Velocity-tuned resonances between different ground-state sublevels in two (σ^+ and σ^- polarized) standing waves

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We demonstrate the existence of velocity-tuned (Doppleron) resonances in two σ^+- and σ^--polarized standing waves, between the two ground-state sublevels of a Λ-type system. We derive the resonance conditions, and the properties of these resonances. We show that fast population transfer between these states may take place, while excited atomic states are only marginally populated. We derive an interesting dependence of the transfer time on the relative spatial phase of the two standing waves. [S1050-2947(97)05108-1]

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I. INTRODUCTION

The diffraction of atomic beams on the gratings formed by near-resonant electromagnetic standing-wave radiation has been investigated extensively both theoretically and experimentally [1–9]. In particular, resonance effects between electrotranslational states of atoms moving in standing-wave light have been discussed [10–15] and demonstrated [16–19] for atom-optical applications. There are two different types of resonances between electrotranslational states of moving, two-level atoms in standing-wave laser fields. The process when an atom is deflected by the laser beams (i.e., its velocity along the beams changes) without a change in its internal state is called a Bragg resonance. Momentum conservation obviously implies that the momentum gained by the atom must be an even number times the photon momentum. Energy conservation restricts the possible change of momentum to that corresponding to the reversal of the speed of the atom. The two together imply that the position of Bragg resonances in velocity space is fixed to translational states corresponding to an integer number times the photon momentum. This position is unaffected by the detuning of the frequency of the radiation from resonance. Doppleron resonances, on the other hand, describe a process when the change in translational state of the atom is accompanied by a change in the internal electronic state. The momentum acquired by the atom is thus an odd number times the photon momentum. Energy conversation again restricts the possible velocities where an atom may be resonantly scattered, but this position in velocity space is not fixed, but is dependent on the value of the detuning. These resonances are termed velocity-tuned resonances, because atoms with a certain speed always satisfy the resonance conditions whatever the value of the detuning. A clear picture of these resonances has been set forth using a band-theoretical approach in Refs. [14,15]. It has also been shown that, in the weak-interaction limit, two-beam resonances are possible.

The advantage of using Doppleron resonances for deflection and splitting of atomic beams over using Bragg resonances is the possibility of achieving much shorter transfer times [13,15], and that a wider range of transitions is possible at different values of the detuning. The main disadvantage is the sensitivity of Doppleron resonances to the incoherent effects introduced by spontaneous emission [15]. While numerous other methods for splitting and deflection of atomic beams have been recently proposed and demonstrated [20–25], Doppleron resonances may still prove to be a useful tool of atom optics, if this handicap could be overcome.

In this paper we discuss the existence and application of velocity-tuned resonances between different ground-state sublevels of a Λ-type system, in the field of two standing waves, one σ^+ polarized, the other σ^- polarized. These resonances, just like ordinary Doppleron resonances, have the potential for small transfer times in clean, two-beam interactions, and the range of possible transitions that may be achieved by changing the detuning is equally wide. At the same time, because excited levels are not populated, these transitions are much less sensitive to the ill effects of spontaneous emission. The paper is divided as follows. We first present the mathematical formalism and the basic assumptions made. We then present a slight generalization of the band-theoretical approach introduced in Ref. [14], and discuss velocity-tuned resonances based on this picture. The precise derivation of the resonance conditions is contained in the Appendix.

II. MATHEMATICAL FORMALISM

We are considering a Λ-type system (i.e., a system with an excited internal state having $J=0$ and two Zeeman-degenerate ground states with $J=1$ and magnetic quantum numbers $m=\pm 1$), that is, moving in the field of two standing waves with opposite circular polarizations. The motion of the atom along the laser beams (along the z axis) is considered quantum mechanically, while the motion perpendicular to the beams is treated classically. For the sake of simplicity, the amplitudes of the laser beams are considered to be constant across the beams. This means that the problem is essentially one dimensional; the perpendicular motion affects only the transit time, and hence the interaction time with the laser beams. The Hamiltonian for the problem is

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$$\hat{H}=\hat{H}_0+\hat{V}=\frac{\hat{p}^2}{2M}+\hat{H}_a+\hat{V}. \tag{2.1}$$

$\hat{H}_a$ is the atomic Hamiltonian describing the internal degrees of freedom, and obeying the equations $\hat{H}_a|g_\pm\rangle=\hbar\omega_g|g_\pm\rangle$ and $\hat{H}_a|e\rangle=\hbar\omega_e|e\rangle$. Note that, while the different ground-state sublevels were assumed to be degenerate for simplicity, this assumption may easily be relaxed, and the results remain valid if this is not the case. The interaction Hamiltonian may be written as $\hat{V}=-\hat{\mathbf{d}}\cdot\mathbf{E}$ in the electric dipole approximation. The electrical field vector associated with two standing waves having opposite circular polarizations may be written as

$$\begin{aligned}\mathbf{E}=&\mathbf{e}_+A_+\exp(-i\omega_+t)\cos(k_+z+\phi)+\text{c.c.}\\&+\mathbf{e}_-A_-\exp(-i\omega_-t)\cos(k_-z)+\text{c.c.}\end{aligned} \tag{2.2}$$

