Nonrelativistic Bohmian Quantum Mechanics Special Relativistic Bohmian Quantum Mechanics General Relativistic Bohmian Quantum Mechanics Naturalness: restored! Summary

Dark Energy from Bohmian Gravitational Binding

Einstein's Schrödinger Equation

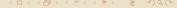
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Naturalness: where are you?

The "naturalness" problem:

Why is the cosmological constant so small?

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in natural units: $\mathbf{L}_P^2 \cdot \Lambda \approx 10^{-122}$



Outline

- 1 Nonrelativistic Bohmian Quantum Mechanics
- 2 Special Relativistic Bohmian Quantum Mechanics
- 3 General Relativistic Bohmian Quantum Mechanics
- 4 Naturalness: restored!



Outline

- Nonrelativistic Bohmian Quantum Mechanics
 - Wave function as a radius-angle complex
 - Schrödinger eq. from action principle
- 2 Special Relativistic Bohmian Quantum Mechanics
- General Relativistic Bohmian Quantum Mechanics
- A Naturalness: restored

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Wigner ...

Schrödinger eq. in magnitude-phase representation

$$-\frac{\hbar^2}{2m}\nabla^2\varphi + V(x)\varphi = i\hbar\frac{\partial}{\partial t}\varphi \tag{1}$$

General radial form for the complex wave function:

$$\varphi = Re^{\frac{i}{\hbar}\alpha}$$

Classical action and momentum

$$\frac{\partial \alpha}{\partial t} = -E, \qquad \nabla \alpha = P \tag{2}$$



Logarithmic Derivatives

$$\frac{\partial}{\partial t}\varphi = \left(\frac{1}{R}\frac{\partial R}{\partial t} - \frac{i}{\hbar}E\right)\varphi, \qquad \nabla\varphi = \left(\frac{1}{R}\nabla R + \frac{i}{\hbar}P\right)\varphi \quad (3)$$

Laplacian

$$\nabla^{2}\varphi = \left|\nabla\left(\frac{\nabla R}{R} + \frac{i}{\hbar}P\right) + \left(\frac{\nabla R}{R} + \frac{i}{\hbar}P\right)^{2}\right|\varphi \tag{4}$$



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Real and Imaginary Part

$$E = V - \frac{\hbar^2}{2m} \left[\nabla \frac{\nabla R}{R} + \left(\frac{\nabla R}{R} \right)^2 - \frac{P^2}{\hbar^2} \right]$$
 (5)

$$\frac{i\hbar}{R}\frac{\partial R}{\partial t} = -\frac{\hbar^2}{2m}\frac{i}{\hbar}\left[\nabla P + \frac{2}{R}P\cdot\nabla R\right] \tag{6}$$



Interpretation

Re: Energy = Classical + Quantum (Bohm potential)

$$E = \frac{P^2}{2m} + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \tag{7}$$

Im: Mass density current continuity

$$m\frac{\partial R^2}{\partial t} + \nabla \left(R^2 P\right) = 0 \tag{8}$$



Action Principle

Variational Principle behind the Schrödinger equation

$$\mathfrak{S} = \int \left(\frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} + V \right) |\varphi|^2 d^3x dt$$
 (9)

"Boltzmannian" eikonal ansatz: $S = \frac{\hbar}{i} \ln \varphi$ Using this ansatz:

$$\mathfrak{S} = \int \left[\frac{\hbar}{i} \varphi^* \frac{\partial \varphi}{\partial t} + \frac{\hbar^2}{2m} \nabla \varphi^* \cdot \nabla \varphi + V \varphi^* \varphi \right] d^3 x \, dt \qquad (10)$$

Variation against φ^* delivers

$$\frac{\delta\mathfrak{S}}{\delta\varphi^*} = \frac{\hbar}{i}\frac{\partial\varphi}{\partial t} - \frac{\hbar^2}{2m}\nabla^2\varphi + V\varphi = 0 \tag{11}$$



