

Apparent Flow from pp to AA due to Radiation

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Overview

- Unruh temperature, K-Bessel spectrum, 'flow'-integral
- Finite time acceleration, Bjorken and Landau illusion
- v_2 photon fits to experiments
- Jacobi-Anger formula, v_n from interference

Radiation from moving charge

Photon number distribution is **Poisson** around the classical expectation value,

$$d^3N = \frac{1}{2\omega} d^3k (2\pi)^3 \sum |\epsilon^{(i)} \cdot J(k)|^2 \quad (1)$$

with point charge source moving on $x(\tau)$ we have

$$J(k) = q \int e^{ik \cdot x(\tau)} u^i(\tau) d\tau. \quad (2)$$

After partial integration

$$\epsilon \cdot J(k) = q \int_1^2 e^{ik \cdot x(\tau)} \frac{d}{d\tau} \left(\frac{\epsilon \cdot u}{k \cdot u} \right) \quad (3)$$

Invariant photon spectrum

$$\frac{dN}{k_\perp dk_\perp d\eta} = \frac{2\alpha}{\pi} \sum |\mathcal{A}|^2 \quad (4)$$

with $\mathcal{A} = \epsilon \cdot J(k)/q$. On straight line trajectories

$$k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = \frac{\alpha}{\pi} \left| \int_{v_1}^{v_2} e^{ik_\perp \gamma v/g} dv \right|^2. \quad (5)$$

Long acceleration scenarios

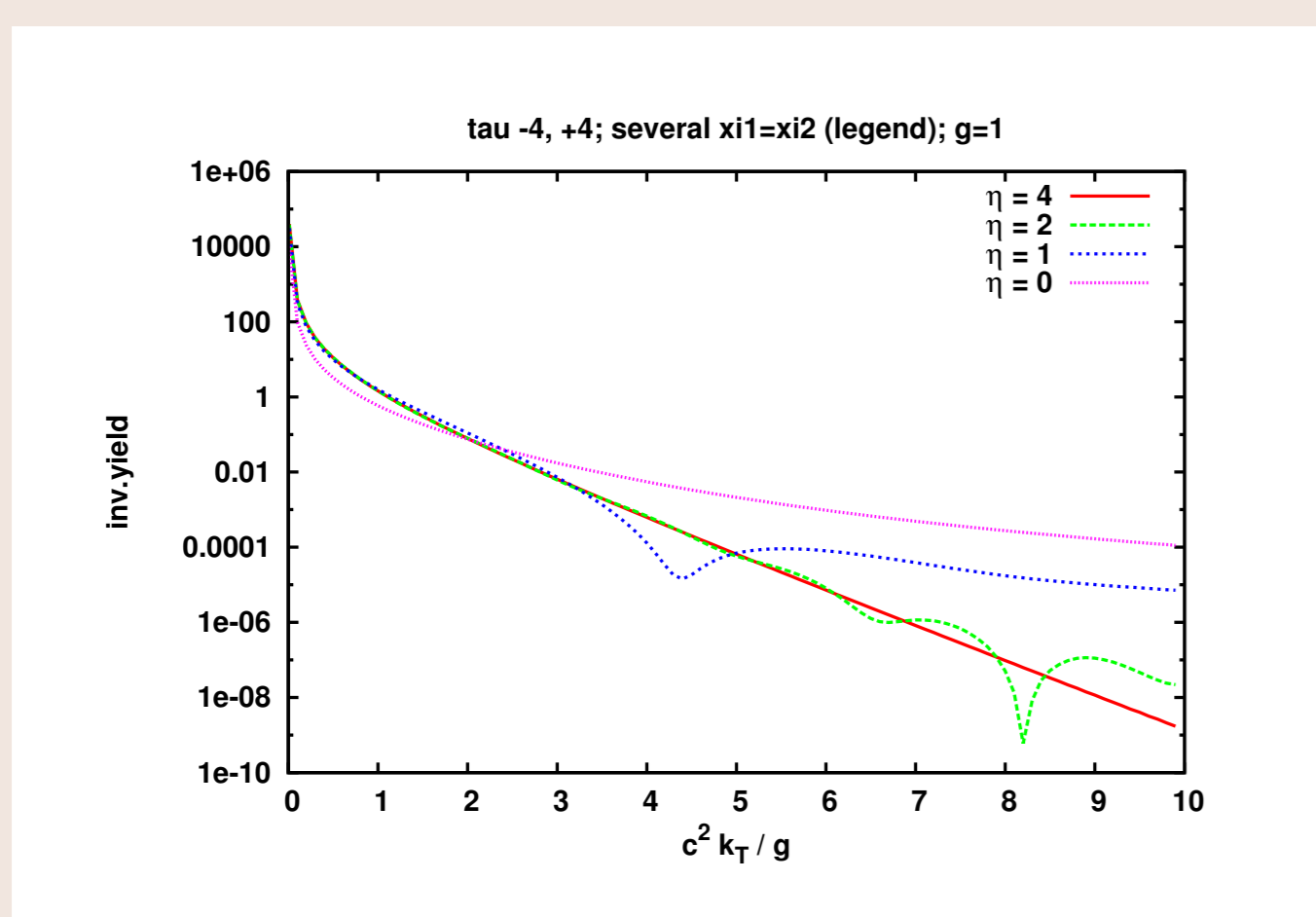
An in its own system constantly accelerating monochromatic source occurs to have a **Planck** black body spectrum for the far observer with a so called **Unruh temperature** of $T = g/2\pi$ in Planck units.

