Propagation of frequency-chirped laser pulses in a medium of atoms with a Λ -level scheme

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We study the propagation of frequency-chirped laser pulses in optically thick media. We consider a medium of atoms with a Λ level-scheme (Lambda atoms) and also, for comparison, a medium of two-level atoms. Frequency-chirped laser pulses that induce adiabatic population transfer between the atomic levels are considered. They induce transitions between the two lower (metastable) levels of the Λ -atoms and between the ground and excited states of the two-level atoms. We show that associated with this adiabatic population transfer in Λ -atoms, there is a regime of enhanced transparency of the medium—the pulses are distorted much less than in the medium of two-level atoms and retain their ability to transfer the atomic population much longer during propagation.

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I. INTRODUCTION

There are numerous techniques that use laser pulses to manipulate the quantum state of atoms coherently in atomic physics. Some of the best known ones include the usage of resonant π -pulses to achieve population transfer between two selected levels of an atom, or the use of frequencychirped laser pulses for similar population transfer via adiabatic rapid passage (ARP) [1]. For atoms with a Λ levelscheme (or Λ -atoms for short), two laser pulses that fulfill the two-photon resonance condition can be used to transfer the atomic population from one of the lower (metastable) states to the other one in the adiabatic regime. The pulses have to overlap partially and must be applied in a counterintuitive order-this is the well-known stimulated Raman adiabatic passage (STIRAP) [2-4]. An important property of this interaction is that the population of the excited state during the process may be negligibly small.

The propagation of laser pulses is also frequently investigated in optically thick media, which consist of atoms with a transition frequency close to that of the laser. The propagation problem is an interesting question in its own right especially in a medium of coherently prepared atoms that may have peculiar and useful nonlinear optical properties [5–7]. A medium like this can be prepared, for example, using laser pulses that perform some coherent manipulation of the atomic quantum states, like a complete transfer of the atomic population from one atomic state to another, or the creation of a coherent superposition of quantum states. When laser pulses are used to prepare such an atomic medium, or when investigating its optical properties, it is important to know whether the pulse that performs a certain type of coherent quantum state manipulation will retain this property throughout the optically thick sample.

Some of the coherent processes that laser pulses induce in atoms can give rise to startling propagation phenomena. For example, the coherent excitation and return to the ground state experienced by two-level atoms exposed to precisely resonant laser pulses with an area of 2π gives rise to the well-known self-induced transparency (SIT) phenomenon-a stable propagation of "slow" light pulses without considerable attenuation [8]. In the case of a medium of Λ -atoms, an intense field close to resonance with one transition can have notable effects on the propagation of a weak probe pulse that is close to resonance with the other transition. The so-called electromagnetically-induced transparency (EIT) phenomenon can give rise to anomalously long propagation lengths, the slowing down, and even the storage of the light pulse [9,10]. In the case of two laser pulses with similar amplitudes traveling in a medium composed of Λ -atoms, stable propagation of pulse pairs is possible under various conditions. Matched pulses may propagate in a coherently prepared medium, or may be generated by the medium itself as a response to a single input pulse under appropriate conditions [11,12]. Similar pulse pairs known as simultons have also been discovered [13]. These latter phenomena are all associated with the adiabatic evolution of the atoms in a dark state under the influence of the laser pulse [14,15].

For frequency-chirped laser pulses that excite two-level atoms via ARP, there is no transparency phenomenon analogous to SIT while propagating through a medium of twolevel atoms. One obvious reason (but not the only one) is that these pulses always transfer the atoms to the excited state, so they continue to lose energy during propagation. This is contrary to resonant SIT pulses that return atoms to the ground state at the end of the interaction and can therefore propagate without loss (provided that relaxation due to spontaneous emission during the interaction can be neglected).

Recently it was discovered that robust adiabatic population transfer between the two lower states of a Λ -atom can also be induced by a single frequency-chirped laser pulse [16,17]. In these works, the population dynamics of Λ -atoms under the influence of chirped pulses have been investigated, without considering the problem of pulse propagation. It was shown that under certain conditions, the population transfer between the two lower states is quite similar to STIRAP. The atomic population may be transferred completely from one stable state to the other, and the excited state of the atom is populated only weakly during the interaction. Thus in a me-

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dium of Λ -atoms one may anticipate a transparency phenomenon to exist for chirped laser pulses that fulfill the conditions for such an adiabatic population transfer.

