

# The hadronization line in stringy quark matter



# [Dictionary.reference.com/search?q=stringy](http://Dictionary.reference.com/search?q=stringy)

- Modern Language Association,  
Random House Inc. 2008

1. Resembling a string or strings, consisting of strings or stringlike pieces: stringy weeds; a stringy fiber
2. Coarsely or toughly fibrous, as meat
3. Sinewy or wiry, as a person
4. Ropy, as a glutinous liquid

- The American Heritage  
Dictionary of the English  
Language, Houghton Mifflin Co.  
2006

1. Consisting of, resembling or containing strings or a string
2. Slender and sinewy; wiry
3. Forming strings, as a viscous liquid; ropy.

# **Dictionary.reference.com/search?q=stringy**

- **On-line Medical Dictionary, 1997-98 Academic Medical Publishing & CancerWEB**
- 1. Consisting of strings or small thraeds; fibrous; filamentous; as a stringy root.**
  - 2. Capable of being drawn into a string; as a glutinous substance; **ropy; viscid; gluely**.**
  - 3. Stringy bark, a name given in Australia to several trees of the genus Eucalyptus (as E. amygdalina, obliqua, capitellata, macrorhyncha, piperita, pilularis and tetradonta), which have a fibrous bark used by the aborigines for making cordage and cloth.**

# The hadronization line in stringy quark matter

T.S.Bíró, MTA KFKI RMKI Budapest, Hungary

J. Cleymans, University of Cape Town, South Africa

- How can be  $E / N = 6 T$  ?
- Stringy corrections to QGP
- High- $T$  equation of state
- The zero pressure line

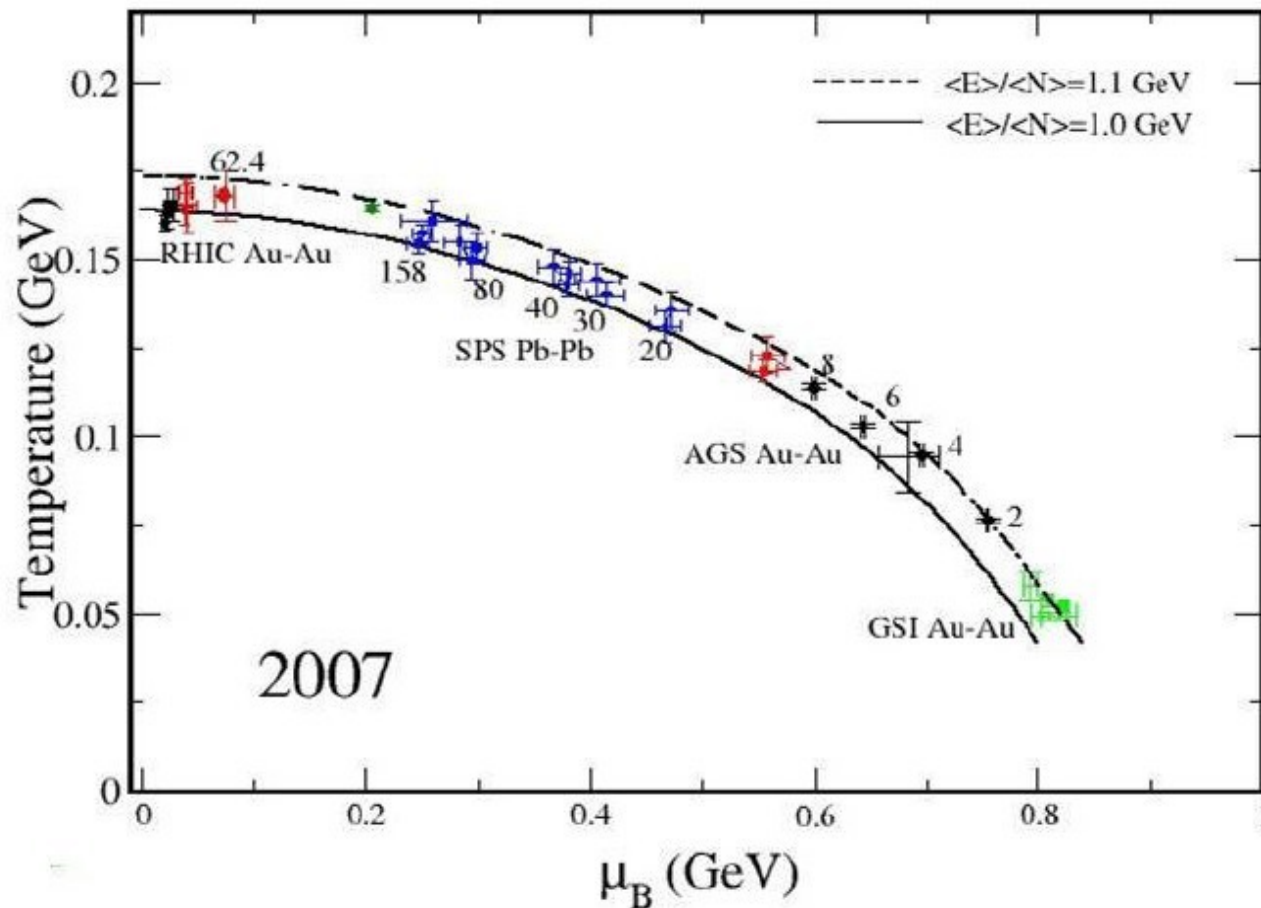


t h e o r y

50% OFF

国内  
イポート





# How can be $E / N = 6 T$ ?

$$T = 167 \text{ MeV}$$

$$E / N = 1 \text{ GeV}$$



Statistical Model: hadronization point around  $\mu = 0$  (RHIC, LHC)

How can be  $E / N = 6 T$  ?

$$E / N = m + \frac{3}{2} T$$

$$m = 750 \text{ MeV}$$

Massive hadrons (rho?)



How can be  $E / N = 6 T$  ?

$$E / N = 3T$$

$$m = 0$$

Ideal gas of radiation

# How can be $E / N = 6 T$ ?

$$e = \sigma T^4 + B$$

$$p = \frac{1}{3} \sigma T^4 - B \geq 0$$

$$n \approx \frac{1}{3} \sigma T^3$$

$$E / N = e / n \leq 4T$$

Bag Model for QGP



# How can be $E / N = 6 T$ ?

## Stringy Massless QGP

$$f = f_{id}(n, T) + \frac{3}{2} A n^{2/3}$$

$$e = e_{id} + \frac{3}{2} A n^{2/3}$$

$$p = p_{id} - \frac{1}{2} A n^{2/3}$$

$$\left. \frac{e}{n} \right|_{p=0} = \left. \frac{e_{id} + 3p_{id}}{n} \right|_{m=0} = \left. \frac{6p_{id}}{n} \right|_{\text{Boltzmann}} = 6T$$



# How can be $E / N = 6 T$ ?

## Stringy Massless QGP

$$f = f_{id}(n, T) + \frac{3}{2} A n^{2/3} \quad n \cdot \sigma \langle \ell \rangle = \sigma g n^{-1/3} n = \frac{3}{2} A n^{2/3}$$

$$e = e_{id} + \frac{3}{2} A n^{2/3}$$

$$p = p_{id} - \frac{1}{2} A n^{2/3}$$

$$\left. \frac{e}{n} \right|_{p=0} = \left. \frac{e_{id} + 3p_{id}}{n} \right|_{m=0} = \left. \frac{6p_{id}}{n} \right|_{\text{Boltzmann}} = 6T$$

# How can be $E / N = 6 T$ ?

**This is more generally true!**

- **correction depends on**

$$C = \sum_i c_i n_i$$

- **constituents are massless**

$$p_{id} = T^4 \cdot \phi\left(\mu_B / T\right)$$

- **Boltzmann approximation is acceptable**

$$p_{id} = T \cdot \sum_i n_i$$

# Stringy corrections to QGP

$$c = \sum_i c_i n_i$$

color density

$$f = f_{id} + \frac{1}{1-\gamma} A c^{1-\gamma}$$

free energy density  
fractional power

$$\mu_i = \mu_{id} + A c^{-\gamma} c_i = q_i \mu_B$$

chemical potential

$$p = p_{id} - \frac{\gamma}{1-\gamma} A c^{1-\gamma}$$

pressure

$$e = e_{id} + \frac{1}{1-\gamma} A c^{1-\gamma}$$

energy density



# Stringy corrections to QGP

Consistent solution for color density in equilibrium:

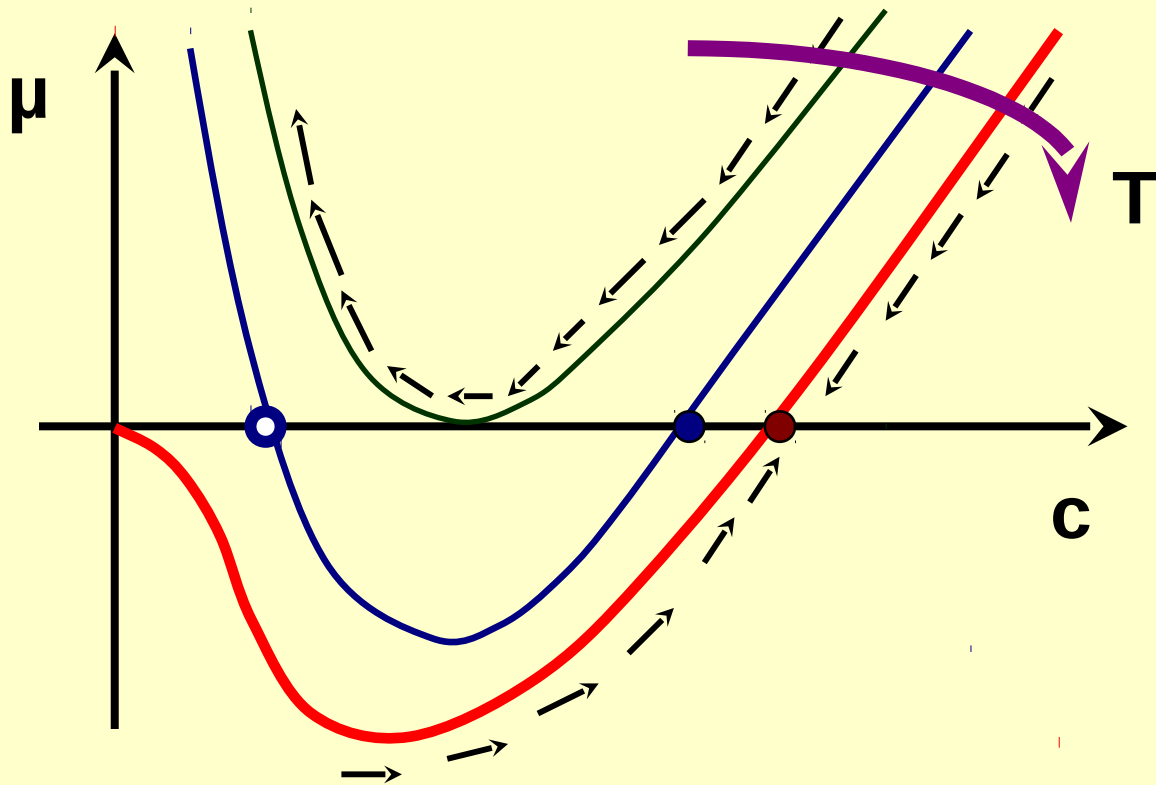
$$c = \sum_i c_i d_i n_{id} \left( T, q_i \mu_B - A c_i c^{-\gamma} \right)$$

$$\Rightarrow c = c(T, \mu_B)$$

It exists only beyond an endline in  $T - \mu$

# Stringy corrections to QGP

Consistent solution for color density in equilibrium:



It exists only beyond an endline in  $T - \mu$

# Stringy corrections to QGP

One massless component, Boltzmann approximation:

$$c = \frac{d}{\pi^2} T^3 e^{q\mu_B/T} e^{-Ac^{-\gamma}/T}$$

$$z = \frac{A}{T} c^{-\gamma}, \quad \alpha = \frac{A}{T} \left( \frac{d}{\pi^2} T^3 e^{q\mu_B/T} \right)^{-\gamma}$$

$$z = \alpha e^{\gamma z}, \quad z = -\frac{1}{\gamma} W(-\gamma\alpha)$$

The solution is related to Lambert's function



# Stringy corrections to QGP

Endline: last possible solution

*Boltzmann approximation*

$$z = \alpha e^{\gamma z}, \quad 1 = \gamma \alpha e^{\gamma z}$$

$$\Rightarrow z_E = \frac{1}{\gamma} \quad \Rightarrow \quad \alpha_E = \frac{1}{\gamma} e^{-1}$$

$$\mu_B = \frac{3\gamma + 1}{\gamma q} T \ln \frac{T_E}{T}$$

It is near to the zero pressure line for low chemical potential

# Stringy corrections to QGP

End temperature: last possible solution at  $\mu = 0$

*Boltzmann approximation*

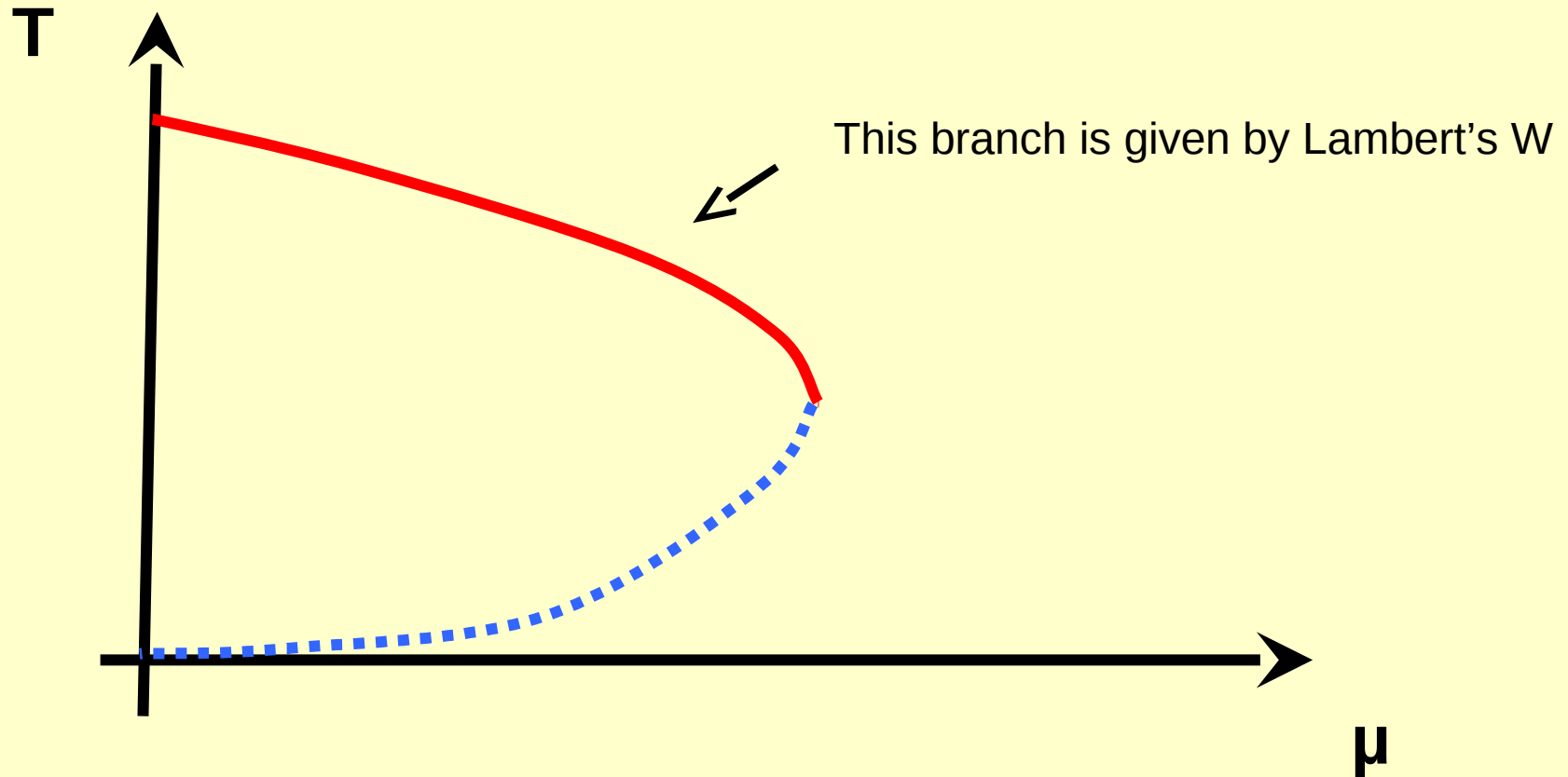
$$T_E = \left[ \left( \frac{d}{\pi^2} \right)^{-\gamma} \gamma e A \right]^{\frac{1}{1+3\gamma}}$$