A forever constantly accelerating single charge emits a (semi-classical) radiation as eq.(5) describes integrated between $v_1 = -1$ and $v_2 = +1$:

$$\frac{dN}{k_\perp dk_\perp d\eta} = \frac{4\alpha}{\pi} \ell^2 K_1^2(\ell k_\perp), \quad (6)$$

with K_1 **K-Bessel** function, showing exponential tail, and $\ell = 1/g$ characteristic length scale. Note that

$$\int_{-\infty}^{+\infty} d\zeta K_2(k \cosh \zeta) = K_1^2(k/2).$$



Transverse photon spectra for long ($g\tau = 4 > \pi$) acceleration at different pseudorapidities.

Rapidity spectra

In the infrared limit $k_\perp \rightarrow 0$ the rapidity dependence can be obtained analytically.

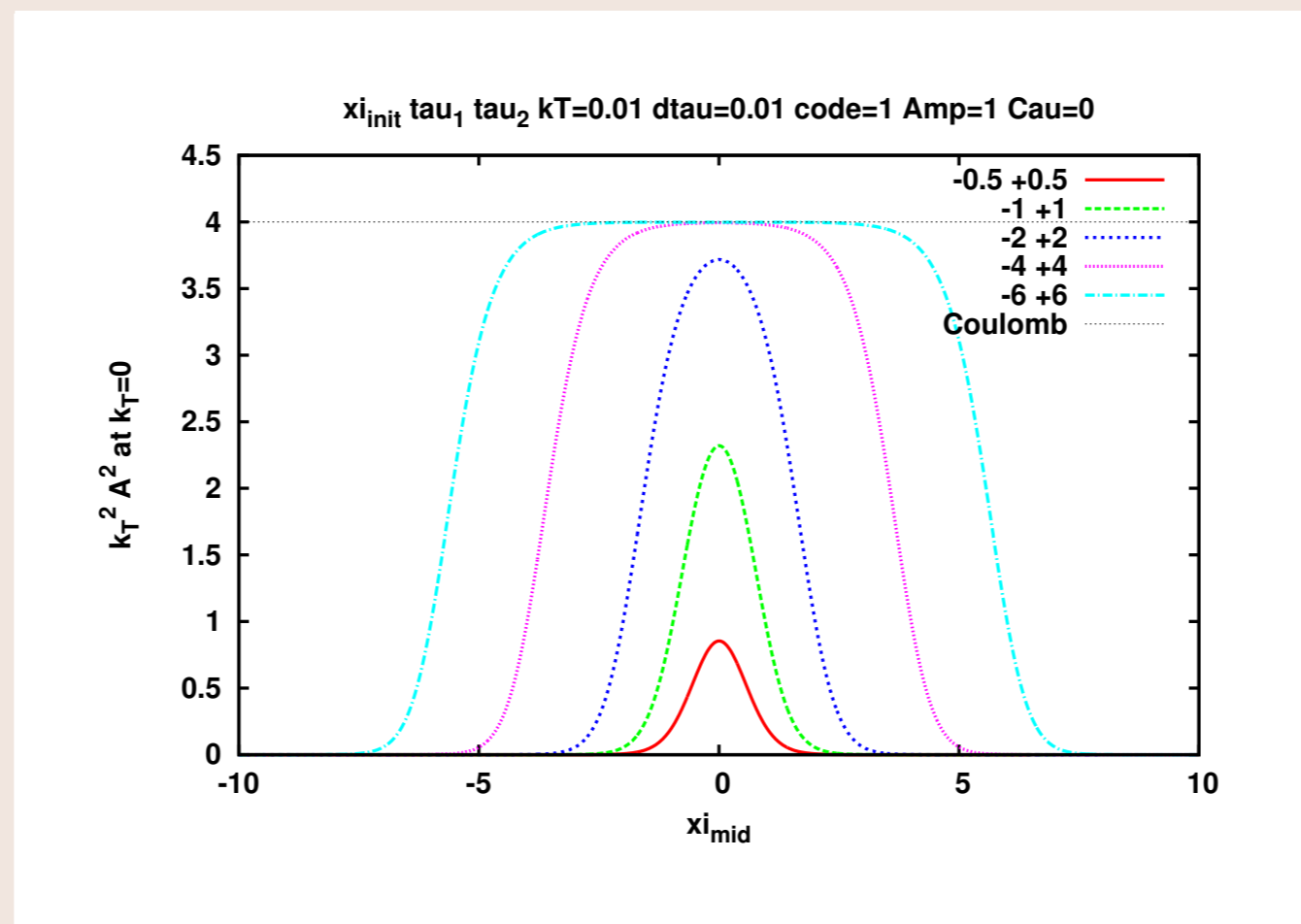
$$\lim_{k_\perp \rightarrow 0} k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = \frac{4\alpha}{\pi} \left(\frac{\text{sh } \delta \text{ ch } \delta}{\text{ch}^2(\xi - \eta) + \text{sh}^2 \delta} \right)^2 \quad (7)$$

with $\xi = (\xi_1 + \xi_2)/2$, $\delta = (\xi_2 - \xi_1)/2$ and $v_i = \tanh(\xi_i)$.

Short braking small δ , long braking large δ .

$$\lim_{k_\perp \rightarrow 0} k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = \frac{4\alpha}{\pi} \frac{\delta^2}{\text{ch}^4(\eta - \xi)} \quad (8)$$

$$\lim_{k_\perp \rightarrow 0} k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = \frac{4\alpha}{\pi} \frac{1}{(1 + e^{-2\delta \text{sh}^2(\eta - \xi)})^2} \quad (9)$$



Infrared photon rapidity distributions for long and short deceleration periods (see legend) obtained numerically at $k_\perp = 0.01g$.

Our numerical studies at finite k_\perp revealed that starting around $k_\perp \approx g/2$ an emerging edge effect develops over the rapidity plateau [1].