Here we have denoted the amplitudes of the σ^+ and σ^- standing waves by A_+ and A_-, respectively, and

$$\mathbf{e}_+=\frac{1}{\sqrt{2}}(\mathbf{e}_\mathbf{x}+i\mathbf{e}_\mathbf{y}) \quad \text{and} \quad \mathbf{e}_-=\frac{1}{\sqrt{2}}(\mathbf{e}_\mathbf{x}-i\mathbf{e}_\mathbf{y}) \tag{2.3}$$

are the polarization vectors of the standing waves. The phase ϕ describes the possible spatial shift of the nodes of the two standing waves compared to each other, while allowing A_+ and A_- to be complex accounts for a possible difference in temporal phase. We may choose a complete set of orthogonal states consisting of the electrotranslational states $|\kappa,g_+\rangle$, $|\kappa,g_-\rangle$, and $|\kappa,e\rangle$, which are eigenstates of the total energy of the free atom. Here κ denotes the wave number of the atomic matter wave, $p=\hbar\kappa$. Using this base we may describe the physical state of the system by three wave functions $\alpha_+(\kappa,t)$, $\alpha_-(\kappa,t)$, and $\beta(\kappa,t)$, which give the probability amplitude for the system to be at position κ in velocity space and at the same time to be in internal state $|g_\pm\rangle$, $|e\rangle$, respectively. Using these wave functions, the physical state of the system may be expressed as

$$\begin{aligned}|\rangle=\int_{-\infty}^{+\infty}&[\alpha_+(\kappa,t)|\kappa,g_+\rangle e^{-i\omega_g t}+\alpha_-(\kappa,t)|\kappa,g_-\rangle e^{-i\omega_g t}\\&+\beta(\kappa,t)|\kappa,e\rangle e^{-i\omega_e t}]d\kappa.\end{aligned} \tag{2.4}$$

Substitution of this expansion in Schrödinger's equation, and using Eq. (2.2) for the electric field yields the following set of equations for the wave functions in the rotating-wave approximation:

$$\begin{aligned}i\hbar\frac{\partial\alpha_+(\kappa,t)}{\partial t}=&\frac{\hbar^2\kappa^2}{2M}\alpha_+(\kappa,t)\\&-d_{g_+,e}\frac{1}{\sqrt{2}}e^{i\epsilon_- t}A_-^*[\beta(\kappa-k,t)+\beta(\kappa+k,t)],\end{aligned}$$

$$\begin{aligned}i\hbar\frac{\partial\alpha_-(\kappa,t)}{\partial t}=&\frac{\hbar^2\kappa^2}{2M}\alpha_-(\kappa,t)\\&-d_{g_-,e}\frac{1}{\sqrt{2}}e^{i\epsilon_+ t}A_+^*[e^{i\phi}\beta(\kappa-k,t)\\&+e^{-i\phi}\beta(\kappa+k,t)],\end{aligned} \tag{2.5}$$

$$\begin{aligned}i\hbar\frac{\partial\beta(\kappa,t)}{\partial t}=&\frac{\hbar^2\kappa^2}{2M}\beta(\kappa,t)\\&-d_{g_+,e}^*\frac{1}{\sqrt{2}}e^{-i\epsilon_- t}A_-[\alpha_+(\kappa-k,t)\\&+\alpha_+(\kappa+k,t)]-d_{g_-,e}^*\frac{1}{\sqrt{2}}e^{-i\epsilon_+ t}A_+\\&\times[e^{i\phi}\alpha_-(\kappa-k,t)+e^{-i\phi}\alpha_-(\kappa+k,t)].\end{aligned}$$

In these equations k is the wave number of the two standing waves (considered to be identical), M is the mass of the atom; $d_{g_+,e}=\langle g_+|\hat{d}_x|e\rangle=i\langle g_+|\hat{d}_y|e\rangle$ and $d_{g_-,e}=\langle g_-|\hat{d}_x|e\rangle=-i\langle g_-|\hat{d}_y|e\rangle$ are the matrix elements of the dipole moment operator between the internal states and $\epsilon_\pm=\omega_\pm-(\omega_e-\omega_g)$ is the detuning of the two waves from the atomic resonance. Note that, while $\epsilon_+\neq\epsilon_-$ strictly implies $k_+\neq k_-$, the rotating-wave approximation justifies writing k for both wave numbers.

As a slight generalization of the band-theoretical approach of Refs. [14,15], we may introduce the notations

$$\begin{aligned}c_n^\pm(q)=\alpha_\pm([n+q]k)\exp(-i\epsilon_\mp t) \quad &\text{if } n \text{ is even},\\ c_n^0(q)=\beta([n+q]k) \quad &\text{if } n \text{ is odd}.\end{aligned} \tag{2.6}$$

This means that we are changing the momentum variable κ of the wave functions to an index $n\in\{\ldots,-2^+,-2^-,0^+,0^-,2^+,2^-,4^+,4^-,\ldots\}\cup\{\cdots-5,-3,-1,1,3,5,\ldots\}$, and a quasimomentum parameter $q\in[-1,1)$. While there are two different indices for each even n, namely, n^+ and n^-, we shall use the phrase parity of the index to distinguish between the three types of indices: odd, even$^+$, and even$^-$. The value of q is restricted (to the first Brillouin zone) to avoid redundancy.

With this notation, and using the definitions

$$\Omega_+=\frac{A_+^*d_{g_-,e}}{\hbar\sqrt{2}} \quad \text{and} \quad \Omega_-=\frac{A_-^*d_{g_+,e}}{\hbar\sqrt{2}} \tag{2.7}$$

for the Rabi frequencies of the two standing waves, we may write Eqs. (2.5) as

$$\begin{aligned}i\dot{c}_n^+(q)=&\left[\frac{\hbar k^2(n+q)^2}{2M}+\epsilon_-\right]c_n^+-\Omega_-[c_{n-1}^0(q)+c_{n+1}^0(q)],\\ i\dot{c}_n^-(q)=&\left[\frac{\hbar k^2(n+q)^2}{2M}+\epsilon_+\right]c_n^--\Omega_+[e^{i\phi}c_{n-1}^0(q)\\&+e^{-i\phi}c_{n+1}^0(q)],\end{aligned} \tag{2.8}$$

$$i\dot{c}_n^0(q)=\frac{\hbar k^2(n+q)^2}{2M}c_n^0-\Omega_+^*[e^{i\phi}c_{n-1}^-(q)+e^{-i\phi}c_{n+1}^-(q)]$$
$$-\Omega_-^*[c_{n-1}^+(q)+c_{n+1}^+(q)].$$