Action with Magnitude - Phase Variables

Split to Quantum + Classical parts

$$\mathfrak{S} = \int \left[\frac{\hbar^2}{2m} (\nabla R)^2 + R^2 \left(\frac{(\nabla \alpha)^2}{2m} + V + \frac{\partial \alpha}{\partial t} \right) \right] d^3 x \, dt \quad (12)$$

Structure of Quantum Principle:

$$\mathfrak{S} = \hbar^2$$
 (quantum kinetic) + R^2 (classically zero)

Path integral, tunneling: $S = \alpha - i\hbar \ln R$



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Outline

- Nonrelativistic Bohmian Quantum Mechanics
- Special Relativistic Bohmian Quantum Mechanics
 - Klein-Gordon Lagrangian
 - Action principle with Bohmian variables
 - Two expressions for the conserved energy
 - Relativistic Bohmian EM Tensor
- General Relativistic Bohmian Quantum Mechanics
- A Naturalness: restored



Klein-Gordon Lagrangian

Two expressions for the conserved energy
Relativistic Rohmian FM Tensor



Quantum Lagrangian

Lagrange density

$$\mathcal{L} = \frac{1}{2} \partial_i \psi^* \, \partial^i \psi \, - \frac{1}{2} \left(\frac{mc}{\hbar} \right)^2 \psi^* \, \psi. \tag{13}$$

Action and other conventions

$$\mathfrak{S} = \int \mathcal{L} \, d^4 x \tag{14}$$

with $dx^i = (cdt, d\vec{r})$. Physical units $[\mathcal{L}] = \text{energy density } / c = [mc/L^3]$.

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Klein-Gordon Lagrangian

Action principle with Bohmian variables
Two expressions for the conserved energy
Relativistic Bohmian EM Tensor



Madelung ansatz

Complex scalar field (related to the wave function)

$$\psi = \frac{\hbar}{\sqrt{mc}} R e^{i\alpha/\hbar}.$$
 (15)

Here α is only the (real) classical action. Units of R from

$$\left(mc/\hbar\right)^2 \psi^* \psi = mc R^2 \tag{16}$$

which is part of \mathcal{L} ; it follows that R^2 is a **number density**. Compare this with Maupertuis action for a classical mass point:

$$-\frac{1}{2}\int \left(\int mc^2R^2d^3x\right)dt = -\int mc^2d\tau \tag{17}$$

It follows a normalization $\int R^2 d^3x = 2$ in the comoving frame.

Klein-Gordon Lagrangian

Two expressions for the conserved energy
Relativistic Bohmian EM Tensor



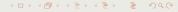
Derivative

First derivative of ψ :

$$\partial_i \psi = \left(\frac{\partial_i R}{R} + \frac{i}{\hbar} \partial_i \alpha\right) \psi \tag{18}$$

The derivative of the classical action is a classical momentum:

$$P_i = \partial_i \alpha, \qquad u_i = P_i/(mc).$$
 (19)



Two expressions for the conserved energy Relativistic Bohmian EM Tensor



Quantum Action

$$\mathfrak{S} = \frac{\hbar^2}{2mc} \int \left[\partial_i R \, \partial^i R + \frac{R^2}{\hbar^2} \left(\partial_i \alpha \, \partial^i \alpha - (mc)^2 \right) \right] \, d^4 x \quad (20)$$

Rewritten as a sum of a quantum and a classical part:

$$\mathfrak{S} = \int \left[\frac{\hbar^2}{2mc} \partial_i R \, \partial^i R \right] + \frac{R^2}{2mc} \left(P_i P^i - (mc)^2 \right) \right] d^4 x \quad (21)$$



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Two expressions for the conserved energy Relativistic Bohmian EM Tensor



U(1) Noether charge

$$J^{k} = \frac{i}{2\hbar} \left(\psi \partial^{k} \psi^{*} - \psi^{*} \partial^{k} \psi \right) = \frac{1}{mc} R^{2} P^{k} = R^{2} u^{k}$$
 (22)

is a number density 4-current $R^2u^k = \rho u^k$.

