}\xi_0^1\varrho_{g_0,g_0}) + Z^*\cdot(\Omega_-^{p}\xi_0^1\varrho_{g_0,g_0} + \Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_0})\}, \tag{18c}$$

$$\varrho_{e_{+1},g_{+2}} = \frac{1}{D}\{-X\cdot(\Omega_-^{p}\xi_2^1\varrho_{g_{+2},g_{+2}} + \Omega_+^{p}\xi_0^1\varrho_{g_0,g_{+2}}) + Z^*\cdot(\Omega_-^{p}\xi_0^1\varrho_{g_0,g_{+2}} + \Omega_+^{p}\xi_2^1\varrho_{g_{-2},g_{+2}})\}, \tag{18d}$$

where $D = 2B(AB + |\Omega^c|^2/4)$, $X = AB + |\Omega_-^c|^2/4$, $Y = AB + |\Omega_+^c|^2/4$, $Z = \Omega_-^c\Omega_+^{c*}/4$, with $A = \delta + i\gamma_{ag}^{\phi}$, $B = \Delta_p - i\gamma_{eg}^{\phi}$, and $|\Omega^c|^2 = |\Omega_-^c|^2 + |\Omega_+^c|^2$. In the above equations for the density matrix elements ϱ_{eg}, we have a linear dependence for the probe field and the ground-state populations and coherences are also included.

We can insert the coherences of Eqs. (18) into (15). Comparing Eqs. (15) and (16) one finds the following explicit expressions for the susceptibility matrix elements:

$$\chi_{--} = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\{X\cdot(\xi_2^1)^2\varrho_{g_{+2},g_{+2}} + Y\cdot(\xi_0^1)^2\varrho_{g_0,g_0} - Z\xi_0^1\xi_2^1\varrho_{g_{+2},g_0} - Z^*\xi_0^1\xi_2^1\varrho_{g_0,g_{+2}}\}, \tag{19a}$$

$$\chi_{-+} = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\{X\xi_2^1\xi_0^1\varrho_{g_0,g_{+2}} + Y\xi_0^1\xi_2^1\varrho_{g_{-2},g_0} - Z\cdot(\xi_0^1)^2\varrho_{g_0,g_0} - Z^*\cdot(\xi_2^1)^2\varrho_{g_{-2},g_{+2}}\}, \tag{19b}$$

$$\chi_{+-} = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\{X\xi_2^1\xi_0^1\varrho_{g_{+2},g_0} + Y\xi_0^1\xi_2^1\varrho_{g_0,g_{-2}} - Z\cdot(\xi_2^1)^2\varrho_{g_{+2},g_{-2}} - Z^*\cdot(\xi_0^1)^2\varrho_{g_0,g_0}\}, \tag{19c}$$

$$\chi_{++} = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\{X\cdot(\xi_0^1)^2\varrho_{g_0,g_0} + Y\cdot(\xi_2^1)^2\varrho_{g_{-2},g_{-2}} - Z\xi_0^1\xi_2^1\varrho_{g_0,g_{-2}} - Z^*\xi_0^1\xi_2^1\varrho_{g_{-2},g_0}\}. \tag{19d}$$

For the propagation of electromagnetic waves in a medium consisting of the multilevel system under consideration, we employ the slowly varying envelope approximation and the emerging equation is transformed to a retarded frame, $\tau = t - z/c$. We arrive at the expression

$$\frac{\partial}{\partial z}\begin{bmatrix} \mathcal{E}^p_-(\tau,z;\Delta_p) \\ \mathcal{E}^p_+(\tau,z;\Delta_p) \end{bmatrix} = i\frac{k_0}{2}\begin{bmatrix} \chi_{--} & \chi_{-+} \\ \chi_{+-} & \chi_{++} \end{bmatrix}\begin{bmatrix} \mathcal{E}^p_-(\tau,z;\Delta_p) \\ \mathcal{E}^p_+(\tau,z;\Delta_p) \end{bmatrix}. \quad (20)$$