We therefore have an infinite set of coupled, ordinary differential equations enumerated by the index n for every value of the continuous parameter q. The fact that the variables $c_n(q)$ are only coupled to $c_{n\pm1}(q)$ reflects the law of momentum conservation. An act of absorption or stimulated emission of a photon changes the momentum of the atom by $\hbar k$ (hence $n\rightarrow n\pm1$), and the quasimomentum q is a constant of the motion. Note that the fact that the state g_+ interacts with e only through σ^-, and g_- only through σ^+, makes it possible to incorporate the detunings into the energy of these states. The unperturbed energies of the electrotranslational states are thus given by

$$E_n^{\pm}(q)=\frac{(n+q)^2\hbar^2k^2}{2M}+\hbar\epsilon_{\mp},$$

$$E_n^0(q)=\frac{(n+q)^2\hbar^2k^2}{2M} \tag{2.9}$$

in terms of the index n and the quasimomentum parameter q.

III. DOPPLERON RESONANCES

The solutions of Eqs. (2.8) may be expressed as solutions of an eigenvalue problem. We must find the eigenstates of the perturbed Hamiltonian (2.1), expressing them as a linear combination of the electrotranslational states that are eigenstates of the unperturbed Hamiltonian $\hat{H}_0$. Since the interaction only couples electrotranslational states with the same quasimomentum parameter [Eqs. (2.8)], the eigenstates of the full Hamiltonian will be given by

$$|\Phi_\nu(q)\rangle=\sum_n p_{\nu,n}(q)|(n+q)k,j\rangle, \tag{3.1}$$

where the index j may be $g_\pm$, e depending on the parity of the index n. ν is the band index that enumerates the (infinite number of) solutions of the eigenvalue equation $\hat{H}|\Phi_\nu(q)\rangle=E_\nu(q)|\Phi_\nu(q)\rangle$. In the limit of zero field (i.e., no interaction), the coefficients will be given by $p_{\nu,n}(q)=\delta_{\nu,n}$, and the states $|\Phi_n(q)\rangle$ will simply be $|(n+q)k,j\rangle$, the band index ν coinciding with the level index n.

The first few branches of the dispersion relation of the free atom in the band-theoretical picture are shown in Fig. 1 (see Eq. (2.9)]. There are three types of branches, corresponding to the three possible parities for the index n. Degeneracies appear wherever two states belonging to different branches have equal unperturbed energies for some value of q, i.e.,

$$E_{n_0}^{\pm,0}(q)=E_{n_1}^{\pm,0}(q). \tag{3.2}$$

Relation (3.2) is termed the zeroth-order resonance condition, as it is exact only in the limit of zero-field strengths.

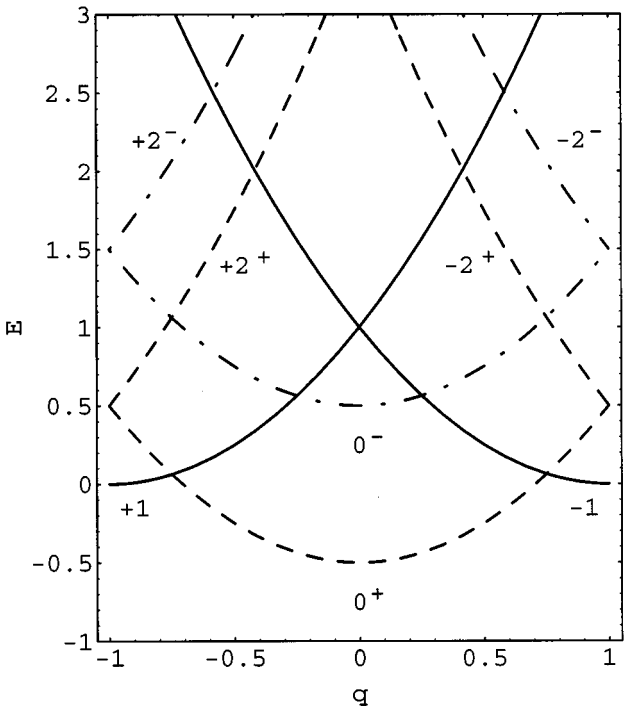

FIG. 1. The first few branches of the dispersion relation of the free atom as a function of the quasimomentum parameter q. The corresponding indices are written beside the branches. The solid lines with odd indices refer to an atom in internal state $|e\rangle$. The dashed lines with indices even$^+$ refer to an atom in internal state $|g_+\rangle$, while the dot-dashed lines with indices even$^-$ refer to an atom in internal state $|g_-\rangle$. Energy is in recoil units ($\hbar\omega_r=\hbar^2k^2/2M$). The detunings are $\epsilon_+=0.5\omega_r$ and $\epsilon_-=-0.5\omega_r$. Intersections between branches of the same index parity give rise to Bragg resonances, while those between branches of different index parity give rise to Doppleron resonances. These include resonances between electrotranslational states containing different ground-state sublevels.

Branches of the same parity are fixed with respect to each other, and intersections only occur in the center and at the boundary of the Brillouin zone. These are the Bragg resonances that are familiar from two-level atoms. They appear between the branches n and $-n$ in the center of the Brillouin zone, and n and $-n\pm2$ at $q=\mp1$. As the position of the branches with a parity even$^+$ (even$^-$) may be tuned by changing the detuning $\epsilon_-(\epsilon_+)$, intersections between branches of different parity may appear at any value of the quasimomentum q. These are the intersections that give rise to Doppleron resonances. It is obvious that these resonances appear not only between ground and excited electronic levels of the atom, but also between the different ground-state sublevels. Note that in this case the difference in momentum between the two states is an even multiple of $\hbar k$, as opposed to an odd multiple in ordinary Doppleron resonances.