Conserved by variation of \mathfrak{S} wrsp. α :

$$\frac{\delta \mathfrak{S}}{\delta \alpha} = -\partial_k \left(\frac{1}{mc} R^2 \partial^k \alpha \right) = 0 \tag{23}$$

Facit:

$$\frac{\delta \mathfrak{S}}{\delta \alpha} = \frac{\delta \mathbf{S}_{\text{quantum}}}{\delta \mathbf{S}_{\text{classical}}} = -\partial_k \mathbf{J}^k = -\partial_k (\rho \mathbf{u}^k) = 0. \tag{24}$$

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Off Mass Shell

Variation against R delivers

$$\frac{\delta \mathfrak{S}}{\delta R} = -\frac{\hbar^2}{mc} \Box R + \frac{R}{mc} \left(P_i P^i - (mc)^2 \right) = 0. \tag{25}$$

Off-mass shell dispersion relation for the classical 4-momentum

$$P_i P^i - (mc)^2 = \hbar^2 \frac{\square R}{R} \tag{26}$$

Metric view:

$$g_{ij}u^{i}u^{j}=1+\left(\frac{\hbar}{mc}\right)^{2}\frac{\Box R}{R} \tag{27}$$

Note: this is a Compton wavelength scaled, locally Lorentzian spacetime metric.



Energy-Momentum Tensor I

(textbook)

Derivation I: using ψ and ψ^* .

(... as in textbooks)

$$\Pi_{k} = \frac{\delta \mathcal{L}}{\delta \partial^{k} \psi} = \frac{1}{2} \partial_{k} \psi^{*}$$
 (28)

$$T_{ij} = \Pi_i \partial_j \psi + \Pi_i^* \partial_j \psi^* - g_{ij} \mathcal{L}. \tag{29}$$

in terms of R and α :

$$T_{ij} = mcR^{2} w_{ij} + \frac{\hbar^{2}}{mc} U_{ij},$$

$$w_{ij} = u_{i}u_{j} - \frac{1}{2}g_{ij} (u_{k}u^{k} - 1),$$

$$U_{ij} = \partial_{i}R \partial_{j}R - \frac{1}{2}g_{ij} \partial_{k}R \partial^{k}R.$$
(30)

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Energy-Momentum Tensor I

(textbook)

Using the off-mass-shell relation (26) leads to:

$$T_{ij} = mcR^{2}u_{i}u_{j} + \frac{\hbar^{2}}{mc} \left(\partial_{i}R \, \partial_{j}R - \frac{1}{2}g_{ij} \left(\partial_{k}R \partial^{k}R + R\Box R \right) \right)$$
(31)

Here the $\hbar^2($) part is the quantum contribution, the rest is just $\frac{\text{dust}}{}$.

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Energy-Momentum Tensor II

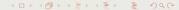
(Takabayashi)

Take the derivative of the off-mass-shell eqation (27):

$$\frac{R^2}{2}\,\partial_i\left[u_ku^k-1-\frac{\hbar^2}{(mc)^2}\frac{\Box R}{R}\right]=0\tag{32}$$

Use Compton wavelength $L_C = \hbar/mc$ and expand:

$$R^2 u^k \partial_i u_k - \frac{1}{2} L_C^2 R^2 \partial_i \left(\frac{\Box R}{R} \right) = 0$$
 (33)



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Energy-Momentum Tensor II

(Takabayashi)

The Madelung fluid is irrotational:

$$\partial_i u_k = \frac{1}{mc} \partial_i \partial_k \alpha = \frac{1}{mc} \partial_k \partial_i \alpha = \partial_k u_i$$
 (34)

Therefore

$$R^2 u^k \partial_i u_k = R^2 u^k \partial_k u_i = \partial_k (R^2 u^k u_i) - u_i \partial_k (R^2 u^k)$$
 (35)

and due to continuity the last term vanishes.