It is near to the zero pressure line for low chemical potential

# Stringy corrections to QGP

*Boltzmann approximation*

Endline: last possible solution





# The zero pressure line

Zero pressure in the Boltzmann approximation

$$p = cT \left( 1 - \frac{\gamma}{1-\gamma} z \right)$$

$$\Rightarrow z_0 = \frac{1-\gamma}{\gamma} \quad \Rightarrow \quad \alpha_0 = \left( \frac{1}{\gamma} - 1 \right) e^{\gamma-1}$$

$$\mu_B = \frac{3\gamma+1}{\gamma q} T \ln \frac{T_0}{T}, \quad \frac{T_0}{T_E} = (1-\gamma)^{-\frac{1}{3\gamma+1}} e^{-\frac{\gamma}{3\gamma+1}}$$

**Endline and zero pressure line are relatively close! ( $T_0 \approx 1.04 T_E$ )**

# E/N at the zero pressure line

## General relations

$$\left. \frac{e}{n} \right|_{p=0} = \frac{\sum [e_{id} + \frac{1}{\gamma} p_{id}]}{\sum n_{id}}$$

$$\left. \frac{e}{n} \right|_{p=m=0} = \left( 3 + \frac{1}{\gamma} \right) T \frac{\sum d_i \Phi_i(\tilde{\mu}_i / T)}{\sum d_i \varphi_i(\tilde{\mu}_i / T)}$$

$$\left. \frac{e}{n} \right|_{p=m=0, \text{ Boltzmann}} = \left( 3 + \frac{1}{\gamma} \right) T$$

This is independent of the value of the string tension!

# High – T equation of state

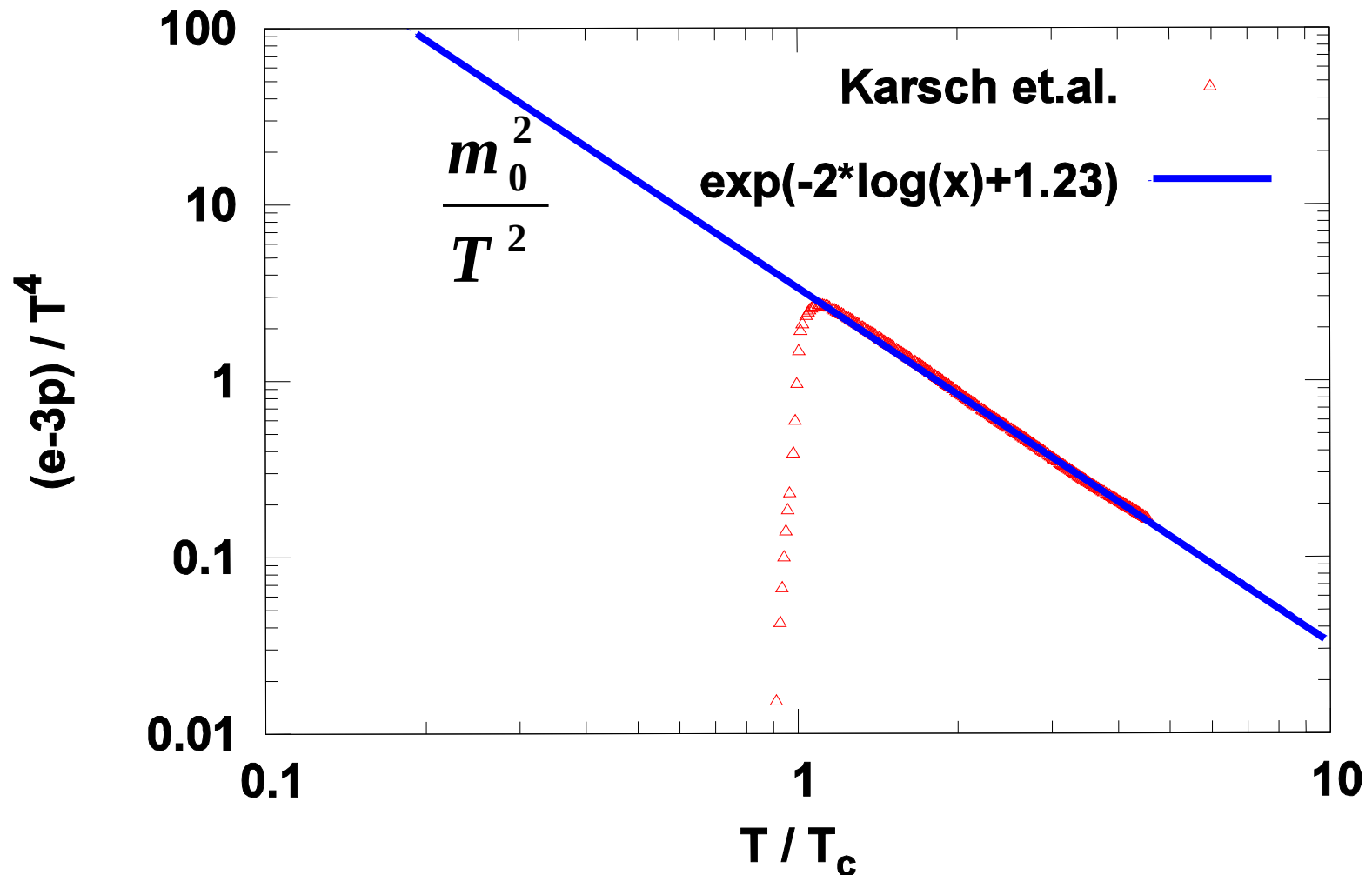
$$\frac{e - 3p}{T^4} = \frac{1 + 3\gamma}{1 - \gamma} \frac{A}{T^4} c^{1-\gamma} \sim T^{-3\gamma-1}$$

lattice QCD :