The effect of a weak interaction with the standing-wave radiation changes the appearance of the dispersion relation, as shown in Fig. 2. The interaction is considered weak if $\varepsilon=\hbar\Omega_\pm/(E_n-E_m)<1$ where m denotes the nearest nonresonant level to n.

Far from the intersections, the branches of the dispersion relation change little. The levels are Stark shifted due to a

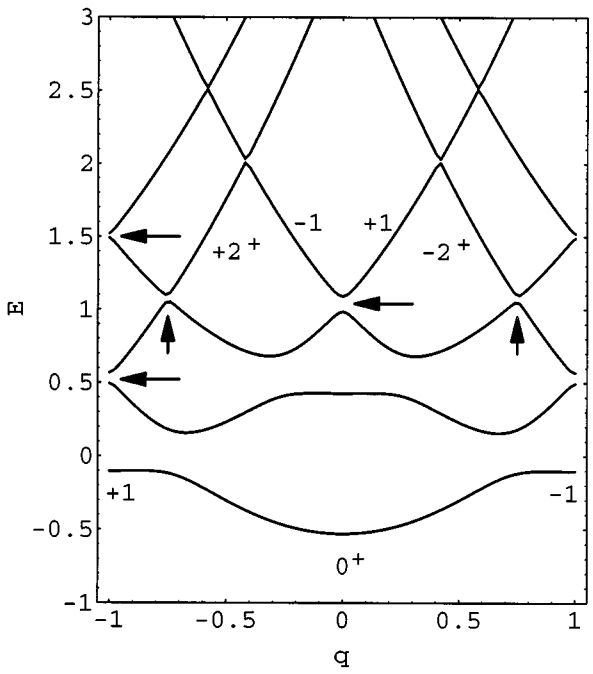

FIG. 2. The same as Fig. 1 but with the field strengths $\Omega_+=\Omega_-=0.15\omega_r$ and spatial phase $\phi=0$. The diagram shows splitting and level repulsion near the degeneracies of the noninteracting dispersion relation. Horizontal arrows pinpoint Bragg resonances, while vertical arrows point to Doppleron resonances between different ground-state sublevels. The indices of electrotranslational states that are written beside some parts of two different branches indicate the state that dominates that particular part of the branch. Note that different electrotranslational states dominate in different parts of a single branch. These parts are separated by resonances where the electrotranslational states are strongly mixed.

mixing of neighboring levels by the laser fields. They still, however, consist of predominantly one electrotranslational state in the weak-interaction limit [14].

In the vicinity of intersections, however, the crossings between the different branches are lifted into anticrossings, and a band gap appears between the levels. (See the Appendix for the derivation of these results.) The splitting between two degenerate levels involving different ground-state sublevels n_0^+ and n_1^- is approximately given by $\Delta E=2|G|$, where

$$G=-\hbar\Omega_-\hbar\Omega_+^* e^{-i\phi}\prod_{\substack{i=n_0+1\\ i\,odd}}^{n_1-1}\frac{1}{E-E_i^0}\times\prod_{\substack{j=n_0+2\\ j\,even}}^{n_1-2}\left(\frac{|\hbar\Omega_-|^2}{E-E_j^+}+\frac{|\hbar\Omega_+|^2e^{-i2\phi}}{E-E_j^-}\right) \tag{3.3}$$

[see Eqs. (A13b and A14c)]. E is the energy of the resonant levels, that may be approximated by $E_{n_0}^+$ in this expression. (See the Appendix.) There is also a renormalization of the resonance condition Eq. (3.2) [see Eqs. (A13a), (A14a), and (A14b)]. What this means is that the values of the detunings at which the states n_0^+ and n_1^- at a given q are exactly resonant depend on the values of the field strengths. [The explicit dependance of the energies on q has been supressed for notational simplicity; see (Eq. 2.9).] It is now clear that the simplification introduced by considering beams with constant field strengths across is important. Since the resonance conditions change with the field strengths, maintaining these conditions throughout the interaction in beams with other than a constant field strength along the cross section is much more complicated.

The eigenstates of the Hamiltonian at a resonance will be to a good approximation given by (see the Appendix)

$$|\Phi_{1,2}(q)\rangle\approx\frac{1}{\sqrt{2}}[|(n_0+q)k,g_+\rangle\pm e^{i\gamma}|(n_1+q)k,g_-\rangle], \tag{3.4}$$

where γ is an unimportant phase arising from the complex nature of the eigenvalue problem. This means that, if the atom is in one of the electrotranslational states (e.g., n_0) at $t=0$, we may expect Pendellösung-type oscillations between n_0 and n_1 whose frequency is proportional to ΔE. ΔE can be seen from Eq. (3.3) to be $O(\hbar\Omega\varepsilon^{n_1-n_0-1})$. Noting that the energy difference between the resonant levels and the nearest nonresonant levels $D_i^0\approx 2n_0\hbar^2k^2/2M$, if $n_0\gg 1$, we may conclude that, for sufficiently large n_0, the splitting of the resonant levels may be $\Delta E>\hbar^2k^2/2M$. Therefore the transfer times in these resonances may be much smaller than the inverse recoil frequency. At the same time, because $\varepsilon<1$, a two-beam resonance may be preserved. The significance of this is that in Bragg resonances the transfer time is always much larger than the inverse recoil frequency in the case of a two-beam resonance. This property has already been shown for ordinary Doppleron resonances [14,15]. While in a two-level atom these processes necessarily involve the excited state of the atom, and are therefore sensitive to incoherence introduced by spontaneous emission, with a Λ-type system we may have such resonances between different ground states of the atom. Since the levels containing the excited electronic states of the atom are only marginally populated during these resonances, they are much less sensitive to spontaneous emission.