The solution of this equation is simply

$$\vec{\mathcal{E}}^p(\tau,z;\Delta_p) = \exp\left(i\frac{k_0}{2}\chi\cdot(z-z_0)\right)\vec{\mathcal{E}}^p(\tau,z_0;\Delta_p), \quad (21)$$

where $\vec{\mathcal{E}}^p(\tau,z;\Delta_p) = [\mathcal{E}^p_-(\tau,z;\Delta_p) \quad \mathcal{E}^p_+(\tau,z;\Delta_p)]^T$. Using the eigensystem of the susceptibility matrix χ, the evaluation of the above equation can always be done in a straightforward manner. In general, the susceptibility matrix Eq. (19) is non-Hermitian. Therefore, it has different right and left eigensystems, which form an orthonormal basis

$$\chi(\Delta_p)\vec{S}^R_i(\Delta_p) = \chi_i\vec{S}^R_i(\Delta_p), \quad (22a)$$

$$\vec{S}^L_i(\Delta_p)\chi(\Delta_p) = \chi_i\vec{S}^L_i(\Delta_p), \quad (22b)$$

$$\vec{S}^L_i(\Delta_p)\cdot\vec{S}^R_i(\Delta_p) = \delta_{ij}, \quad (22c)$$

for $i = 0, 1$. Expanding the susceptibility matrix in this basis, the solution Eq. (21) of the propagation equation for a pulsed initial field reads

$$\vec{\mathcal{E}}^p(\tau,z) = \int \mathrm{d}\Delta_p\left\{\sum_{j=0}^{1}\exp\left(i\frac{k_0}{2}\chi_j(\Delta_p)\cdot(z-z_0)\right)\vec{S}^R_j(\Delta_p)\circ\vec{S}^L_j(\Delta_p)\right\} \cdot\vec{\mathcal{E}}^p(\tau,z_0;\Delta_p), \quad (23)$$

where $\vec{x}\circ\vec{y}$ denotes the diadic product of two vectors. The mode functions $\vec{S}^Q_i(\Delta_p)$ depend on the probe field detuning Δ_p; therefore, if a narrow bandwidth pulse is injected into the medium, the transmitted component of the field will have a frequency-dependent polarization. In the next section we shall study some special examples to field propagation in our system.

5. FIELD PROPAGATION

The susceptibility matrix of Eq. (19) can be controlled in two ways: (1) changing the polarization state of the control field, (2) changing the initial quantum state on the ground-state manifold. There are a number of proposals and some experimental works for preparing pure states in Zeeman sublevel manifolds [45–48,53–58]. Hence, first we study some special cases of the susceptibility matrix for pure initial ground state, then we turn to the case of general, nonpure initial state.

A. Pure Initial Ground State

For pure superposition initial ground state $\varrho_{gg} = |\psi_g\rangle\langle\psi_g|$, where $\psi_g = [c_{g_{-2}} c_{g_0} c_{g_{+2}}]^T$ and the state is normalized $\sum_m |c_{g_m}|^2 = 1$, the susceptibility matrix Eq. (19) can be written in the form

$$\chi = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\begin{bmatrix} ABv_1 + |a|^2 & ABv_2 - a^*b \\ ABv_2^* - ab^* & ABv_3 + |b|^2 \end{bmatrix} \quad (24)$$

with $v_1 = (\xi^1_2)^2|c_{g_{+2}}|^2 + (\xi^1_0)^2|c_{g_0}|^2$, $v_3 = (\xi^1_2)^2|c_{g_{-2}}|^2 + (\xi^1_0)^2|c_{g_0}|^2$, $v_2 = \xi^1_0\xi^1_2(c_{g_0}c^*_{g_{+2}} + c_{g_{-2}}c^*_{g_0})$, $a = (\Omega^c_+\xi^1_0 c_{g_0} - \Omega^c_-\xi^1_2 c_{g_{+2}})/2$, $b = (\Omega^c_-\xi^1_0 c_{g_0} - \Omega^c_+\xi^1_2 c_{g_{-2}})/2$. The characteristic polynomial of the susceptibility matrix in Eq. (24) determines the main features of the propagating modes