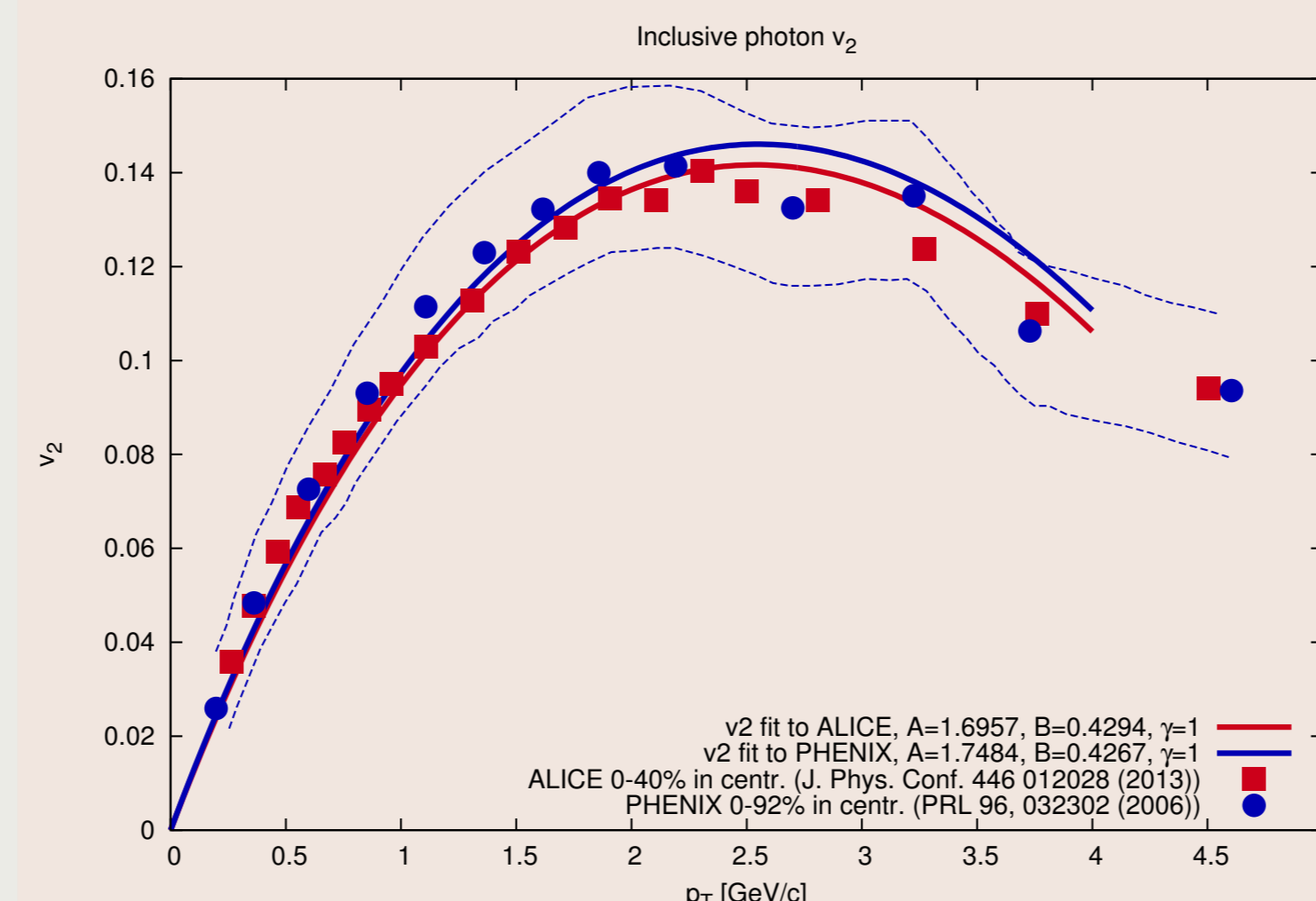
Fits to experimental data

In the simplest (two-antenna arrays) scenario we fit:

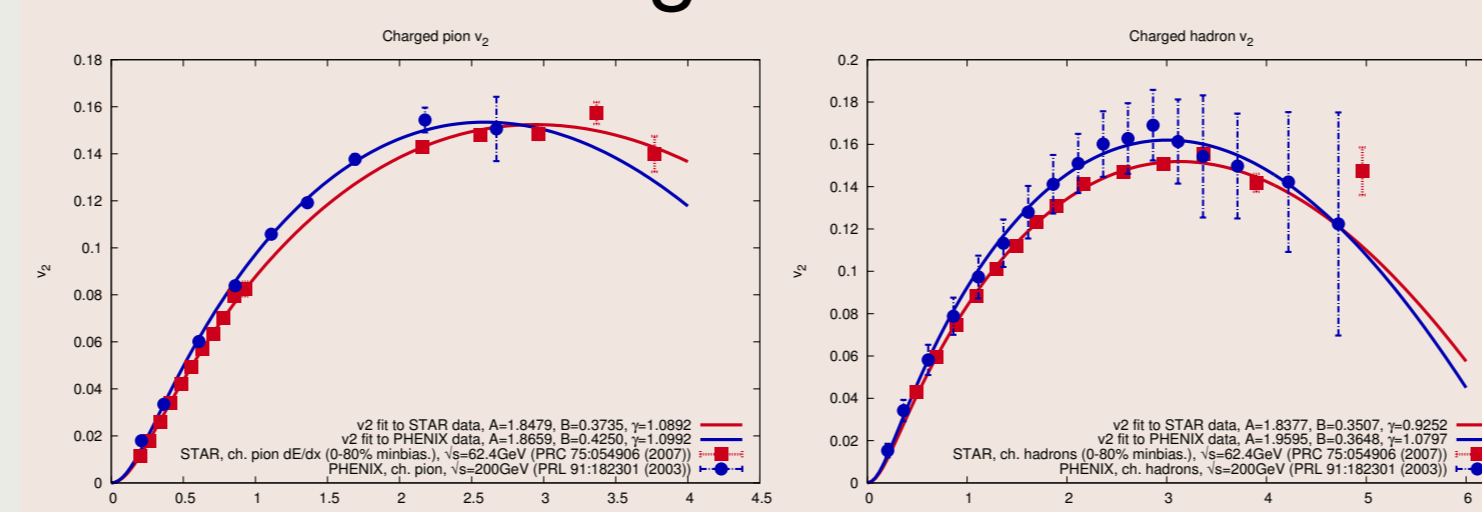
- $\epsilon = \frac{2\gamma}{1+\gamma^2}$, $\gamma = |A_1|/|A_2|$ magnitude ratio parameter
- $B = d$ antenna distance parameter
- $A = F_2$ geometric form factor

We assume that F_2 depends on centrality, but not on the momentum k_\perp .

Photon v_2



Pions and charged hadrons



Elliptic Flow from Waves

Jacobi-Anger Formula

$$e^{ix \cos \Theta} = J_0(x) + 2 \sum_{n=1}^{\infty} i^n J_n(x) \cos(n\Theta). \quad (10)$$

Interference Term in 1-Photon Yield is proportional to

$$Y \propto |A_1 e^{ik \cdot x_1} + A_2 e^{ik \cdot x_2}|^2 \quad (11)$$

Detector angle α , distance angle ψ , distance d result in

$$Y \propto |A_1 e^{ik_\perp \frac{d}{2} \cos(\alpha - \psi)} + A_2 e^{-ik_\perp \frac{d}{2} \cos(\alpha - \psi)}|^2 \quad (12)$$

Expanding the square we arrive at

$$Y \propto |A_1|^2 + |A_2|^2 + 2 \Re \left(A_1 A_2^* e^{ik_\perp d \cos(\alpha - \psi)} \right) \quad (13)$$

Flow coefficients are defined by relative amplitudes of $\cos(n\Theta)$ terms to the zeroth order term.

$$v_n = \frac{2R_n J_n(k_\perp d)}{|A_1|^2 + |A_2|^2 + R_0 J_0(k_\perp d)} \quad (14)$$

with

$$R_n := 2 \Re(i^n A_1 A_2^*) = 2 |A_1| |A_2| \cos\left(\Delta\varphi + n\frac{\pi}{2}\right).$$

In relative ratios of this *Young* interference k_\perp powers cancel in the ratio of A -squares!!!

We define the **interference ratio**:

$$r_n := \frac{2 |A_1| |A_2|}{|A_1|^2 + |A_2|^2} \cos\left(\Delta\varphi + \frac{n\pi}{2}\right). \quad (15)$$

We get

$$v_n = \frac{2r_n J_n(k_\perp d)}{1 + r_0 J_0(k_\perp d)} \quad (16)$$

Considering the phase difference $\Delta\varphi$

$$v_n = \frac{2\epsilon J_n \cos\left(\Delta\varphi + n\frac{\pi}{2}\right)}{1 + \epsilon J_0 \cos(\Delta\varphi)}. \quad (17)$$

Integrating over $\Delta\varphi$ **uniformly** we obtain

$$\langle v_n \rangle = 2 \cos\left(n\frac{\pi}{2}\right) \frac{J_n}{J_0} \left(1 - \frac{1}{\sqrt{1 - \epsilon^2 J_0^2}} \right) \quad (18)$$