$$3\gamma + 1 = 2 \quad \rightarrow \quad \gamma = \frac{1}{3}$$

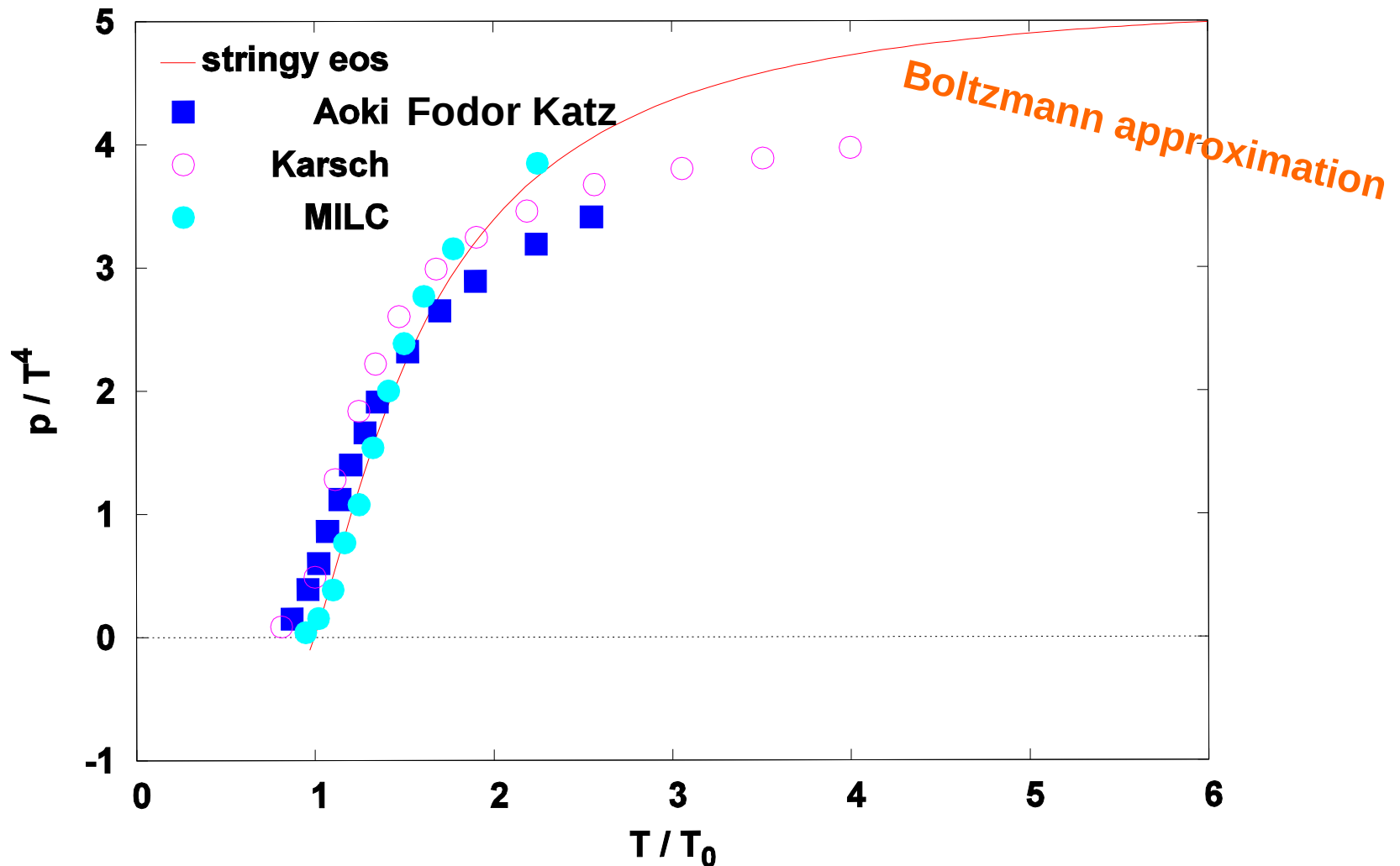
# High-T behavior of lattice eos

Interaction measure log-log plot

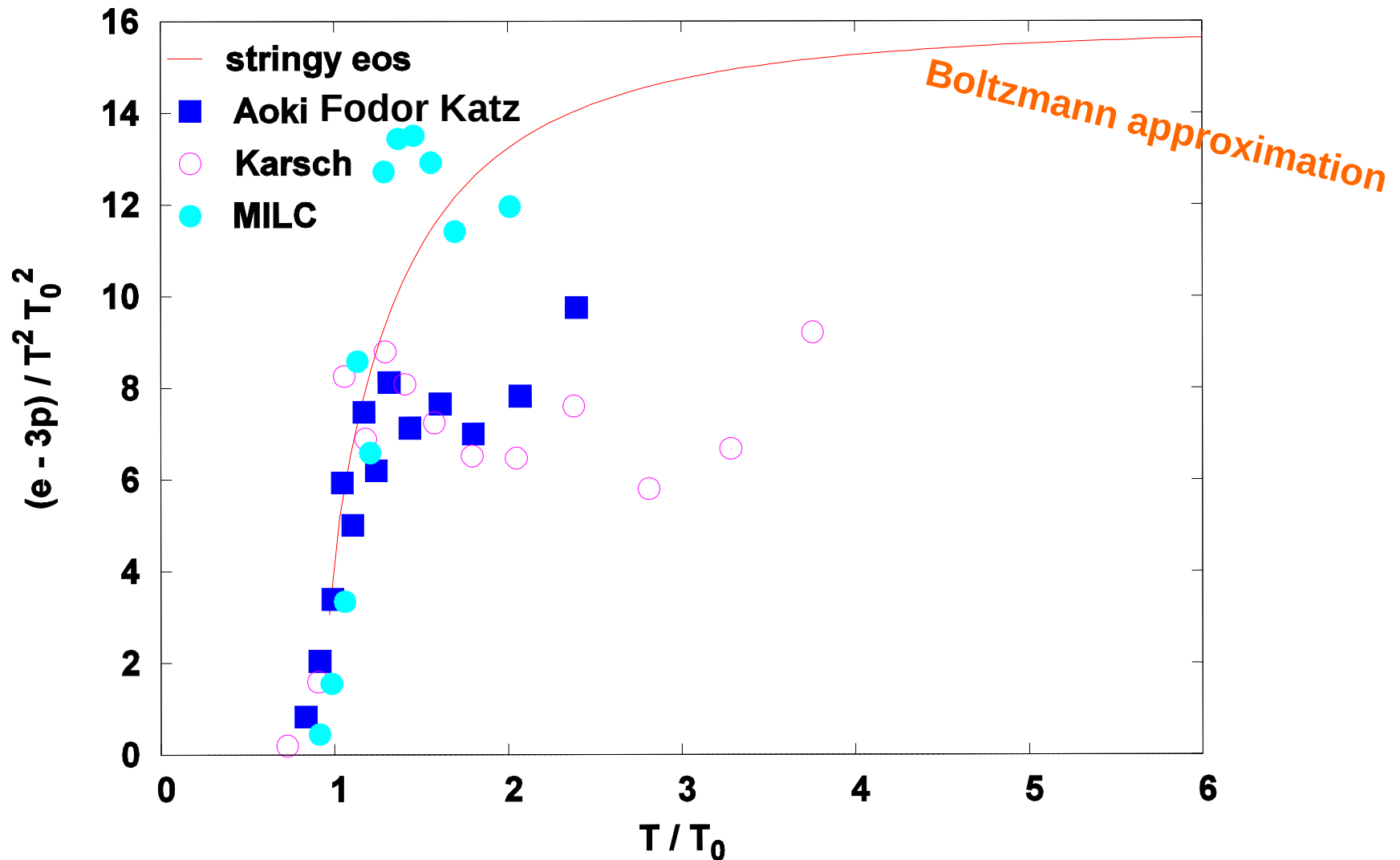




# High-T behavior of lattice eos



# High-T behavior of lattice eos



# Where to stop with the Boltzmann approximation?

Fermi integral  $\approx$  Boltzmann integral for negative or zero  $\mu$ .

$$q\mu_B - Ac^{-\gamma} \leq 0$$

$$\frac{q\mu_B}{T} = \left(3 + \frac{1}{\gamma}\right) \ln \frac{T_E}{T} \leq z_E = \frac{1}{\gamma}$$