$$\chi^2 - \chi\cdot[AB\cdot(v_1+v_3) + |a|^2 + |b|^2] + (AB)^2[v_1v_3 - |v_2|^2] + AB\cdot[v_3|a|^2 + v_1|b|^2 + v_2ab^* + v_2^*a^*b] = 0, \quad (25)$$

where the prefactor $\mathcal{N}d^2/\hbar\varepsilon_0 D$ is omitted. In many experimental cases it is a good approximation to set $\gamma^\phi_{ag} = 0$. In this limit, the parameter A is equal to the two-photon detuning, $A = \delta$. Hence, at two-photon resonance $A = 0$, the characteristic polynomial of Eq. (25) takes the simple form

$$\chi^2 - \chi\cdot[|a|^2 + |b|^2] = 0. \quad (26)$$

It follows that the eigensystem for the susceptibility matrix of Eq. (24) at two-photon resonance reads

$$\chi_0 = 0, \qquad \vec{S}_0 = \frac{1}{\sqrt{|a|^2+|b|^2}}\begin{bmatrix} b \\ a \end{bmatrix}, \quad (27a)$$

and

$$\chi_1 = \frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}(|a|^2+|b|^2), \qquad \vec{S}_1 = \frac{1}{\sqrt{|a|^2+|b|^2}}\begin{bmatrix} a^* \\ -b^* \end{bmatrix}, \quad (27b)$$

provided that a and b do not vanish simultaneously. Since in this special case the susceptibility matrix is symmetric, the left- and right-hand-side eigenvectors coincide. The eigenvectors define the polarization state of the propagating modes. In accordance with the qualitative analysis in Section 3, we find a zero and a nonzero eigenvalue. The field mode assigned to the eigenvalue zero is characterized by the associated eigenvector $\vec{S}_0$. This field mode propagates without attenuation. The other field mode is assigned with a nonzero, complex eigenvalue χ_1. The real part of χ_1 describes a phase shift, whereas the imaginary part an attenuation, such as in a two-level system.

The two field modes are orthogonal to each other; hence, in principle they can be distinguished with measurements. The upper/lower components of the eigenvectors in Eqs. (27a) and (27b) define the left-/right-circular-polarization components of the modes, respectively. In Table 1 we show the Stokes parameters associated with the modes in Eqs. (27a) and (27b) for unit field intensity. We use the definitions $Q = -2\Re\{\mathcal{E}^{p*}_-\mathcal{E}^p_+\}/\mathcal{I}^p$, $U = 2\Im\{\mathcal{E}^{p*}_-\mathcal{E}^p_+\}/\mathcal{I}^p$, and $V = (|\mathcal{E}^p_-|^2 - |\mathcal{E}^p_+|^2)/\mathcal{I}^p$, with $\mathcal{I}^p = |\mathcal{E}^p_-|^2 + |\mathcal{E}^p_+|^2$. We conclude that the two modes are clearly distinguishable.

If both a and b vanish simultaneously, the characteristic polynomial Eq. (25) takes the form

$$\chi^2 - \chi\cdot[AB\cdot(v_1+v_3)] + (AB)^2[v_1v_3 - |v_2|^2] = 0. \quad (28)$$

The condition for this situation can be summarized in the relation

Table 1. Stokes Parameters Associated with the Polarization States in Eqs. (27a) and (27b)

	mode_0	mode_1
Q	$-[2/(\|a\|^2+\|b\|^2)]\Re\{b^*a\}$	$-Q_0$
U	$[2/(\|a\|^2+\|b\|^2)]\Im\{b^*a\}$	$-U_0$
V	$(\|b\|^2-\|a\|^2)/(\|a\|^2+\|b\|^2)$	$-V_0$

$$\frac{\Omega_+^c}{\Omega_-^c}=\frac{\xi_0^1 c_{g_0}}{\xi_2^1 c_{g_{-2}}}=\frac{\xi_2^1 c_{g_{+2}}}{\xi_0^1 c_{g_0}}. \tag{29}$$

The second equation implies $6c_{g_{-2}}c_{g_{+2}}=c_{g_0}^2 5$. At two-photon resonance $A=0$, Eq. (28) simplifies to $\chi^2=0$, i.e., we have full transparency: the probe field propagates without attenuation irrespective to its polarization state.