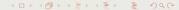
Energy-Momentum Tensor II

By these manipulations we obtain

$$\partial_k \left(R^2 u^k u_i \right) = \frac{1}{2} L_C^2 R^2 \partial_i \left(\frac{\square R}{R} \right) \tag{36}$$

Further use of the Leibniz rule leads to

$$R^{2}\partial_{i}\left(\frac{\square R}{R}\right) = R\square\partial_{i}R - \partial_{i}R\square R$$
$$= \partial^{k}\left(R\partial_{k}\partial_{i}R - \partial_{k}R\partial_{i}R\right) \tag{37}$$





Energy-Momentum Tensor II

This reveals a vanishing divergence of the Bohm-Takabayashi tensor

$$\mathcal{T}_{ij} = mcR^2 u_i u_j - \frac{\hbar^2}{2mc} \left(R \partial_i \partial_j R - \partial_i R \partial_j R \right)$$
 (38)

It differs from the Klein-Gordon one (31) by

$$\Delta_{ij} = T_{ij} - \mathcal{T}_{ij} = \frac{\hbar^2}{2mc} \left(\partial_i R \partial_j R + R \partial_i \partial_j R - g_{ij} (\partial_k R \partial^k R + R \Box R) \right)$$
(39)



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Difference

We note that

$$(g_{ij}\Box - \partial_i\partial_j)\frac{R^2}{2} = g_{ij}(\partial_k R \partial^k R + R\Box R) - \partial_i R \partial_j R - R \partial_i \partial_j R \tag{40}$$

One realizes that

$$\Delta_{ij} = \frac{\hbar^2}{4mc} \left(\partial_i \partial_j - g_{ij} \Box \right) R^2 \tag{41}$$

has a vanishing divergence.

Note:
$$\Delta_{ij} = \partial^a f_{aij}$$
 with $f_{aij} = \frac{\hbar^2 R}{2mc} (g_{ai} \partial_j R - g_{ij} \partial_a R)$.



Full Quantum Energy-Momentum Tensor III

It can be derived in two ways: from Klein-Gordon Lagrangian and from Madelung fluid hydrodynamics.

They differ in a tensor part with *vanishing divergence*. The general tensor contains parameter μ multiplying the divergenceless part:

$$\mathfrak{T}_{ij} = mcR^2 u_i u_j + \mathfrak{U}_{ij} + \mu \Delta_{ij}$$
 (42)

with the general Bohm-Takabayashi term

$$\mathfrak{U}_{ij} = \frac{\hbar^2}{2mc} \left(\partial_i R \partial_j R - R \partial_i \partial_j R \right). \tag{43}$$

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Trace of Energy-Momentum Tensor

$$\mathfrak{T}_{i}^{i} = \left\{1 + L_{C}^{2} \frac{1 - 3\mu}{4} \square\right\} (mcR^{2}) \tag{44}$$

For $\mu = 0$ original Bohm potential.

For $\mu = 1/3$ only classical dust contributes to trace.

For $\mu = 1$ the original Klein-Gordon case.



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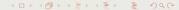
Energy-Momentum Tensor in scaling variables

Use $R = e^{\sigma}/\sqrt{V}$, then

$$\mathfrak{T}_{j}^{i} = \frac{mc}{V}e^{2\sigma}u^{i}u_{j} + \hbar^{2}\mathfrak{W}_{j}^{i}$$
 (45)

with

$$\mathfrak{W}_{j}^{i} = \frac{1}{2mcV} e^{2\sigma} \left[2\mu \partial^{i} \sigma \, \partial_{j} \sigma + (\mu - 1) \partial^{i} \partial_{j} \sigma - \mu \delta_{j}^{i} \left(2\partial_{k} \sigma \partial^{k} \sigma + \Box \sigma \right) \right]$$
(46)