$$T_E > T > T_E e^{-1/2} \approx 0.6 T_E$$

**The endline does not turn back!**

$$T_{\max} = T_E e^{-1}$$

# The zero pressure line

Fermions (quarks only) at  $T = 0$

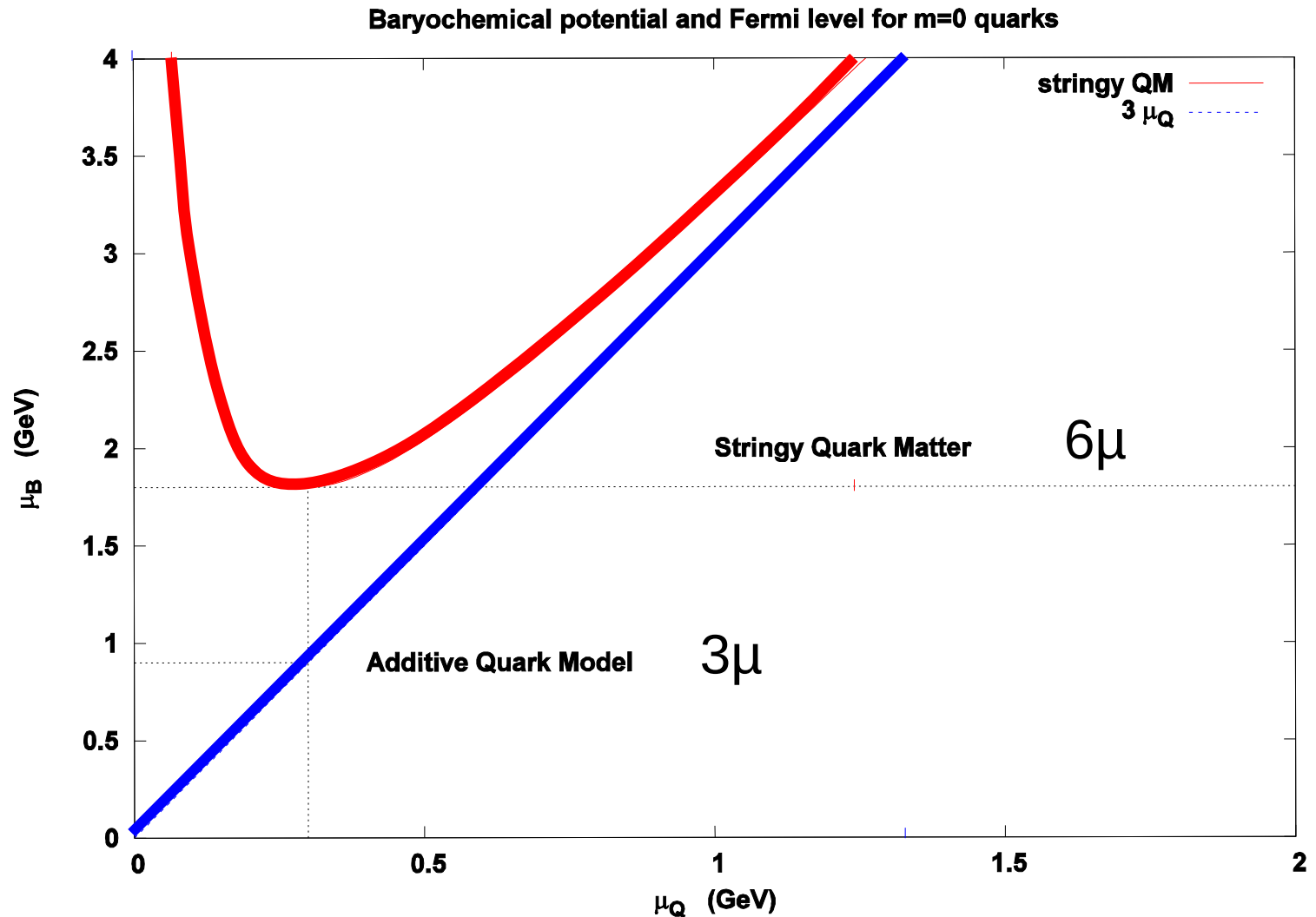
$$c = \frac{dc_q}{6\pi^2} \left( q\mu_B - Ac^{-1/3}c_q \right)^3$$

$$c = c_q \left[ \frac{q\mu_B}{2B} + \sqrt{\frac{q^2\mu_B^2}{4B^2} - \frac{\tilde{A}}{B}} \right]^3, \quad \left. \frac{e}{n} \right|_{p=0, T=0} = q\mu_B$$

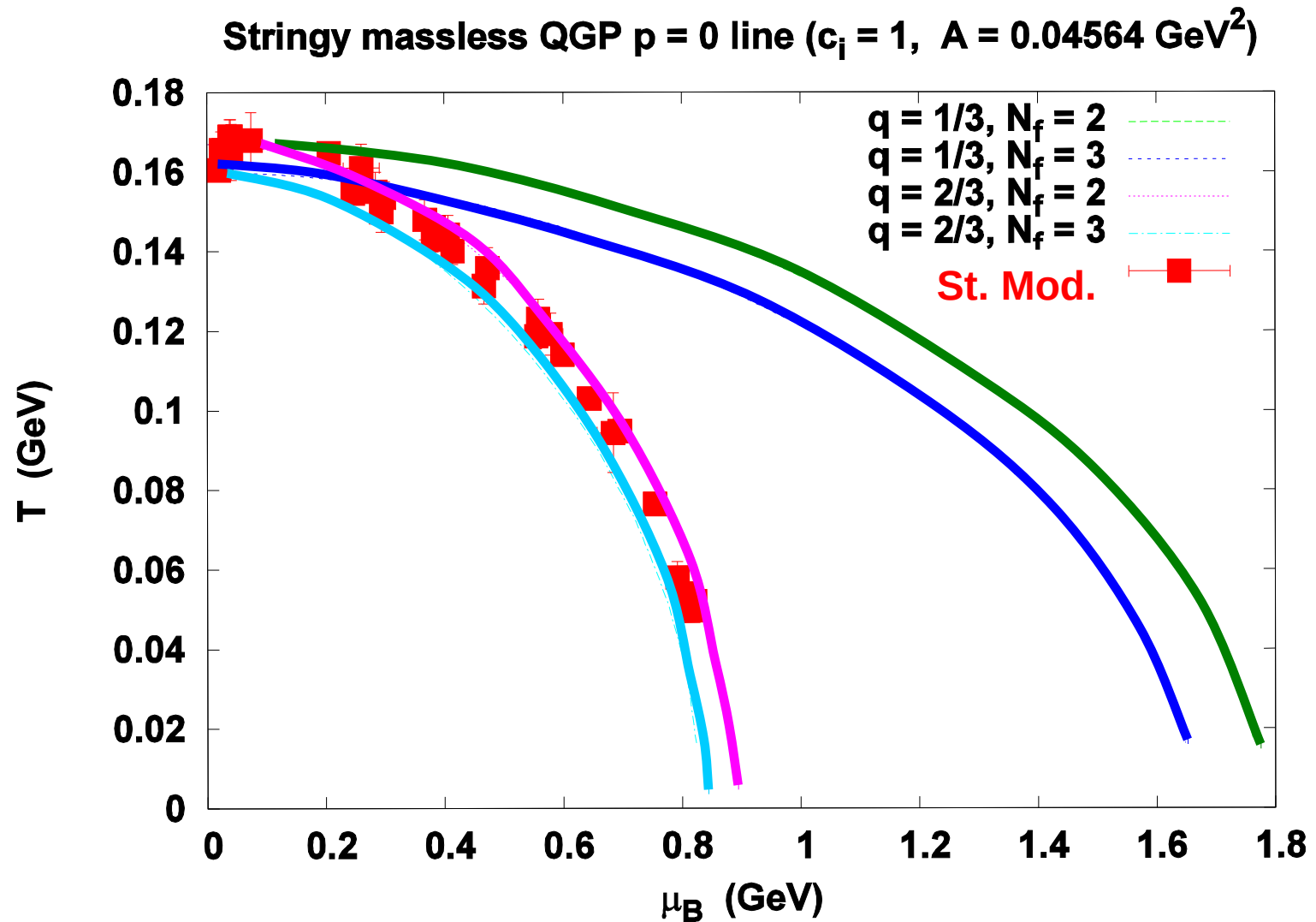
$$\mu_0^Q = \frac{3}{2\sqrt{2}} \mu_E \approx 1.06 \mu_E \approx 10.4 T_0^{QGP} \approx 1.74 \text{ GeV}$$