An interesting special case occurs when $\xi_2^1 c_{g_{-2}}=\xi_2^1 c_{g_{+2}}=\pm\xi_0^1 c_{g_0}$, i.e., $c_{g_{-2}}=c_{g_{+2}}=\sqrt{1/8}$ and $c_{g_0}=\pm\sqrt{6/8}$. Then $a=\pm(\Omega_+^c\mp\Omega_-^c)\sqrt{3/160}=\mp b$, $v_1=v_3=\pm v_2\equiv v=3/20$, and the characteristic polynomial in Eq. (25) simplifies to

$$\chi^2-\chi\cdot[AB\cdot 3/10+(\Omega_+^c\mp\Omega_-^c)^2\cdot 3/80]=0. \tag{30}$$

This equation is valid for any value of A (δ); therefore, one of the modes propagates without attenuation and phase shift ($\chi_0=0$ for any δ), i.e., the group velocity is equal to the speed of light. The polarization state of the $\chi_0=0$ mode reads

$$\vec{S}_0=\frac{1}{\sqrt{2(1\mp\sin(2\varphi^c)\cos(\vartheta^c))}}\begin{bmatrix}\sin\varphi^c\mp\cos\varphi^c e^{i\vartheta^c}\\ \mp(\sin\varphi^c\mp\cos\varphi^c e^{i\vartheta^c})\end{bmatrix}\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \mp 1\end{bmatrix}, \tag{31}$$

where $\vartheta^c=\vartheta_+^c-\vartheta_-^c$. The Stokes vector associated to this polarization state is given by $[Q,U,V]=[\pm 1,0,0]$, i.e., we have a linearly polarized field, parallel to the $\hat{y}$ ($Q=-1$) or $\hat{x}$ ($Q=+1$) axis, respectively. This result is valid for $\Omega_-^c\neq\pm\Omega_+^c$. The situation $\Omega_-^c=\pm\Omega_+^c$ is the special case of vanishing a and b, which was studied in the previous paragraph. In this latter case, one of the eigenvalues is equal to zero $\chi_0=0$ for any Δ_p, this mode is decoupled from the atomic system. The other one for $\Delta_c=0$ reads

$$\chi_1=\frac{\mathcal{N}d^2AB}{\hbar\varepsilon_0 D}=\frac{3\mathcal{N}d^2}{20\hbar\varepsilon_0}\times\frac{\Delta_p\left(\Delta_p^2-\frac{|\Omega^c|^2}{4}\right)+i\Delta_p^2\gamma_{eg}^\phi}{\Delta_p^2(\gamma_{eg}^\phi)^2+\left(\frac{|\Omega^c|^2}{4}-\Delta_p^2\right)^2}, \tag{32}$$

which, apart from a constant multiplier, is precisely the same as the susceptibility for the three-level nondegenerate EIT. In summary, for the special atomic superposition $c_{g_{-2}}=c_{g_{+2}}=\sqrt{1/8}$ and $c_{g_0}=\pm\sqrt{6/8}$, and specific control field $\Omega_-^c=\pm\Omega_+^c$, there is a fast and a slow mode in the medium. The group velocity for the fast mode is the speed of light, whereas for the slow mode it can be significantly less than the speed of light, determined by Eq. (32). This behavior is very similar to the pulse propagation in a tripod system under EIT conditions [35,36].