In the present paper, we examine the propagation of frequency-chirped laser pulses through a medium of Λ -atoms, pulses that are capable of producing adiabatic population transfer between the two lower (metastable) states. We compare the propagation of these pulses to their propagation in a medium of two-level atoms, and show that the distortion of the pulses is greatly reduced in the threelevel case. In effect, there exists a transparency phenomenon for frequency-chirped laser pulses propagating in a medium of Λ -atoms that is somewhat similar to electromagnetically induced transparency. The most important difference is that in this case the chirped laser pulse itself takes care of the coupling to the third level of the atom, so it renders the medium to a good measure transparent for itself. It could thus be called chirped-pulse induced transparency. We calculate propagation through both an inhomogeneously broadened medium and a medium with only homogeneous broadening. One question to address is, how long do the frequency-chirped pulses propagating in an optically thick medium retain their ability to induce ARP between the atomic levels? This question is very important when considering chirped pulses as a tool for the manipulation of atoms in an extended region. We show that the pulse traveling in a medium of Λ -atoms will maintain its capability to produce adiabatic population transfer for a much larger distance than the same pulse traveling in a medium of two-level atoms. We also discuss the similarities and differences of the present process with EIT and the propagation of matched pulses.

II. THEORETICAL FRAMEWORK

To describe the propagation of frequency-chirped laser pulses in a medium of Λ -atoms we follow the wellestablished route of using the classical Maxwell equations for the electric field of the pulses and the Schroedinger equation of quantum mechanics for the time evolution of the atomic variables [18]. For our purposes, it is convenient to write the electric field of the laser pulse as

$$E(x,t) = \widetilde{A}(x,t)e^{i(kx-\omega_L t)} + \text{c.c.},$$

where A(x,t) is the complex field amplitude, which varies slowly in time and space compared to the central laser frequency ω_L and k (assumed to be constants). The frequencychirp is contained in the phase of the complex amplitude $\tilde{A}(x,t)$. The polarization of the medium is written as

$$P(x,t) = N[u(x,t) + iv(x,t)]e^{i(kx-\omega_L t)} + \text{c.c.},$$

where *N* is the atomic density and u(x,t), v(x,t) are the real, single-atom polarization functions. Using this notation, the wave equation for the propagation of the pulse in the slowly-varying envelope approximation will become

$$\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\widetilde{A}(x,t) = i\frac{\mu_0\omega_L c}{2}N[u(x,t) + iv(x,t)].$$
 (1)

Assuming the usual dipole interaction between a classical electric field and a quantum-mechanical atom, the Schroedinger equation for the elements of the atomic densitymatrix can be written as

$$\begin{aligned} \frac{\partial \rho_{11}}{\partial t} &= i(\Omega \sigma_{12}^* - \Omega^* \sigma_{12}) + \frac{\Gamma}{1 + |D|^2} \rho_{22}, \\ \frac{\partial \rho_{22}}{\partial t} &= -i(\Omega \sigma_{12}^* - \Omega^* \sigma_{12} + D\Omega \sigma_{32}^* - D^* \Omega^* \sigma_{32}) - \Gamma \rho_{22}, \\ \frac{\partial \rho_{33}}{\partial t} &= i(D\Omega \sigma_{32}^* - D^* \Omega^* \sigma_{32}) + \frac{\Gamma |D|^2}{1 + |D|^2} \rho_{22}, \\ \frac{\partial \sigma_{12}}{\partial t} &= -i\Delta \sigma_{12} + i\Omega (\rho_{22} - \rho_{11}) - iD\Omega \rho_{13} - \frac{\Gamma}{2} \sigma_{12}, \\ \frac{\partial \sigma_{32}}{\partial t} &= -i(\Delta + \omega_{31})\sigma_{32} + iD\Omega (\rho_{22} - \rho_{33}) - i\Omega \rho_{13}^* - \frac{\Gamma}{2} \sigma_{32}, \\ \frac{\partial \rho_{13}}{\partial t} &= -i\omega_{31}\rho_{13} + i(\Omega \sigma_{32}^* - D^* \Omega^* \sigma_{12}) - \frac{\Gamma |D|^2}{1 + |D|^2} \rho_{13}, \end{aligned}$$