# The endpoint at T=0

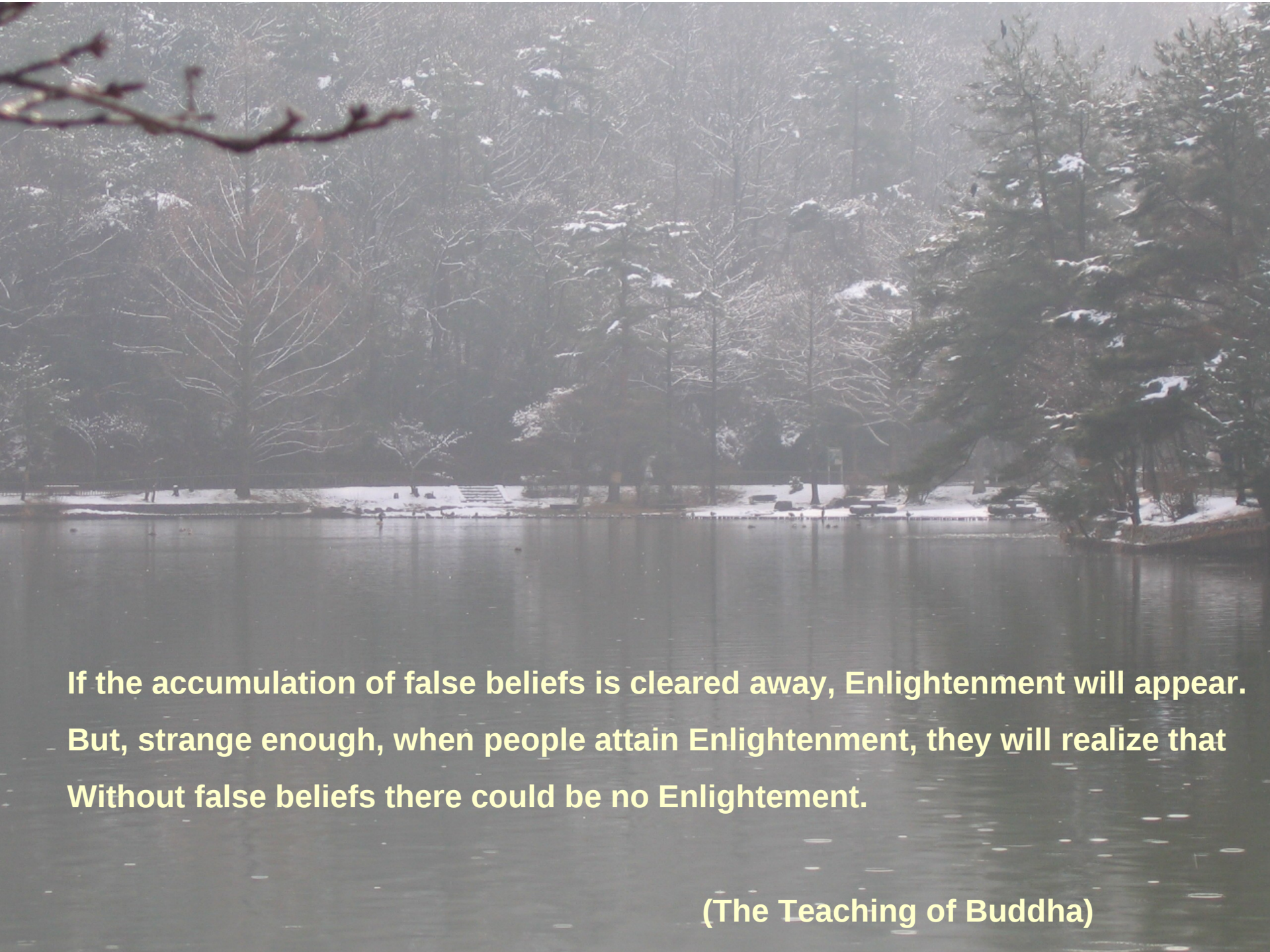


# The zero pressure line



# Summary

- $E / N = 6T$  only from stringy massless QGP
- fractional power is  $1/3$  from high- $T$  lattice eos
- zero pressure line and endpoint of solution are close
- at finite baryon density and zero temperature  
the hadronization line is at  $\mu_B = 3 k_F$
- at any temperature IR effects may be present  
(already a small string tension makes  $E / N = 6T$ )



If the accumulation of false beliefs is cleared away, Enlightenment will appear.  
But, strange enough, when people attain Enlightenment, they will realize that  
Without false beliefs there could be no Enlightenment.

(The Teaching of Buddha)



# Bibliography

- **arXiv: 0801.3998**, T.S.Biro and J. Cleymans: The hadronization line in stringy matter
- **hep-ph / 98083941**, T.S.Biro, P.Levai, J.Zimanyi and C.Traxler: Hadronization in heavy ion collisions, PRC59, ....., 1999
- T.S.Biro, A.A.Shanenko and V.D.Toneev: Toward Thermodynamic Consistency of Quasiparticle Picture, Phys.Atomic Nuclei 66, 982, 2003
- J.Cleymans, H.Oeschler, K.Redlich, S.Wheaton, PRC73, 034905, 2006.





**Discussion**

Handwritten text on a small white label, likely identifying the carving.

Handwritten text on a blue label, likely identifying the carving.



# FAQ

- **What is actually non-perturbative at  $T > T_c$  ? (low  $Q^2$  pairs)**
- **What is non-particle like (stringy) at  $T > T_c$  ? (interaction)**
- **Are all color charges stringed at  $T > T_c$ ? No! (a small fraction suffices for the effect.)**
- **Asymptotic freedom is incomplete at any finite temperature. Why and what is the quantitative measure of this effect? →**

# Arguments

Non-perturbative effects at arbitrary high temperature:

1. Thermal distribution of  $Q^2$
  2. NP order parameter: cut-off in  $Q^2$
  3. Its thermal expectation value  $\rightarrow$  order of NP effects
  4. high-T expansion
- $\square\square \rightarrow$  high-T NP terms in EoS (pressure, int.measure)



# Pressure: NP effects at any T

$$p = p_P \int_{\Lambda^2}^{\infty} P(Q^2) dQ^2 + p_{NP} \int_0^{\Lambda^2} P(Q^2) dQ^2$$

$$\int_0^{\Lambda^2} P(Q^2) dQ^2 = \int_0^{\Lambda^2/T^2} f(x) dx$$

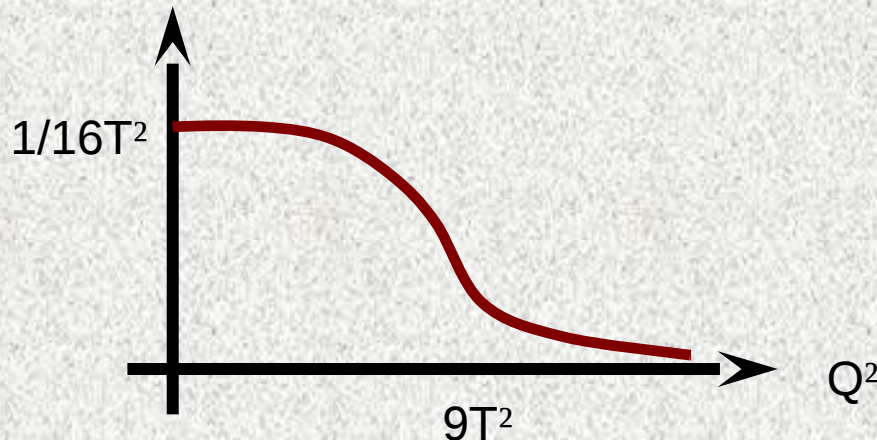
$$p = p_p - (p_p - p_{NP}) \begin{cases} 1 & (T \ll \Lambda) \\ \Lambda^2/T^2 f(0) & (T \gg \Lambda) \end{cases}$$

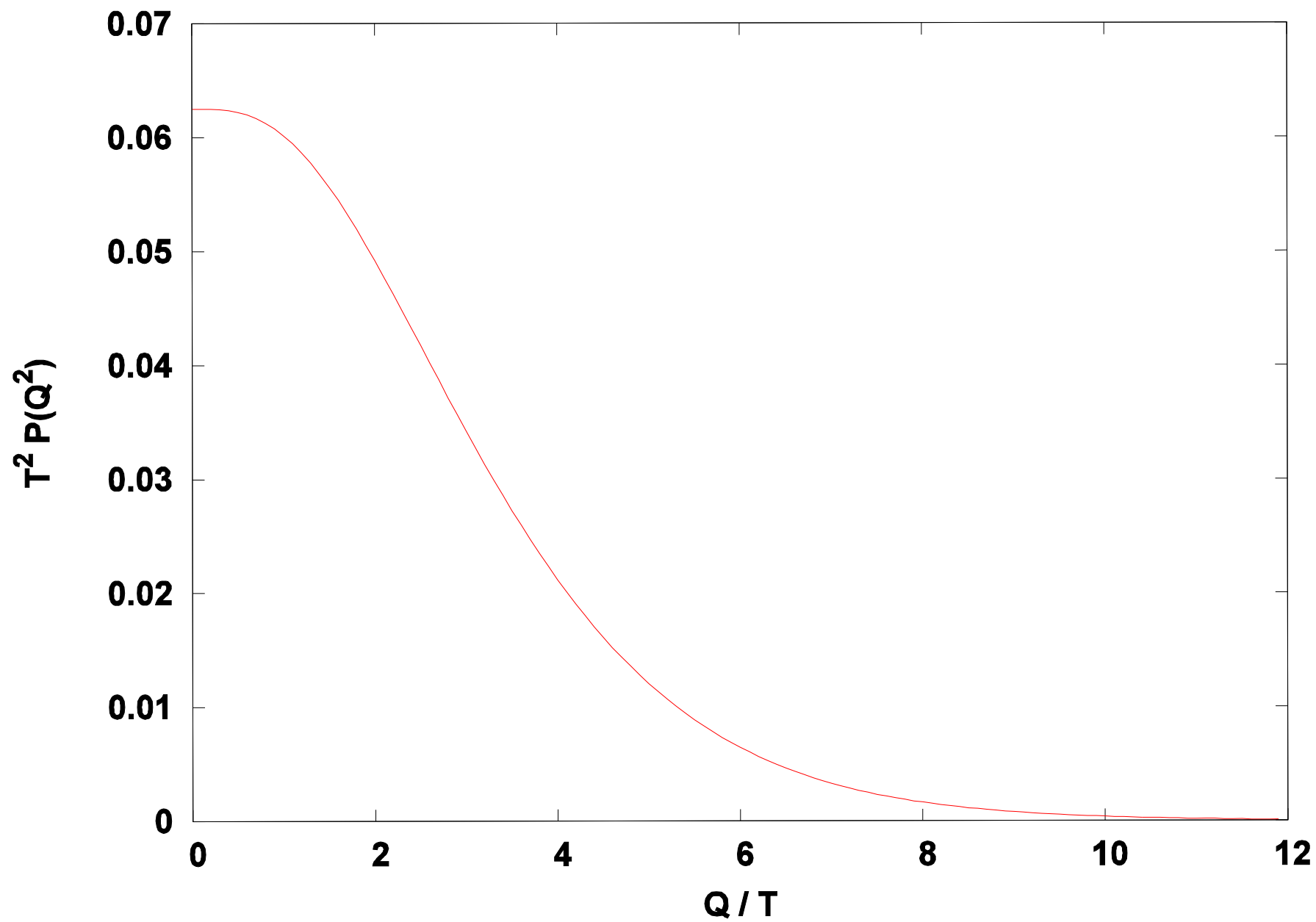
If it were  $f(0) = 0$ , then the QGP pressure would be free of NP effects!