In order to illustrate the behavior of the susceptibility we made a series of plots shown in Fig. 3. For the simulations we chose the parameters $\Gamma_{eg}=0.4\Gamma_e$, $\Gamma_{ea}=0.6\Gamma_e$, $\Omega^c=2\Gamma_e$, $\gamma_{ag}^\phi=0$, and $\Delta_c=0$. First we have checked the validity of the approximate analytical solutions for the density matrix elements in Eqs. (18a)–(18d): to this end, we solved numerically the full Master equation (4), and compared the obtained density matrix elements with the analytical ones Eqs. (18a)–(18d). In the test runs we have varied the polarization state of both the control and probe fields. The polarization state of the control field is defined through the relations $\mathcal{E}_-^c=\mathcal{E}^c\cos(\varphi^c)e^{i\vartheta_-^c}$ and $\mathcal{E}_+^c=\mathcal{E}^c\sin(\phi^c)e^{i\vartheta_+^c}$. We chose φ^c in the range $(0.05\pi\cdots 0.45\pi)$, so between a nearly circularly polarized and linearly polarized fields, and we set $\vartheta_\pm^c=0$. In these runs we have set $\Omega^p=0.05\Gamma_e$. We have performed several dozen of runs and found agreement between the analytical and numerical solutions within a few percents. Then we turned to the computation of the susceptibility of the system. In the ground state we chose the maximal coherence superposition state $\psi_g=[1/\sqrt{3}\quad 1/\sqrt{3}\quad 1/\sqrt{3}]^T$. In Fig. 3(a) the control field is nearly circularly polarized $\varphi^c=0.05\pi$, whereas in Fig. 3(b) the control field is linearly polarized $\varphi^c=0.25\pi$. In both cases there is a mode which gets absorbed and another mode which exhibits the typical EIT characteristics. In the first part of this section we have derived an analytical expression for the susceptibility at two-photon resonance ($\delta=0$): these results fully support those findings. In Fig. 3(c) we have set $\psi_g=[1/\sqrt{8}\quad\sqrt{6/8}\quad 1/\sqrt{8}]^T$ and $\varphi^c=0.25\pi$. As we have described earlier in this section, for this special choice of the ground-state superposition state, one of the probe field modes is decoupled from the atomic system for any value of the probe field detuning Δ_p. The susceptibility for the other mode exhibits the EIT characteristic behavior.

We also show the dependence of the eigenmode vectors $\vec{S}_i(\Delta_p)$ on the probe field detuning in Fig. 4. We made these plots for $\varphi^c=\pi/8$, the initial ground-state superposition is $\psi_g=[1/\sqrt{3}\quad 1/\sqrt{3}\quad 1/\sqrt{3}]^T$, the rest of the parameters are defined as in Fig. 3(a). In column (a), the absorption and phase shift curves are shown, respectively. In column (b), we depict the V component (ellipticity) of the Stokes vectors associated with the polarization vectors of the two modes, the Q and U components (relative phase) do not represent relevant information now. Around two-photon resonance, the change of the ellipticity is quadratic, hence its dependence is weak on field detuning.

B. General Case

If the ground state of the atomic system is not in a pure state, then the susceptibility matrix Eq. (19) cannot be decomposed to the product form of Eq. (24). Instead, one has to keep the density matrix elements ϱ_{m_a,m_b}, and the susceptibility matrix reads

$$\chi=\frac{\mathcal{N}d^2}{\hbar\varepsilon_0 D}\begin{bmatrix}ABv_1+R_{aa^*} & ABv_2-R_{a^*b}\\ ABv_2^*-R_{ab^*} & ABv_3+R_{bb^*}\end{bmatrix}, \tag{33}$$

where we made the replacements

$$v_1=(\xi_2^1)^2\varrho_{g_{+2},g_{+2}}+(\xi_0^1)^2\varrho_{g_0,g_0}, \tag{34a}$$

$$v_3=(\xi_2^1)^2\varrho_{g_{-2},g_{-2}}+(\xi_0^1)^2\varrho_{g_0,g_0}, \tag{34b}$$