From Eq. (3.3) one can easily see an interesting dependence of the energy splitting (and hence the oscillation frequency) on the relative spatial phase ϕ of the two standing waves. The various terms in this product are associated with the intermediate states that lie between the two resonant states. Since there are two different ways an atom can go from an intermediate excited state n_i^0 to n_{i+2}^0, (through the state n_{i+1}^+ by the emission and the absorption of a σ_- photon, and through the state n_{i+1}^- by the emission and absorption of a σ_+ photon), the amplitudes for these two possibilities must be coherently summed. The energy splitting is proportional to the product of the norms of terms containing such coherent sums. Since the relative phase of the terms in these sums can be seen from Eq. (3.3) to be dependent on ϕ, it is obvious that, at particular values of $\Omega^\pm$ and ϕ, there may be destructive interference between the two possibilities in one of these sums, and the energy splitting given by Eq. (3.3) may be zero. More precisely, assuming $0<n_0,n_1$, the energies $E_n^\pm(q)$ are monotonically increasing functions of n, and because $E\approx E_{n_0}^+\approx E_{n_1}^-$ for a resonance, for a pair of

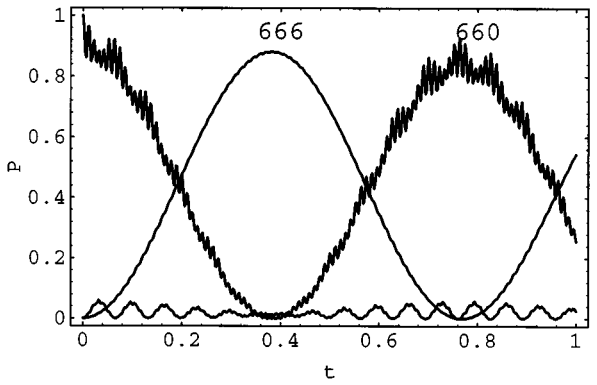

FIG. 3. Time evolution of the state probabilities in velocity space of a system initially in state $n_0=660^+$, showing oscillatory behavior between levels 660^+ and 666^-. Time on the horizontal axis is measured in units of inverse recoil frequency. The figure was obtained by numerically solving Eqs. (2.8). The parameters used were $\Omega_+=2200\omega_r$, $\Omega_-=1500\omega_r$, $\epsilon_+=-17827.1\omega_r$, $\epsilon_-=-10\,000\omega_r$, and $\phi=\pi/2$. Both the frequency of the oscillations, and the place of the resonance are in good agreement with the values calculated from Eq. (A13). It can be seen from the figure that around 90% of the population is transferred in a six-photon transition to the state 666^-, while 10% is scattered into neighboring nonresonant levels. The state probability of one of these levels is also illustrated at the bottom of the figure for comparison. The maximum probability of this level is around 6%. Note that the frequency of the oscillations in the situation above with $\phi=0$ would be less than one-tenth of the frequency of oscillation on the figure.

intermediate ground-state levels n_j^+ and n_j^- the two denominators $E-E_j^+$ and $E-E_j^-$ will be of opposite sign. For this reason, the norm of the sum of these terms will be largest if $e^{-i2\phi}=-1$, i.e., if $\phi=\pm\pi/2$, and smallest if $\phi=0,\pi$. Between the maximum and the minimum, the energy splitting given by Eq. (3.3) is a monotonic function of ϕ. Thus we have the important result that the resonant coupling between the two ground-state sublevels is always strongest when the nodes of one standing wave coincide with the maxima of the other. If the nodes of the two standing waves coincide, the energy splitting given by Eq. (3.3) may disappear altogether at specific values of the field intensities. This does not mean that the resonances will be completely suppressed, as there may still be a significant splitting in a more precise approximation (see the Appendix for the details), but the coupling can be orders of magnitude larger at $\phi=\pm\pi/2$ than at $\phi=0,\pi$. Note that the exact value of the resonant energies and detunings also varies slightly with ϕ.

Figure 3 depicts the time evolution of the velocity space state probabilities of a system initially in state $n_0^+=660^+$. The figure shows Pendellösung-type oscillations between the states 660^+ and 666^-. These values were chosen to correspond with those in Ref. [15] for a good comparison. The transfer time is around 0.4 in recoil units in a six-photon resonance. The transfer efficiency is almost 90%, with the remaining 10% of the population being scattered into various neighboring nonresonant levels. The maximum probabilities of neighboring excited states are approximately 6%. The data for the figure were obtained by numerically solving Eqs. (2.8). The frequency of the oscillations, as well as the value of the detunings where the resonance appears, is in good agreement with the values calculated from Eq. (A13).

Resonances that are very similar in nature to the ones decribed above were mentioned in Ref. [26]. The authors considered a three-level system with a Zeeman-degenerate excited state, and, in the presence of a magnetic field and a standing light wave with a polarization gradient, identified velocity-tuned resonances between different excited-state sublevels. They termed these Raman-type resonances. Considering a three-level system with two Zeeman-degenerate ground-state sublevels in this latter field configuration brings even closer the similarity between these resonances, and the ones considered in this paper.

Another work discussing similar resonances is Ref. [27]. In this paper, multiphoton resonances between the ground-state sublevels of a Λ-type system in two bichromatic waves is discussed. While only the internal states of the atom are considered in the treatment, it is stressed that an atom moving along a standing wave will ''see'' a bichromatic wave because of the Doppler shift, hence at given detunings of the two standing waves the resonances can be regarded to be velocity-tuned resonances. The resonances described in that paper and those in the present paper are only equivalent, however, if recoil effects may be completely neglected. This means that unless $\Delta E\gg l^2\hbar\omega_r$ (where ω_r is the recoil frequency and $l=n_1-n_0$ the number of photons absorbed and emitted), the full electrotranslational description must be used for the discussion of the resonances. With the additional requirement of clean, two-beam resonances, the simplified treatment of that paper is valid only in the limit of strong fields and very large velocities.