# Thermal distribution of $Q^2$

$$P(Q^2) = \frac{\iint dE_1 dE_2 d\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)} \delta(Q^2 - 2E_1 E_2 (1 - \cos \theta))}{\iint dE_1 dE_2 d\theta E_1^2 E_2^2 e^{-\beta(E_1+E_2)}}$$

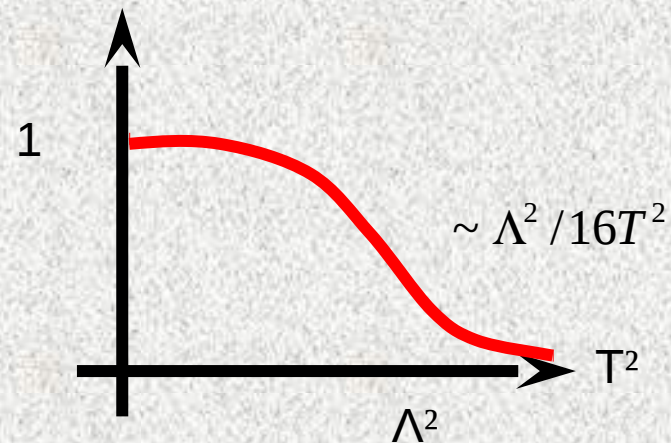
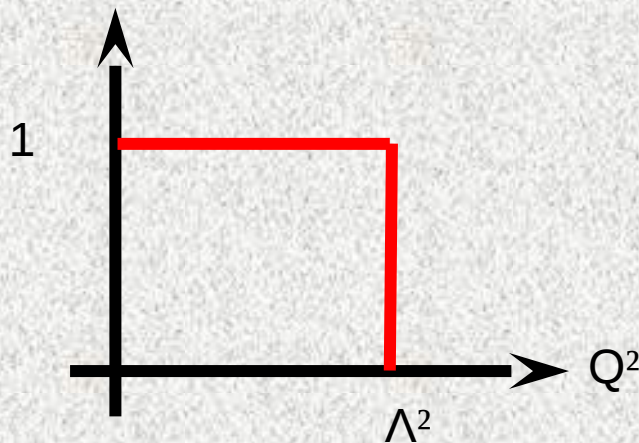
$$P(Q^2) = \frac{1}{64T^2} \left( \frac{Q^3}{T^3} K_1\left(\frac{Q}{T}\right) + 2 \frac{Q^2}{T^2} K_2\left(\frac{Q}{T}\right) \right)$$