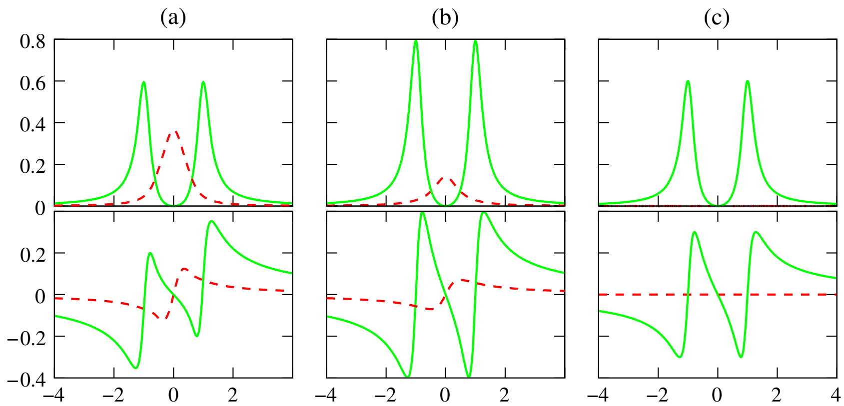

Fig. 3. (Color online) Linear susceptibility of the six-level system for the probe field. In the horizontal axis the probe field detuning Δ_p is measured in Γ_e units. The absorption (upper panels) and phase shift (lower panels) associated with the modes are shown in arbitrary units. The susceptibility curves for one of the modes are drawn with solid green lines, whereas that for the other modes are drawn with dashed red lines. The parameters for the plots are given in the text.

$$v_2 = \xi_0^1 \xi_2^1 (\varrho_{g_0,g_{+2}} + \varrho_{g_{-2},g_0}), \tag{34c}$$

$$R_{aa} = [|\Omega_+^c|^2 (\xi_0^1)^2 \varrho_{g_0,g_0} + |\Omega_-^c|^2 (\xi_2^1)^2 \varrho_{g_{+2},g_{+2}} - (Z \xi_0^1 \xi_2^1 \varrho_{g_{+2},g_0} + \mathrm{c.c.})]/4, \tag{34d}$$

$$R_{bb} = [|\Omega_-^c|^2 (\xi_0^1)^2 \varrho_{g_0,g_0} + |\Omega_+^c|^2 (\xi_2^1)^2 \varrho_{g_{-2},g_{-2}} - (Z \xi_0^1 \xi_2^1 \varrho_{g_0,g_{-2}} + \mathrm{c.c.})]/4, \tag{34e}$$

$$R_{ab^*} = [-|\Omega_-^c|^2 \xi_0^1 \xi_2^1 \varrho_{g_{+2},g_0} - |\Omega_+^c|^2 \xi_0^1 \xi_2^1 \varrho_{g_0,g_{-2}} + Z (\xi_2^1)^2 \varrho_{g_{+2},g_{-2}} + Z^* (\xi_0^1)^2 \varrho_{g_0,g_0}]/4, \tag{34f}$$

and $R_{a^*b} = R_{ab^*}^*$. The characteristic polynomial of the susceptibility matrix in Eq. (33) is given by

$$\chi^2 - \chi \cdot [AB \cdot (v_1 + v_3) + R_{aa^*} + R_{bb^*}] + (AB)^2 [v_1 v_3 - |v_2|^2] + AB \cdot [v_3 R_{aa*} + v_1 R_{bb^*} + v_2 R_{ab^*} + v_2^* R_{a^*b}] + R_{aa^*} R_{bb^*} - R_{a^*b} R_{ab^*} = 0, \tag{35}$$

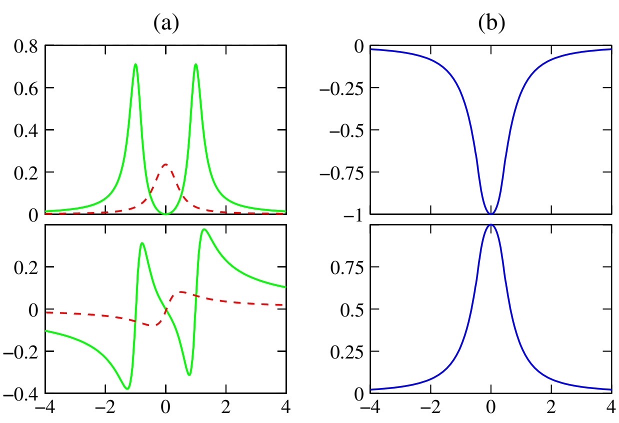

Fig. 4. (Color online) In column (a) the absorption and phase shift curves are shown for the two modes in a system with $\varphi^c = \pi/8$. The rest of the parameters are the same as in Fig. 3(a). In column (b) the ellipticity parameter V is depicted for the modes.