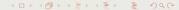


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Outline

- Nonrelativistic Bohmian Quantum Mechanics
- Special Relativistic Bohmian Quantum Mechanics
- General Relativistic Bohmian Quantum Mechanics
 - Einstein equation
 - Consequences of a conformal transformation
 - Identification of the Bohmian terms
- 4 Naturalness: restored





Einstein equation

Consider the downscaled μ -Madelung-Bohm-Takabayashi energy-momentum as a source of gravity:

$$G_j^i - \Lambda \delta_j^i = \frac{8\pi G}{c^3} e^{-2\sigma} \mathfrak{T}_j^i. \tag{47}$$

Use the Schwarzschild length (half-radius) $L_S = \frac{Gm}{c^2}$, to achieve

$$G_{j}^{i} - \Lambda \delta_{j}^{i} = \frac{8\pi L_{S}}{V} u^{i} u_{j} + \frac{4\pi L_{S} L_{C}^{2}}{V} \mathfrak{Q}_{j}^{i}$$

$$\mathfrak{Q}_{j}^{i} = 2\mu \partial^{i} \sigma \partial_{j} \sigma + (\mu - 1) \partial^{i} \partial_{j} \sigma - \mu \delta_{j}^{i} (2\partial_{k} \sigma \partial^{k} \sigma + \Box \sigma).$$



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Conformal transformation by $\Omega(x)$

From (originally flat) spacetime to a curved one:

$$g'_{ik} = e^{2s} \eta_{ik} \tag{48}$$

(induced additive change in) Christoffel symbol

$$\delta\Gamma^{j}_{ik} = \partial_{i}s \,\delta^{j}_{k} + \partial_{k}s \,\delta^{j}_{i} - \partial^{j}s \,\eta_{ik} \tag{49}$$

(induced additive change in) Ricci tensor

$$\delta R_{ik} = 2 \left(\partial_i s \, \partial_k s - \partial_i \partial_k s \right) - \left(\Box s + 2 \, \partial_j s \, \partial^j s \right) \, \eta_{ik} \tag{50}$$

(induced additive change in) Ricci scalar

$$\delta \mathcal{R} = -6 e^{-2s} \left(\Box s + \partial_j s \, \partial^j s \right) \tag{51}$$

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Transformed Einstein equation

Regard a conformally transformed Einstein tensor.

$$\overline{G}_{j}^{i} - \Lambda \delta_{j}^{i} = \frac{8\pi G}{c^{3}} e^{-2\sigma} \mathfrak{T}_{j}^{i}$$
 (52)

after conformal transformation becomes:

$$\left[G_{j}^{i} + \left(2\Box s + \partial_{k} s \partial^{k} s - \Lambda\right) \delta_{j}^{i} - 2\partial^{i} \partial_{j} s + 2\partial^{i} s \partial_{j} s\right] = \frac{8\pi G}{c^{3}} e^{-2\sigma} \mathfrak{T}_{j}^{i}.$$
(53)

We want to connect s and σ . As a source we insert our μ -Madelung Energy-Momentum-Tensor as obtained above.



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Term by term identification

$$G_{j}^{i} = \frac{8\pi L_{S}}{V} u^{i} u_{j}$$

$$\left(2\Box s + \partial_{k} s \partial^{k} s - \Lambda\right) = -\mu \frac{4\pi L_{S} L_{C}^{2}}{V} \left(2\partial_{k} \sigma \partial^{k} \sigma + \Box \sigma\right)$$

$$-2\partial^{i} \partial_{j} s = (\mu - 1) \frac{4\pi L_{S} L_{C}^{2}}{V} \partial^{i} \partial_{j} \sigma$$

$$2\partial^{i} s \partial_{j} s = 2\mu \frac{4\pi L_{S} L_{C}^{2}}{V} \partial^{i} \sigma \partial_{j} \sigma$$



Obviously: $s = \sigma$!