where we have defined $\Omega = A^* d_{12}/\hbar$ the Rabi frequency with d_{12} being the dipole matrix element between $|1\rangle$ and $|2\rangle$. $\Delta = \omega_L - \omega_{12}$ is the detuning of the laser from the $|1\rangle \rightarrow |2\rangle$ transition of the atom, $\omega_{31} = \omega_{32} - \omega_{12}$ the Raman frequency, $D = d_{32}/d_{12}$ the ratio of the dipole matrix elements, and $\Gamma = 1/\tau_{\text{spont}}$ the inverse lifetime of the excited state. We have also defined $\sigma_{12} = \rho_{12} e^{i(kx-\omega_L t)}$ and $\sigma_{32} = \rho_{32} e^{i(kx-\omega_L t)}$. To calculate u(x,t), v(x,t) that yield the polarization of the medium needed in the wave equation, we have to average over the inhomogeneous line shape function $g(\Delta)$:

$$u(x,t) = \int \operatorname{Re}[\sigma_{21}(\Delta)d_{12} + \sigma_{23}(\Delta)d_{32}]g(\Delta)d\Delta,$$
$$v(x,t) = \int \operatorname{Im}[\sigma_{21}(\Delta)d_{12} + \sigma_{23}(\Delta)d_{32}]g(\Delta)d\Delta.$$
(3)

The inhomogeneous line shape function is assumed to be Gaussian $g(\Delta) = \exp[-(\Delta + \Delta_0)^2/2\sigma_{\Delta}^2]/\sqrt{2\pi\sigma_{\Delta}^2}$, with width σ_{Δ} , and a possible offset between the central frequency and the laser Δ_0 , taken hereafter to be zero. In the limit where $\Gamma \gg \sigma_{\Delta}$ (the case when homogeneous broadening dominates) u(x,t), v(x,t) are given simply by

$$u(x,t) + iv(x,t) = \sigma_{21}(0)d_{12} + \sigma_{23}(0)d_{32}.$$
 (4)

We then introduce the retarded time t' = t - x/c, and assume Gaussian pulses with a linear frequency-chirp $\Omega(t') = \Omega_0 \exp(-i\beta t'^2)\exp(-t'^2/2\tau_p^2)$. Note that the frequency-sweep rate is thus 2β . It is convenient to introduce dimensionless space and time coordinates with $\tau = t'/\tau_p$ and $\xi = x/\xi_0$ where

$$\xi_0 = \frac{\varepsilon_0 \hbar c}{N \omega_L |d_{12}|^2 T^*} \tag{5}$$

is the so-called absorption length, which contains the dephasing time of the atoms T^* and their spatial density N [18]. In the limit when inhomogeneous broadening of the transition is dominant, T^* is connected with the width of the inhomogeneous linewidth function as $T^* = \pi g(0) = \sqrt{\pi/2\sigma_{\Delta}^2}$. On the other hand, when inhomogeneous broadening can be neglected, $T^* = \tau_{\text{spont}} = 1/\Gamma$. Using these units, the wave equation (1) becomes

$$\frac{\partial\Omega}{\partial\xi} = -i\frac{1}{2\tau^*} \int \left[\sigma_{12}(\Delta) + D^*\sigma_{32}(\Delta)\right] g(\Delta) d\Delta \tag{6}$$

when inhomogeneous broadening dominates, or

$$\frac{\partial\Omega}{\partial\xi} = -i\frac{1}{2\tau^*}(\sigma_{12} + D^*\sigma_{32}) \tag{7}$$

when homogeneous broadening is dominant. Here $\tau^* = T^*/\tau_p$ is the dimensionless relaxation time. Note that the current choice of the length scale (5) cannot be extended to the infinitely sharp-line limit. In that case, one may, for example, use $\xi_0 = c \cdot \tau_p$, the spatial length of the laser pulse in vacuum as a length scale. However, the current choice is better suited to the more realistic case of finite atomic lifetime because the atomic density is also absorbed into ξ_0 .