IV. SUMMARY

We have investigated the properties of velocity-tuned resonances between ground-state sublevels of a Λ-type system. We have shown that these resonances, just like ordinary Doppleron resonances, have the potential for small transfer times for beam splitting or deflection, while being largely immune to the incoherent effects of spontaneous emission. This fact may make such resonances a useful tool of atom optics.

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APPENDIX

We now set out to derive the resonant solutions of Eqs. (2.8). We first introduce the dimensionless quantities $g_\pm=\Omega_\pm/\omega_r$ and $\delta_\pm=\epsilon_\pm/\omega_r$, where we have used $\omega_r=\hbar k^2/2M$, the recoil frequency, as a unit. The dimensionless equations to be solved are thus

$$i\dot{c}_n^+(q)=[(n+q)^2+\delta_-]c_n^+(q)-g_-[c_{n-1}^0(q)+c_{n+1}^0(q)],$$

$$i\dot{c}_n^-(q)=[(n+q)^2+\delta_+]c_n^-(q)-g_+[e^{i\phi}c_{n-1}^0(q)+e^{-i\phi}c_{n+1}^0(q)], \tag{A1}$$

$$i\dot{c}_n^0(q)=(n+q)^2c_n^0(q)-g_-^*[c_{n-1}^+(q)+c_{n+1}^+(q)]$$
$$-g_+^*[e^{i\phi}c_{n-1}^-(q)+e^{-i\phi}c_{n+1}^-(q)].$$

For the sake of simplicity, we shall suppress the dependence on the parameter q in what follows, with the exception of the kinetic energy. As the dimensionless counterpart of Eqs. (2.9), we introduce the notations $E_n^{\pm}=(n+q)^2+\delta_{\mp}$ and $E_n^0=(n+q)^2$ for the unperturbed energies of the levels with even and odd n, respectively. Writing the variables in vector form,

$$\begin{pmatrix} \vdots \\ c_n^+ \\ c_n^- \\ c_{n+1}^0 \\ c_{n+2}^+ \\ c_{n+2}^- \\ c_{n+3}^0 \\ \vdots \end{pmatrix}, \tag{A2}$$

the set of Eqs. (A1) may be tackled by solving the eigenvalue problem

$$\begin{pmatrix} \cdot & \cdot & \cdot & & & \\ g_- & D_n^+ & 0 & g_- & 0 & \\ g_+e^{i\phi} & 0 & D_n^- & g_+e^{-i\phi} & 0 & \\ & g_-^* & g_+^*e^{i\phi} & D_{n+1}^0 & g_-^* & g_+^*e^{-i\phi} \\ & & & \vdots & \vdots & \vdots \end{pmatrix}$$
$$\times\begin{pmatrix} \vdots \\ c_n^+ \\ c_n^- \\ c_{n+1}^0 \\ \vdots \end{pmatrix}=0, \tag{A3}$$

where we used the notations $D_n^{\pm}(E)=E-E_n^{\pm}$ for even values of n, and $D_n^0(E)=E-E_n^0$ for odd values of n. We must find the energies E for which nontrivial solutions of Eq. (A3) exist, and the corresponding eigenvectors. While this is not a tridiagonal recurrence relation as in the case of the two-level atom, one may still proceed either by using matrix-continued fractions [28], or by using a slight generalization of the continued fraction method adopted in Refs. [14,15]. We first single out the two potentially degenerate states with indices n_0^+ and n_1^- for which

$$E_{n_0}^+\approx E_{n_1}^-. \tag{A4}$$

We assume for now that $n_1-n_0<n_0$, so in the case of $q=0,\pm1$, if any of these two states also possesses a Bragg resonant state of the same parity, it is sufficiently far away to be neglected. Condition, (A4), however, only determines the difference between the two detunings. For any given δ_-, for example, we may find δ_+, so that $(n_0+q)^2+\delta_-=(n_1+q)^2+\delta_+$ is satisfied, and there may exist an odd index m for which $E_m^0=(m+q)^2\approx(n_0+q)^2+\delta_-$; i.e., even in the absence of Bragg resonances, we may have a threefold degeneracy appearing at an infinite number of values of δ_-. In what follows, we assume that this is not the case, i.e., that we have chosen the detunings so that there is no further degeneracy apart from the levels n_0^+ and n_1^-. This assumption, however, needs to be verified for every actual value of $\delta_{\pm}$ used. Pursuing a slight generalization of ordinary continued fractions, we now write

$$c_{n+1}^0=t_{n+1}(g_-^*c_n^++g_+^*e^{i\phi}c_n^-),$$

$$c_{n-1}^0=s_{n-1}(g_-^*c_n^++g_+^*e^{-i\phi}c_n^-)$$

if n is even, and

$$c_{n+1}^+=T_{n+1}^+g_-c_n^0,$$
$$c_{n-1}^+=S_{n-1}^+g_-c_n^0,$$
$$c_{n+1}^-=T_{n+1}^-g_+e^{i\phi}c_n^0, \tag{A5}$$
$$c_{n-1}^-=S_{n-1}^-g_+e^{-i\phi}c_n^0$$

is n is odd.