We have classical dust

$$G_j^i = \frac{8\pi L_S}{V} u^i u_j, \tag{54}$$

we have $\mu = 1/3$, i.e. **conformal** quantum part, and

$$V = \frac{4\pi}{3} L_S L_C^2 \tag{55}$$

is a scaled **Planck volume** (by M_P/m). Finally we have a **cosmolgical** term:

$$\Lambda = 3\left(\Box \sigma + \partial_k \sigma \partial^k \sigma\right) = 3\frac{\Box R}{R} \tag{56}$$

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The Constant Part of the Cosmological Term

For a mass m particle - bound in $-\alpha/r$ potential

$$\langle \Lambda \rangle = -3 \langle \frac{\nabla^2 R}{R} \rangle = \frac{3}{a^2}$$
 (57)

with $a = L_C/\alpha = \hbar/\alpha mc$ being the Bohr radius.

An "onium" of two equal masses of *m* has double Bohr radius (reduced mass is half).

It is nice to use units with c = 1. Then

$$\hbar = L_P \cdot M_P;$$
 $G = \frac{L_P}{M_P}.$



Grav-Onium Scenario: formulas

Gravitational α from

thanks to Antal Jakovác for (re-)forcing this view!

$$\frac{\alpha}{r} = \frac{Gm^2}{\hbar r} \qquad \Longrightarrow \qquad \alpha = \frac{m^2}{M_P^2}.$$

Gravi-Bohr radius: $a_B = \frac{\hbar}{\alpha m} = L_P \left(\frac{M_P}{m}\right)^3$.

Grav-Onium radius: $a = 2a_B$, reduced mass: $m_{red} = m/2$.

Cosmo constant: $L_P^2 \cdot \Lambda = 3L_P^2/a^2 = 0.75 \, (m/M_P)^6$.

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Grav-Onium Scenario: numbers

Planck scale: $L_P \approx 1.6 \cdot 10^{-20} \text{ fm}$ $M_P \approx 1.2 \cdot 10^{19} \text{ GeV}.$

Cosmo constant: $\Lambda \approx 10^{-82} \text{ fm}^{-2}$

in natural units: $L_P^2 \cdot \Lambda \approx 2.56 \cdot 10^{-122}$

The reduced mass ratio: $m/M_P \approx 5.7 \cdot 10^{-21}$.

The grav-onium mass: $2m \approx 137 \text{ MeV}.$



Summary

- QM is off-mass-shell $(\hbar^2 \square R/R)$ for free particles.
- Via conformal trf a cosmological term $(\Lambda = 3\Box R/R)$ appears.
- Natural reference volume: $V = \frac{4\pi}{3}L_SL_C^2$.
- $\mu = 1/3$: classical trace and quantum dilaton .
- Grav-Onium mass is around the π^0 -mass from cosmological constant

BACKUP SLIDES



Outlook

- Outlook
 - Delphenic: Madelung ansatz for Pauli and Dirac.
 - Delphenic: Madelung as a conformal transformation
 - Jackiw: nonabelian external fields.
 - Brans-Dicke etc.: Dilaton type actions.



Madelung fluid density

Canonical momenta from $\mathfrak{S} = \int \mathfrak{L} d^3x dt$:

$$\Pi_{R} = \frac{\partial \mathfrak{L}}{\partial \nabla R} = \frac{\hbar^{2}}{m} \nabla R, \qquad \Pi_{\alpha} = \frac{\partial \mathfrak{L}}{\partial \nabla \alpha} = \frac{R^{2}}{m} \nabla \alpha,
P_{R} = \frac{\partial \mathfrak{L}}{\partial \frac{\partial R}{\partial t}} = 0, \qquad P_{\alpha} = \frac{\partial \mathfrak{L}}{\partial \frac{\partial \alpha}{\partial t}} = R^{2} \qquad (58)$$

Continuity eq:

$$\frac{\partial P_{\alpha}}{\partial t} + \nabla \Pi_{\alpha} = 0$$

fluid density $\rho = P_{\alpha} = R^2$.