Equation (6) or Eq. (7) for the propagation of the light pulse together with Eqs. (2) for the time evolution of the atomic density matrix constitute the starting point of our investigations. [Equations (2) are unchanged by introducing a retarded time and transforming to dimensionless quantities, only *t* has to be replaced with τ .] The equations can be solved numerically to explore the behavior of the interaction between the propagating pulses and the atomic medium. To treat the two-level atom case, they can easily be reduced to the usual Maxwell-Bloch equations [18] by inserting D=0and discarding the superfluous equations for ρ_{33} , σ_{13} , and σ_{32} from Eqs. (2).

III. PROPAGATION OF FREQUENCY-CHIRPED PULSES IN OPTICALLY THICK MEDIA

It is clear from Eqs. (2) and Eq. (6) or Eq. (7), that in general the medium will affect the pulses propagating through it via the polarization they induce. The most obvious mechanism is the absorption of energy from the pulse by the atoms that is then lost due to spontaneous emission; but even if the pulses are much shorter than the spontaneous lifetime of the atoms $\tau_p \ll \tau_{\text{spont}}$, the induced polarization may distort the pulses considerably during propagation. The propagation of resonant sech pulses with a definite pulse area of 2π in a medium of two-level atoms (SIT) is a notable exception to this, when pulses are short enough for spontaneous emission to be neglected. Even though the atoms are excited in this case, and the medium thus has strong polarization, the pulse travels undistorted. The reason for this is that energy absorbed by the atoms from the front of the pulse is reemitted into the tail coherently, so that the shape and amplitude of the



FIG. 1. Level structure of the Λ -atom. Two stable or metastable states are connected to a single excited state.

pulses do not change-the propagation will, however, be slowed down drastically. Clearly, such a mechanism cannot work for a frequency-chirped laser pulse that excites twolevel atoms via ARP, since atoms are left in the excited state. Photons are absorbed continuously from the pulse and correspondingly, its ability to transfer the atomic population to the excited state is expected to decrease gradually. Furthermore, the pulses that are capable of inducing ARP in two-level media always have fairly large intensity. Even though a pulse area analogous to the one defined for resonant, transform limited pulses is not really meaningful for a frequencychirped pulse, the integral of the envelope function corresponds to a resonant pulse area of many times 2π . It is well known that resonant pulses with large areas will, in general, reshape and break up into smaller pulses, with areas of 2π each [19].

A. Population transfer by a single frequency-chirped pulse

Recently, it has been shown that a single frequencychirped laser pulse can, under certain conditions, transfer the atomic population of a Λ -atom from one stable state to the other via adiabatic population transfer [16, 17]. The condition for this to happen can be obtained from the general conditions for adiabatic following in quantum mechanics: the rate of change of the dressed state eigenvalues of the timedependent Hamiltonian must be much smaller than the difference of any of these eigenvalues [20]. Translating this to the particular case of a two- or three-level atom in the field of a frequency-chirped laser pulse, one obtains the requirements that: (a) the time integral of the pulse envelope (the Rabi frequency) should be at least $(40-50)\pi$ and (b) the range of the frequency chirp should be comparable to (the same order of magnitude as) the largest Rabi frequency $\beta \times 5\tau_p \approx \Omega_0$. In addition to this, for a three-level Λ -atom the energy separation between the two stable states $\hbar \omega_{13}$ must be large compared to the transform limited bandwidth of the pulse, but should be smaller than the overall bandwidth covered by the frequency chirp. Furthermore, the chirp direction must be such that the laser pulse becomes resonant with the initially populated transition first. (This latter requirement, which corresponds to an "intuitive" order of resonances, is contrary to the STIRAP scheme, where a counterintuitive pulse sequence is required.) In our case, $\omega_{12} < \omega_{23}$, so a positive chirp is required for the population transfer to take place if the atom is in $|1\rangle$ initially (see Fig. 1). (Note that the first condition that follows from the adiabatic theorem is in fact a requirement for the pulse area, rather than directly the intensity.) As the two first requirements (a) and (b) are the same



FIG. 2. (Color online) (a) Time evolution of the level populations of a Λ -atom during the interaction with the laser pulse that enters the medium. The atom is transferred from $|1\rangle$ to $|3\rangle$. The dimensionless parameters used for the solution of Eqs. (2) are Ω_0 =50, β =10, ω_{31} =7, and Γ =0.025. (b) Time evolution of the level populations of a two-level atom interacting with the same pulse. The atom is excited via ARP.

for two- and three-level atoms, it is clear that under appropriate conditions, the very same pulse that transfers the atomic population to the excited state $|2\rangle$ in a two-level atom, will transfer it to $|3\rangle$ in the Λ -atom.