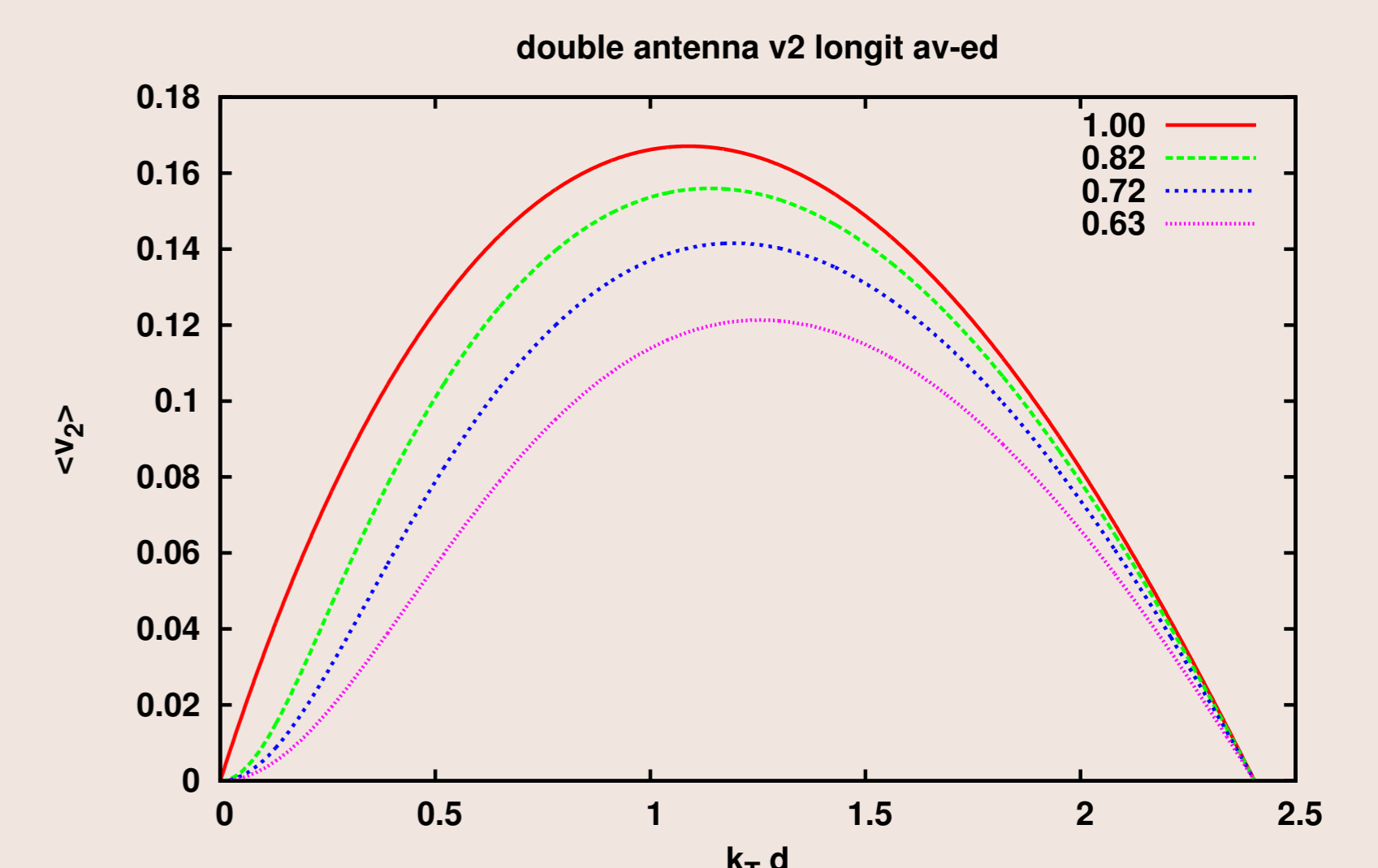


Figure: v_2 dependence on the amplitude ratio starts linear for equal amplitudes, otherwise quadratic.

References

- T.S.Biró, Z.Szendi, Z.Schram, EPJ A 50 (2014) 60.
- T.S.Biró, M.Gyulassy, Z.Schram, Phys.Lett. B 708 (2012) 276.