where the prefactor $\mathcal{N} d^2 / \hbar \varepsilon_0 D$ is omitted, as before. In case of the pure initial state, the last term $R_{aa^*} R_{bb^*} - R_{a^*b} R_{ab^*}$ was missing. At two-photon resonance $A = 0$, the characteristic polynomial Eq. (35) simplifies to

$$\chi^2 - \chi \cdot [R_{aa^*} + R_{bb^*}] + R_{aa^*} R_{bb^*} - R_{a^*b} R_{ab^*} = 0. \tag{36}$$

The two roots of this equation is given by

$$\chi_\pm = \frac{R_{aa^*} + R_{bb^*}}{2} \pm \frac{1}{2} \sqrt{(R_{aa^*} - R_{bb^*})^2 + 4 R_{ab^*} R_{a^*b}}. \tag{37}$$

The discriminator cannot reach the value of the sum $R_{aa^*} + R_{bb^*}$; hence, none of the roots is equal to zero. Therefore, both two modes undergo absorption in the medium. It is also not possible that both R_{aa^*}, R_{bb^*}, and R_{ab^*} get zero simultaneously. Therefore, there is no way to obtain a zero eigenvalue of the susceptibility matrix. We conclude that there is no "perfect" transparency if the ground state of the system is not a pure state.

In order to show the impact of the decrease of atomic coherences on the susceptibility of the medium, we have replotted Figs. (3) in (5), in which the only change is that we reduced the values of the ground-state coherences to half of the pure-state values. As one expects, features related to atomic coherence start to diminish: there is no longer a mode exhibiting "perfect" transparency. The high slope of the corresponding phase shift curves also get reduced, which implies higher group velocity.

6. SUMMARY

We have studied the propagation of a coherent probe light pulse in a medium consisting of effective three-level atoms. The degenerate ground- and excited-state sublevels are coupled by the nearly resonant probe pulse, while the excited-state sublevels are coupled further to an auxiliary state by a strong control (dressing) CW field. The linkage scheme resembles the EIT configuration; however, now the ground and excited levels are degenerate. Both the probe and control fields are elliptically polarized. We have determined the susceptibility matrix of the system for the probe field. We have shown that an incoming field was split to two components with

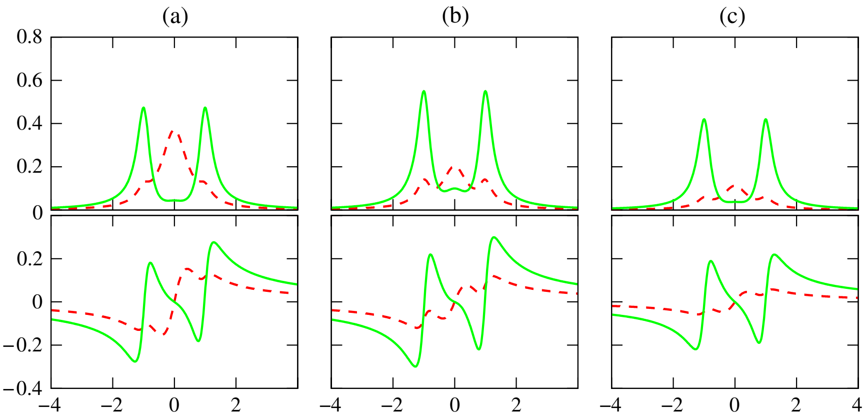

Fig. 5. (Color online) Same as Fig. 3, but the coherences in the ground-state manifold are reduced to half of their original values.

different propagation properties: the components are defined by the eigenmodes (eigenstates) of the susceptibility matrix. We have shown that under rather general conditions, for a pure ground-state superposition, there was one mode exhibiting EIT characteristic behavior. In general, the other mode gets absorbed, or for a special combination of the control field polarization state and the ground-state superposition amplitudes, the mode propagates without attenuation and phase shift for any value of the detuning from resonance. We have also studied the propagation of the probe pulse in the system with nonpure initial atomic state. We have shown that the conditions for “perfect” EIT could not be fulfilled; hence, both components of the probe pulse suffered attenuation.

The transmission properties of the system are sensitive to the initial ground-state superposition state. Hence, the proposed setup can be a useful tool to test the accuracy and reliability of state preparation methods.

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