Figure 2 shows the time evolution of the atomic populations (a) ρ_{11} , ρ_{22} , and ρ_{33} of a three-level Λ -atom and (b) ρ_{11} and ρ_{22} of a two-level atom under the influence of a frequency-chirped laser pulse. The data was obtained from a numerical solution of Eqs. (2) using the following parameters: amplitude of the pulse $\Omega_0 = 50/\tau_p$, the chirp parameter $\beta = 10/\tau_p^2$, the separation of the two lower states of the Λ -atom $\omega_{31} = 7/\tau_p$, and $\Gamma = 0.025/\tau_p$, which corresponds to $T^* = 40\tau_p$, so the effect of spontaneous transitions is relatively small during the interaction. These pulse parameters fulfill the conditions for adiabatic population transfer mentioned in the previous paragraph, and the figures show that the same pulse indeed induces adiabatic population transfer in both cases. (The transfer, however, is not perfect, it is only about 95% effective in both cases due to the finite lifetime of the excited state and the finite pulse length.) The precise numerical values presented here for the pulse amplitude and chirp are not very significant. As adiabatic processes are "robust," a slight variation of the pulse parameters does not compromise it. In fact, for the present value of the amplitude, taking a chirp speed parameter as low as $\beta = 5/\tau_p^2$ or as high as β = 50/ τ_n^2 still yields a pulse that transfers the atomic population adiabatically.

It is also visible from Fig. 2(a) that during the population transfer induced in Λ -atoms the excited state will be populated only slightly. This property is similar to the population transfer induced by a pair of laser pulses in the STIRAP scheme. However, there is one major difference: while in the STIRAP scheme the dressed state that must be followed adiabatically to achieve the population transfer is composed solely of $|1\rangle$ and $|3\rangle$ and is thus "perfectly dark," this is not the case for the chirped-pulse scheme. Detailed theory shows [16,17] that in this case, the dressed state responsible for the population transfer also contains the excited state $|2\rangle$ with a certain small amplitude. Even though this amplitude can be effectively suppressed by increasing the pulse intensity, it can never be completely eliminated. So, the population transfer by a chirped pulse may be degraded slightly by incoherent effects involving the excited state even in the perfectly adiabatic limit.

B. Pulse propagation in homogeneously broadened media

The major difference in the effect of the chirped laser pulse on the time evolution of the atomic populations in the two- and three-level cases is clearly expected to manifest itself also in the propagation of the laser pulse. In the Λ -atom case the atom is no longer left in the excited state after the interaction, but in another stable state whose energy separation from the initial state $|1\rangle$ is much smaller than that of the excited state. Furthermore, since polarization of the medium is associated with populating the excited state (there is no dipole transition between the two stable states $|1\rangle$ and $|3\rangle$) the back-action of the Λ -atoms on the pulse will be much weaker during propagation than that of the two-level atoms.

Figure 3 shows the evolution of the magnitude of the laser pulse envelope $|A(\xi, \tau)|$ as the frequency-chirped laser pulse propagates through a medium of (a) three-level atoms with a Λ level-scheme and (b) two-level atoms. The data for the plots was obtained by solving numerically Eqs. (2) and (7) for the two media with initial pulse parameters identical to the ones used to produce Fig. 2. It can be seen that the modulations of the pulse envelope induced by the atoms are considerably smaller in a medium of Λ -atoms than in the medium of two-level atoms. It is also clear from the figure that the most important effect on the pulses is the modulation, the distortion of the pulse envelope as they progress into the medium and not the decay of the pulses due to absorption. This is evidently also true for the medium of two-level atoms, where one would expect to see a decay of the pulse amplitude as it excites the atoms during propagation. The primary reason is that the pulses are strong and able to saturate the medium. The well-known result of exponential decay of the pulse intensity is therefore not valid in this case. Furthermore, the laser pulse propagating in the medium of two-level atoms loses its ability for producing population transfer to the excited state very quickly (not because of amplitude decay, see later, in particular Fig. 4), so the decrease in amplitude remains undetectable on the scale of Fig. 3(b)). It is also notable that while the modulations of the envelope are concentrated on the trailing edge in the twolevel case, in the three-level one both leading and trailing