Substitution of these equations into Eqs. (A3) yields the following reccurence relations:

$$t_n=\frac{-1}{D_n^0+|g_-|^2T_{n+1}^++|g_+|^2T_{n+1}^-},$$
$$s_n=\frac{-1}{D_n^0+|g_-|^2S_{n-1}^++|g_+|^2S_{n-1}^-}$$

if n is odd, and

$$T_n^+=\frac{-D_n^-+|g_+|^2t_{n+1}(e^{i2\phi}-1)}{D_n^-D_n^++D_n^-|g_-|^2t_{n+1}+D_n^+|g_+|^2t_{n+1}},$$
$$T_n^-=\frac{-D_n^++|g_-|^2t_{n+1}(e^{-i2\phi}-1)}{D_n^-D_n^++D_n^-|g_-|^2t_{n+1}+D_n^+|g_+|^2t_{n+1}},$$
$$S_n^+=\frac{-D_n^-+|g_+|^2s_{n-1}(e^{-i2\phi}-1)}{D_n^-D_n^++D_n^-|g_-|^2s_{n-1}+D_n^+|g_+|^2s_{n-1}}, \tag{A6}$$
$$S_n^-=\frac{-D_n^++|g_-|^2s_{n-1}(e^{i2\phi}-1)}{D_n^-D_n^++D_n^-|g_-|^2s_{n-1}+D_n^+|g_+|^2s_{n-1}}$$

if n is even.

Using these relations to eliminate the levels $l<n_0$ and $l>n_1$, the equations for the potentionally resonant levels, and the nonresonant levels in between then, become

$$D_{n_0}^+c_{n_0}^++s_{n_0-1}|g_-|^2c_{n_0}^++g_-g_+^*e^{-i\phi}s_{n_0-1}c_{n_0}^-+g_-c_{n_0+1}^0=0, \tag{A7a}$$

$$D_{n_1}^{-}c_{n_1}^{-}+t_{n_1+1}|g_+|^2c_{n_1}^{-}+g_-^{*}g_+e^{-i\phi}t_{n_1+1}c_{n_1}^{+}+g_+e^{i\phi}c_{n_1-1}^{0}=0, \tag{A7b}$$

$$\begin{pmatrix} D_{n_0}^{-}+|g_+|^2s_{n_0-1} & g_+e^{-i\phi} & 0 & \cdots & \cdots & \\ g_+^{*}e^{i\phi} & D_{n_0+1}^{0} & g_-^{*} & g_+^{*}e^{-i\phi} & 0 & \cdots \\ & & \ddots & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & \cdots & 0 & g_-^{*} & g_+^{*}e^{i\phi} & D_{n_1-1}^{0} \quad g_-^{*} \\ & & & \cdots & 0 & g_- \quad D_{n_1}^{+}+|g_-|^2t_{n_1+1} \end{pmatrix} \begin{pmatrix} c_{n_0}^{-} \\ c_{n_0+1}^{0} \\ c_{n_0+2}^{+} \\ \vdots \\ c_{n_1-1}^{0} \\ c_{n_1}^{+} \end{pmatrix}$$

$$= \begin{pmatrix} -g_-^{*}g_+e^{i\phi}s_{n_0-1}c_{n_0}^{+} \\ -g_-^{*}c_{n_0}^{+} \\ 0 \\ \vdots \\ 0 \\ -g_+^{*}e^{-i\phi}c_{n_1}^{-} \\ -g_-g_+^{*}e^{i\phi}t_{n_1+1}c_{n_1}^{-} \end{pmatrix}. \tag{A7c}$$

Denoting the matrix in the equation above by **N**, we may eliminate the variables $c_{n_0}^{-}$, $c_{n_0+1}^{0}$, $c_{n_1}^{+}$ and $c_{n_1-1}^{0}$ from Eqs. (A7a) and (A7b) by inverting **N** and expressing them as a linear combination of $c_{n_0}^{+}$ and $c_{n_1}^{-}$. We thus receive the following equations for the two potentially resonant levels:

$$\begin{pmatrix} A(E) & G_1(E) \\ G_2(E) & B(E) \end{pmatrix} \begin{pmatrix} c_{n_0}^{+} \\ c_{n_1}^{-} \end{pmatrix} = 0, \tag{A8}$$

where A and B are given by $A=D_{n_0}^{+}+A'$ and $B=D_{n_1}^{-}+B'$, and the auxillary quantities A', B', G_1, and G_2 are by

$$A'=s_{n_0-1}|g_-|^2-\frac{\Delta_{1,1}}{\Delta_{n_0,n_1}}|g_-|^2|g_+|^2s_{n_0-1}^2 - s_{n0-1}\left(\frac{\Delta_{1,2}}{\Delta_{n_0,n_1}}|g_-|^2g_+^{*}e^{-i\phi}+\text{c.c.}\right)-|g_-|^2\frac{\Delta_{22}}{\Delta_{n_0,n_1}},$$

$$B'=t_{n_1+1}|g_+|^2-\frac{\Delta_{m,m}}{\Delta_{n_0,n_1}}|g_-|^2|g_+|^2t_{n_1+1}^2 - t_{n_1+1}\left(\frac{\Delta_{m,m-1}}{\Delta_{n_0,n_1}}g_-^{*}|g_+|^2e^{-i2\phi}+\text{c.c.}\right) - |g_+|^2\frac{\Delta_{m-1,m-1}}{\Delta_{n_0,n_1}}, \tag{A9}$$

$$G_1=-g_-g_+^{*}e^{-i\phi}\frac{\Delta_{2,m-1}}{\Delta_{n_0,n_1}}-g_-g_+^{*2}e^{-i2\phi}s_{n_0-1}\frac{\Delta_{1,m-1}}{\Delta_{n_0,n_1}} - g_-^2g_+^{*}e^{i\phi}t_{n_1+1}\frac{\Delta_{2,m}}{\Delta_{n_0,n_1}}-g_-^2g_+^{*2}s_{n_0-1}t_{n_1+1}\frac{\Delta_{1,m}}{\Delta_{n_0,n_1}},$$

$$G_2=-g_-^{*}g_+e^{i\phi}\frac{\Delta_{m-1,2}}{\Delta_{n_0,n_1}}-g_-^{*}g_+^2e^{i2\phi}s_{n_0-1}\frac{\Delta_{m-1,1}}{\Delta_{n_0,n_1}} - g_-^{*2}g_+e^{-i\phi}t_{n_1+1}\frac{\Delta_{m,2}}{\Delta_{n_0,n_1}}-g_-^{*2}g_+^2s_{n_0-1}t_{n_1+1}\frac{\Delta_{m,1}}{\Delta_{n_0,n_1}}.$$