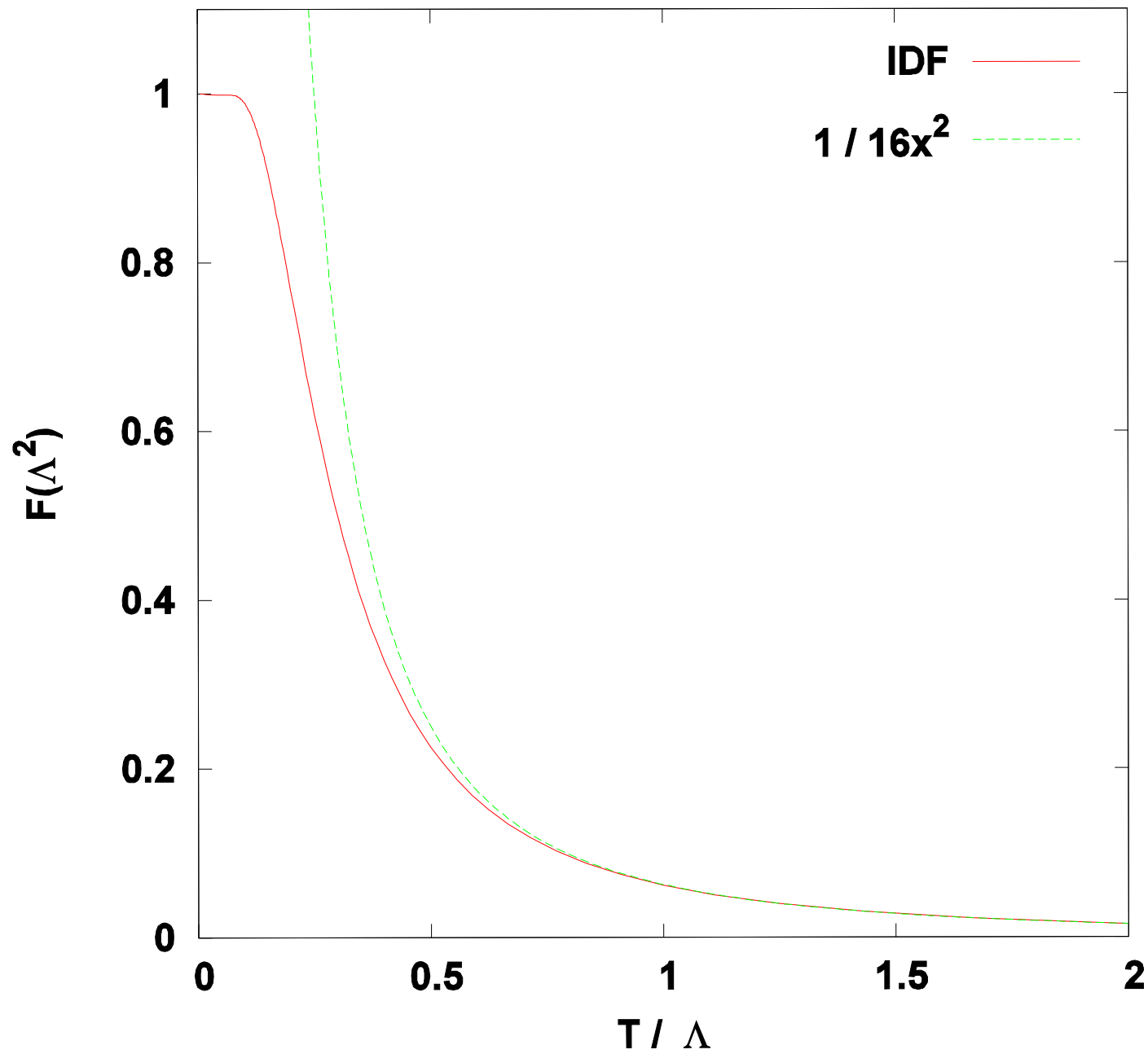




# Thermal expectation of NP order parameter

$$\langle \Theta(\Lambda^2 - Q^2) \rangle = \int_0^{\Lambda^2} P(Q^2) dQ^2$$



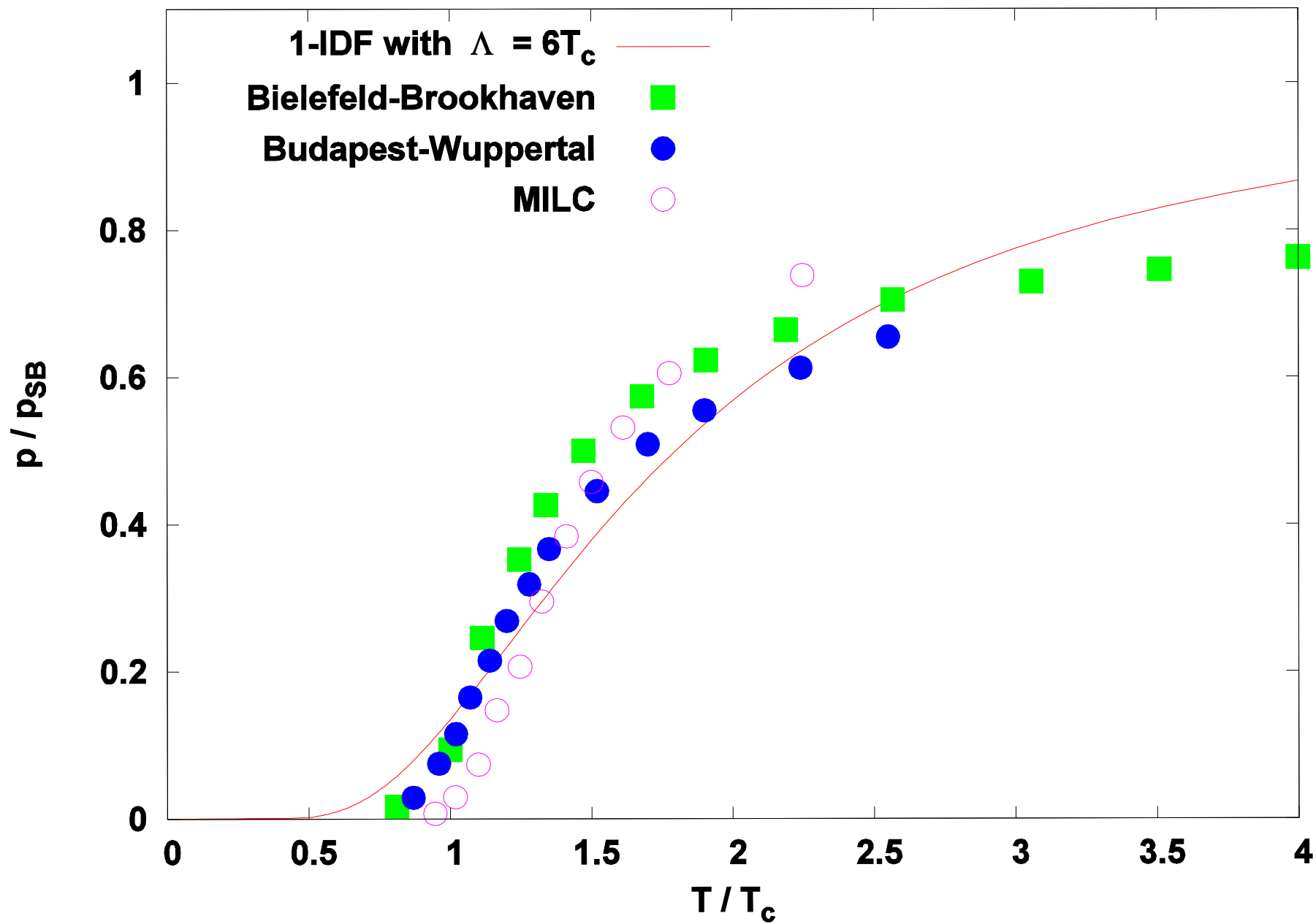


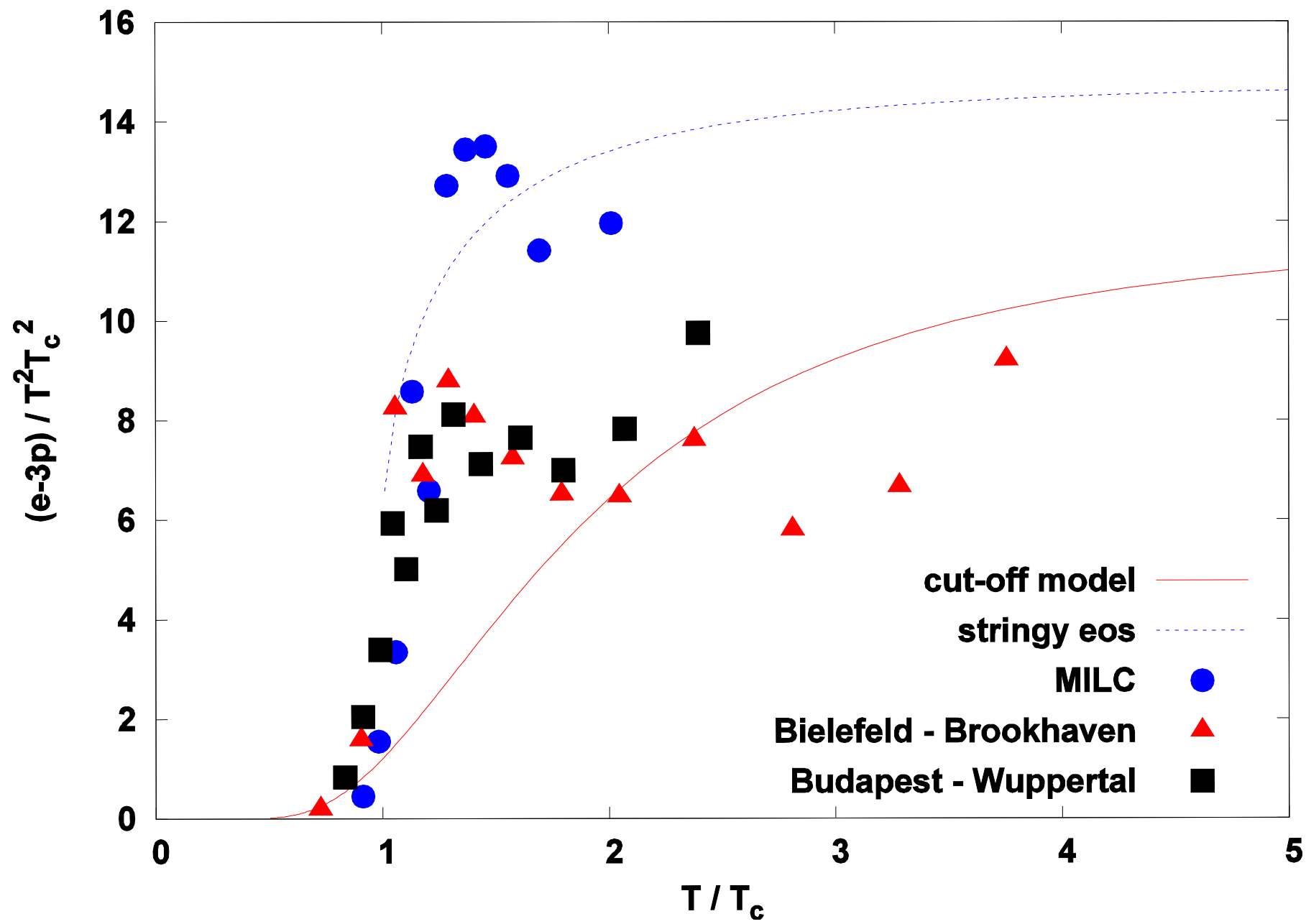


## NP effects at high T in the EoS

$$p = \frac{1}{3} \kappa T^4 - c_{NP} \Lambda^2 T^2$$

$$e - 3p = 2c_{NP} \Lambda^2 T^2$$





# To do list

1. Include massive quarks (strange)
2. Compare stringy matter and quasiparticle mass effects to leading order at high  $T$
3. Compare stringy eos with power-law tail effects
4. Compare with other simple ideas, like  $Q^2$  cutoff