FIG. 3. (a) The evolution of the magnitude of the frequencychirped laser pulse envelope $|\tilde{A}(\xi,\tau)|$ in space and time while propagating through a medium of Λ -atoms. The dimensionless distance ξ is measured in units of absorption length, the dimensionless time τ is measured in units of the pulse width. (b) The evolution of $|\tilde{A}(\xi,\tau)|$ while propagating through a medium of two-level atoms. The parameters used for the solution are $\Omega_0=50$, $\beta=10$, $\omega_{31}=7$, and $\Gamma=0.025 \Rightarrow \tau^*=40$ in both cases.

edges of the pulse are distorted to some extent.

More important from a practical point of view than the relative size of the envelope modulations is the ability of the pulses to produce adiabatic population transfer between the atomic levels. Figure 4 shows the final level populations of (a) a Λ -atom and (b) a two-level atom after the passing of the laser pulse, as a function of the dimensionless penetration depth ξ . In the case of the two-level atoms, final population of the excited state should be close to 1 if adiabatic population transfer functions well, and correspondingly, a final population of $|3\rangle$ close to 1 has the same meaning for the Λ -atom. (At the entry into the media, the pulses do perform adiabatic population transfer to a very good measure, see Fig. 2.) It is evident from Fig. 4(a) that the population transfer induced by the pulse in the medium of Λ -atoms is un-



FIG. 4. (Color online) Final populations of the atomic levels after the passing of the pulse as a function of the dimensionless penetration depth ξ for (a) a medium of three-level Λ -atoms and (b) a medium of two-level atoms.

changed until about ξ =8000, i.e., 8000 times the absorption length defined by the parameters of the medium. After this, the capability of the pulse to perform adiabatic population transfer starts to degrade gradually, and it is then lost relatively quickly. The reason for this behavior can be deduced from Fig. 5, where the time evolution of the atomic populations of a Λ -atom have been plotted at ξ =10 000. While population transfer between the two lower states still works quite well, the population of the excited state during the interaction has increased several times over the initial value [compare with Fig. 2(a)]. As the pulse progresses, the excited state population during the interaction increases gradually.



FIG. 5. (Color online) Time evolution of the level populations of a Λ -atom during the interaction with the laser pulse at ξ =10 000 in the medium.

As this happens, polarization of the medium that distorts the pulse also grows. Since the back-action on the pulse increases, the degradation of the pulses capability to induce adiabatic population transfer speeds up.

In light of this latter result, the comments made on the possible values of the chirp parameter in Sec. III A should be refined somewhat. For a given pulse amplitude, the chirp could be changed over a fairly large interval without compromising the final result of the population transfer. However, for undistorted pulse propagation it is also important that the value of the excited state probability *during* the population transfer process remain small. Thus, of the range of chirp-speed parameters where adiabatic population transfer is satisfactory, that subrange will be most favorable from the point of view of pulse propagation, where atomic excitation during the process will be smallest. The interval where transparency can be observed will still be almost as large as that mentioned in Sec. III A, as this quantity also changes slowly with the pulse parameters.