Here we have denoted the determinant of matrix **N** by Δ_{n_0,n_1}, $\Delta_{i,j}$ is the j,ith element of the adjunct of **N**, and m is the dimension of **N**. A nontrivial solution of the two-by-two problem (A8) exists, if

$$AB-|G|^2=0. \tag{A10}$$

Furthermore, we would like to enforce the resonance condition

$$A=B. \tag{A11}$$

Turning to the weak interaction limit, we define the small parameter

$$\varepsilon=\max\left\{\frac{|g_{\pm}|}{D_i}\right\}, \tag{A12}$$

where the index i runs through all the nonresonant levels between, and in the vicinity of the resonant levels. From the reccurence relations (A6) we may see that the imaginary parts of the quantities s_{n_0-1} and t_{n_1+1} are at least $O(\varepsilon^4)$ times smaller than the real parts, and can thus be taken to be real to an excellent approximation in the weak interaction limit. We may then write $G=G_1=G_2^*$ and Eqs. (A10) and (A11) may easily be rewritten as

$$\delta_+=\delta_--(n_1+q)^2+(n_0+q)^2-A'+B', \quad \text{(A13a)}$$

$$E_{1,2}=\delta_-+(n_0+q)^2-A'\pm|G|, \quad \text{(A13b)}$$

as the conditions for nontrivial resonant solutions of Eq. (A8).

It is instructive to expand the quantities A, B, and G in powers of ε, and see the leading order terms. They turn out to be

$$A=E-(n_0+q)^2-\delta_--\frac{|g_-|^2}{D^0_{n_0-1}}-\frac{|g_-|^2}{D^0_{n_0+1}}+O(g\varepsilon^3), \quad \text{(A14a)}$$

$$B=E-(n_1+q)^2-\delta_+-\frac{|g_+|^2}{D^0_{n_1-1}}-\frac{|g_+|^2}{D^0_{n_1+1}}+O(g\varepsilon^3), \quad \text{(A14b)}$$

$$G=-g_-g_+^*e^{-i\phi}\prod_{\substack{i=n_0+1\\ i\,odd}}^{n_1-1}\frac{1}{D_i^0}\prod_{\substack{j=n_0+2\\ j\,even}}^{n_1-2}\left(\frac{|g_-|^2}{D_j^+}+\frac{|g_+|^2e^{-i2\phi}}{D_j^-}\right). \quad \text{(A14c)}$$

It is evident from Eqs. (A14) that the quantities A', B', and G are only weakly dependent on δ_+ and E (i.e., the dependence is only through terms that are higher order than zero in the parameter $\varepsilon<1$). Given δ_-, Eqs. (A13) are best solved by a simple iteration, noting that they are of the forms $\delta_+=f_1(E_{1,2},\delta_+)$ and $E_{1,2}=f_2(E_{1,2},\delta_+)$, and using the initial values $\delta_+^{(0)}=\delta_--(n_1+q)^2+(n_0+q)^2$ and $E^{(0)}=(n_0+q)^2+\delta_-$.

The splitting between the two resonant levels $E_{1,2}$ from Eq. (A13b) is $\Delta E=2|G|$. While for δ_+ we actually obtain two different solutions with $E_{1,2}$, in the weak-interaction limit this difference is negligible. From Eq. (A13b) and the expansions (A14) it can be seen that the leading-order correction to $E_{1,2}$ comes from the interaction of the resonant levels with the nearest nonresonant ones. This shifts the resonant levels, and brings about a renormalization of the resonance condition (A13a). The leading-order term in the coupling G is easily seen to be $O(g\varepsilon^{n_1-n_0-1})$. Since, to be consistent, we must keep all terms at least as large as this in A and B, in all cases of practical interest ($n_1-n_0\geqslant 4$) we shall have to use a longer expansion of A and B that includes corrections arising from the coupling of the resonant levels to levels such as $n^{\pm}_{n_0\pm 2}$ and $n^{\pm}_{n_1\pm 2}$. In practice, while the different terms in this expansion have clear physical meanings, it is difficult to pursue. Fortunately, calculating the matrix **N** and inverting it numerically to find A, B, and G in each step of the iteration is easily done by computer. We must not forget, however, that the continued fraction expansion to the quantities s_{n_0-1} and t_{n_1+1} should also be followed through to include all terms whose orders of magnitude in ε is larger than, or equal to that of G. With $E_{1,2}$ and δ_+ fulfilling Eqs. (A13a) and (A13b), the eigenvectors corresponding to energies $E_{1,2}$ will be given by $|1,2\rangle\approx(1/\sqrt{2})(|n_0^+\rangle\pm e^{i\gamma}|n_1^-\rangle)$ to corrections the order of ε. [γ is a phase factor that may arise from the complex nature of the eigenvalue problem (A10).]

The problem of solving Eqs. (2.8) in a region where multiple degeneracies appear due to Bragg resonances (i.e., when $q=0$ and n_0 is small, so that the coupling between Bragg-resonant states is not negligible compared to that between Doppleron-resonant states) may be simplified by introducing symmetric and antisymmetric solutions in the form $x_n^{0,\pm}=c_n^{0,\pm}+c_{-n}^{0,\pm}$ and $z_n^{0,\pm}=c_n^{0,\pm}-c_{-n}^{0,\pm}$, and noting that the equations for these variables are separate. By using these variables, one may truncate the redundant equations for $n<0$, and thus formally get rid of the problem of having Bragg resonant states at $-n_0$.

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