Madelung current

The "classical" momentum defines a velocity as $\vec{P} = m\vec{v}$

The continuity equation reads as

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{\mathbf{v}}) = \mathbf{0}. \tag{59}$$



Bohm potential

The quantum correction to the energy can be expressed as

$$-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 \rho}{2\rho} - \frac{(\nabla \rho)^2}{4\rho^2}\right) \tag{60}$$



Fisher entropy

The action expressed by ρ (up to surface terms) becomes

$$\mathfrak{S} = \int \left[\left(\frac{\frac{P^2}{2m} + V - E}{\frac{2m}{4\rho}} \right) + \left(\frac{\frac{\hbar^2}{2m} \frac{(\nabla \rho)^2}{4\rho}}{\frac{4\rho}{4\rho}} \right) \right] d^3x \ dt \tag{61}$$

The last term, the quantum part, looks like Fisher entropy.

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Fisher entropy may also have to do with uncertainity (in phase space, however).



Interlude: Dilaton

The improved Energy-Momentum-Tensor (arxiv:0307199)

- Canonical EMT: infinitesimal shift in x^{μ} generates it
- Neither symmetric nor gauge invariant
- Not traceless even for scale-invariant L.

Cure by adding a term with vanishing divergence

$$T^{\mu\nu} = \Theta^{\mu\nu} + \partial_{\kappa} f^{\kappa\mu\nu}$$

anti-symmetric in κ, ν .





Inf Sym Trf

Canonical Momentum:

$$\Pi_i^{\mu} := \frac{\delta \mathcal{L}}{\delta \partial_{\mu} \varphi^i}.$$

Transformation:

$$x' = \Omega x$$
, $\varphi'(x') = U\varphi(\Omega^{-1}x')$

Infinitesimal:

$$\delta \mathbf{x}^{\mu} = \epsilon_{\mathbf{a}} \mathbf{G}^{\mathbf{a}\mu}, \quad \delta \varphi^{i} = \epsilon_{\mathbf{a}} \left(\mathbf{F}^{\mathbf{a}i} - \mathbf{G}^{\mathbf{a}\mu} \partial_{\mu} \varphi^{i} \right)$$

Conserved Current:

$$\epsilon_{\mathbf{a}} \cdot \mathbf{J}^{\mathbf{a}\mu} = \Pi^{\mu}_{i} \delta \varphi^{i} - \Theta^{\mu}_{\nu} \delta \mathbf{x}^{\nu}$$

Antisymetric *correction* to the Noether current is possible.



Dilatation

$$\delta \varphi^i = \mathbf{d} \cdot \epsilon \, \varphi^i; \qquad \delta \mathbf{x}^{\nu} = \epsilon \, \mathbf{x}^{\nu}.$$

Conserved dilatation Noether current:

$$J^{\mu} = \mathbf{X}^{\nu} \Theta^{\mu}_{\nu} - \mathbf{d} \cdot \varphi^{i} \Pi^{\mu}_{i}.$$

The zero divergence criterion (using that of $\Theta_{\mu\nu}$):

$$\partial_{\mu} J^{\mu} = \Theta^{\mu}_{\ \mu} - d \cdot \partial_{\mu} \left(\varphi^{i} \Pi^{\mu}_{i} \right) = 0.$$

So in order to have scale invariance we add a divergenceless part to achieve zero trace of $T_{\mu\nu}!_{\text{max}}$



Our Case

For the complex KG Lagrangian:

$$arphi^i\Pi_i^\mu=rac{1}{2}\left(\psi\partial^\mu\psi^*+\psi^*\partial^\mu\psi
ight)=rac{\hbar^2}{mc}\,\partial^\mu\,R^2=L_C^2\,\partial^\mu(mcR^2).$$

Correction to trace of EMT:

$$\mathfrak{T}^{\mu}_{\mu} = (\partial_{\mu}J^{\mu})_{\text{non-scaling part}} + d \cdot L^{2}_{C} \square (mcR^{2}).$$

Conclusion:
$$d = (1 - 3\mu)/4 = 0$$
.