The pulse traveling in the medium of two-level atoms on the other hand loses its ability to transfer the atoms to the excited state very quickly. The final value of ρ_{22} drops to almost 0 within a very short distance and then starts to oscillate [Fig. 4(b)]. It is also notable that a series of small pulses seem to be left behind in the medium after the chirped pulse passes [Fig. 3(b)] in the two-level medium. These two facts are strongly connected. A closer look reveals that in the medium of two-level atoms, the distortion of a chirped pulse takes the form of a series of small pulses that appear to catch up to the trailing edge of the original pulse. They originate in (are emitted by) the inverted medium behind the pulse and propagate with a superluminal phase and group velocity [21]. Thus they are a consequence of the unstable state of the medium after the frequency-chirped pulse inverts the atomic populations. It is the effect of these small pulses that is visible in the oscillations of the final populations. The quick degradation of the capability to produce ARP is not due to energy loss from the pulse, but the complex reshaping that results from the excitation of the medium. This is in agreement with experimental observations, which have uncovered complex oscillatory reshaping dynamics and pulse breakup during the propagation of resonant laser pulses in a medium without inhomogeneous broadening [22,23].

In some sense, the enhanced transparence of the medium due to the coupling to a third atomic state is reminiscent of EIT, where the coupling to a third state renders the medium transparent to the probe beam. Of course one major difference is that in our case there is no additional field to do the coupling-the frequency-chirped laser pulse itself couples the excited state with the third state, increasing the transparency of the medium for itself. Thus, in analogy with the phrase electromagnetically induced transparency, it could possibly be termed chirped-pulse induced transparency. However, it is obvious that there are some important differences too compared to EIT. Separating the complex pulse envelope into a real amplitude and a phase function $A(\xi, \tau)$ $=A(\xi,\tau)\exp[i\Phi(\xi,\tau)]$ it is possible to show that the real amplitude is decoupled from the medium when the population transfer is perfectly adiabatic. This is despite the fact that the excited state of the atom is populated during the population transfer even in this limit. On the other hand, the phase function that describes the chirp is distorted due to the backaction of the medium through polarization, and the amplitude is then compromised because the population transfer ceases to be perfectly adiabatic. Therefore it is clear that although there is an enhanced transparency of the medium, the frequency-chirped pulse is not an exact solitonlike solution of the probagation equations like the sech pulses of SIT, or the probe pulse of EIT, or matched pulse pairs [11,13]. Another difference compared to SIT or EIT also visible from Fig. 3 is the fact that the peak of the pulse remains at $\tau=0$ in the retarded time coordinate, i.e., there is no reduction of the group velocity as in SIT and EIT.

It is interesting to note that in practice, pulse propagation in the sharp-line limit is often explored using picosecond pulses [24]. However, then the two-level atom approximation fails, and the true multilevel nature of the atoms must be taken into account. Since the coupling constants are different for various transitions, their contributions to the polarization of the medium will soon get out of phase and pulse distortion and breakup will be much smaller than that calculated from a simple two-level model. Such a mechanism will no doubt be at work also for frequency-chirped laser pulses.

C. Pulse propagation in inhomogeneously broadened media

One of the primary reasons why frequency-chirped laser pulses are convenient for the manipulation of atoms is the ability to transfer the atomic populations even when there is a fairly large inhomogeneous broadening of the transition line. In the case of population transfer between the lower states of a Λ -atom, this broadening can be tolerated as long as it is considerably smaller than the full bandwidth of the pulse. Above that, atoms with a transition frequency far from the center of the inhomogeneously broadened spectrum will experience an interaction that does not fulfill the criteria for an adiabatic process, and therefore the population transfer will be imperfect. In case the broadening is very large, the pulse simply does not interact with a certain part of the atoms—they remain in state $|1\rangle$.

To investigate the effect of inhomogeneous broadening of the atomic transition on the propagation of a frequencychirped laser pulse, we have solved Eqs. (2) together with Eq. (6). For the width of the atomic distribution we used $\sigma_{\Delta}=10/\tau_p$ which corresponds to a dephasing time of τ^* =0.1253. The pulse parameters were the same as in the previous section. With these parameters, we are in a regime where the pulse is capable of inducing adiabatic population transfer over the whole atomic ensemble. To separate the effects of spontaneous emission and dephasing due to inhomogeneous broadening, this time we have assumed $\Gamma=0$.

The evolution of the pulse amplitude $|\tilde{A}(\xi, \tau)|$ is shown in Fig. 6 as the pulses propagate in the medium of (a) Λ -atoms and (b) two-level atoms. Again, the amplitude is plotted as a function of the dimensionless distance ξ . However, since the dephasing time T^* has decreased a great deal (from $T^* = 40\tau_p$ in the homogeneous case to $T^* = 0.1253\tau_p$ in the present case), the unit of distance has also changed compared



FIG. 6. (a) The evolution of the magnitude of the frequencychirped laser pulse envelope $|\tilde{A}(\xi,\tau)|$ in space and time while propagating through a medium of Λ -atoms with inhomogeneous broadening. The dimensionless distance ξ is measured in units of absorption length, the dimensionless time τ is measured in units of the pulse width. (b) The evolution of $|\tilde{A}(\xi,\tau)|$ while propagating through a medium of inhomogeneously broadened two-level atoms. The parameters used for the solution are $\Omega_0=50$, $\beta=10$, $\omega_{31}=7$, and $\sigma_{\Delta}=10 \Rightarrow \tau^*=0.1253$ in both cases.

to that obtained in Sec. III B. To compare Figs. 3 and 6 one must note that ξ =10 000 in the homogeneously broadened medium corresponds to the same physical distance as ξ =31 in the inhomogeneously broadened case (provided that all other parameters like the density of the atoms are the same). Viewing Figs. 6 and 3 one can see that the distortion of the pulse amplitude is somewhat less at roughly double the distance in the inhomogeneous case compared to the homogeneous one. This is also evident if one compares Figs. 7 and 4, which show the final populations of the atomic levels in the two cases as a function of the penetration depth. During pulse propagation, inhomogeneous broadening has an important consequence—it provides a mechanism for the decay of the polarization of the medium through the dephasing of the



FIG. 7. (Color online) Final populations of the atomic levels after the passing of the pulse as a function of the penetration depth for (a) the medium of three-level Λ -atoms and (b) the medium of two-level atoms.

individual atomic dipoles and thus decreases the back-action of the medium on the pulse. Therefore a limited amount of inhomogeneous broadening tends to improve the propagation properties of the pulses. It is still true that the difference between propagation in a medium of three-level Λ -atoms and a medium of two-level atoms is great—the former is much more transparent [see Figs. 7(a) and 7(b)]. Thus the mechanism at work functions well also in the inhomogeneously broadened medium.

It is also interesting to note that it is logical to expect a strong degradation of this transparency when the inhomogeneous broadening of the medium is greater than the full bandwidth of the frequency-chirped laser pulse. This is because then a portion of the atoms will not experience adiabatic population transfer, but will end up partly in the excited state, which means that the medium will be polarized. However, numerical solution of the equations shows that this degradation of the transparency of a medium of Λ -atoms is much smaller than expected. Only a small fraction of atoms have a resonance far enough from the central frequency to experience imperfect population transfer and considerable excitation during the interaction, but not too far to be unaffected by the pulse almost completely. Therefore the frequency-chirped laser pulse will travel undistorted almost to the same extent as before, only the population transfer that it induces in the atomic ensemble will be imperfect due to the large inhomogeneous broadening.

IV. SUMMARY

We have studied the propagation of frequency-chirped laser pulses in a medium of three-level Λ -atoms and compared it to their propagation in a medium of two-level atoms. A frequency-chirped pulse that inverts the populations of the two-level atoms can also produce adiabatic population transfer between the two lower states of the Λ -atom, that is similar to STIRAP. This manifests itself in the propagation properties of the chirped laser pulse through an optically thick sample of atoms. A chirped pulse traveling through a medium of two-level atoms is quickly distorted, and its ability to transfer the atomic population to the excited state via ARP degrades quickly. For a medium of Λ -atoms on the other hand, there is an increased transparency. The pulse envelope is distorted much less and the pulse's ability to induce adiabatic population transfer between the lower (metastable) states of the Λ -atoms is retained much longer. This can be attributed to the existence of the third atomic level, where the atomic population is transferred instead of the excited state, and the fact that the excited state is weakly populated during the population transfer process. The adiabatic state that is followed during the population transfer process is almost completely "dark," so the polarization of the medium of Λ -atoms will be much smaller than in the two-level case. Thus frequency-chirped laser pulses may be useful for the coherent preparation of optically thick samples of Λ -atoms, but not for two-level atoms.

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