

AT HOMEWORKS 2012

- (1) – Calculate the chemical potential of an ideal gas using the derivative of the entropy:

$$T\partial_M S(E, V, M) = \mu(E, V, M)$$

Transform that to more convenient variables, and calculate  $\mu(T, p)$ .

- Calculate the following partial derivative in general:

$$\partial_T \mu(T, p) = ?$$

- The Gibbs free energy is defined as the Legendre transformed form of the internal energy by  $S \rightarrow T$  and  $V \rightarrow p$  partial transformations. Calculate the appropriate form of the Gibbs relation, the differential of  $G$ .

- (2) – Calculate the Gibbs relation of the specific Gibbs free energy  $h = e - Ts + pv$ .  
– Prove the identity

$$\partial_v s(T, v) = \partial_T p(T, v)$$

by choosing an adequate thermodynamic potential.

- Prove that a Van der Waals gas has a thermodynamic potential. The corresponding equations of state are

$$p(T, v) = \frac{RT}{v-b} - \frac{a}{v^2}, \quad T(e, v) = \frac{e}{c} + \frac{a}{cv}.$$

- (3) What are the conditions of thermodynamic stability of an ideal gas?

- (4) Using Bridgeman tables find the following expressions:

- $\partial_v T(v, h) = ?$ ,
- $\partial_v h(v, f) = ?$ ,
- $\partial_p h(p, s) = ?$ .

Prove the first identity.

- (5) – The thermic equation of state of a Callendar gas is

$$p(v, T) = \frac{RT}{v-b+a/T}.$$

Calculate  $T(e, v)$  and  $s(e, v)$ .

- The evolution equation of a simple reversible system is

$$\frac{d}{dt}e = (aT+b)(T-T_a) - \beta p(p-p_a), \quad \frac{d}{dt}v = \beta(p-p_a),$$

Calculate the derivative of the entropy  $s(e, v)$  along the above differential equation.

- (6) A system of two reversible thermodynamic bodies is governed by the following evolution equations

$$\begin{aligned} \frac{d}{dt}e_1 &= \dot{q}_1 - p_1 f_1, & \frac{d}{dt}v_1 &= f_1, \\ \frac{d}{dt}e_2 &= \dot{q}_2 - p_2 f_2, & \frac{d}{dt}v_2 &= f_2 \end{aligned}$$

Calculate the  $(s_1 + s_2)$  derivative of the total entropy of the system along these differential equations if

- $v_1 + v_2 = \text{const.}$  and  $e_1 + e_2 = \text{const.}$

- $v_1 + v_2 = \text{const.}$  and  $q_1 + q_2 = \text{const.}$
- (7) Give the equilibrium of the differential equation below and investigate stability properties:

$$\dot{e} = \alpha_1(T_1 - T) + \alpha_2(T_2 - T).$$

Here  $e = cT$ ,  $c, \alpha_1, \alpha_2, T_1, T_2$  are constants.

- (8) Calculate the following expressions:

$$\partial^i(\rho v^i v^j)$$

where  $\rho, v^i$  are the density and velocity fields of a fluid.

$$\partial^i \left( \frac{x^i}{r} \right),$$

where  $r = \sqrt{x^i x^i}$ .

$$\partial^{ij} \left( \frac{x^i x^j}{r} \right),$$

where  $r = \sqrt{x^i x^i}$ .

- (9) - Give the following expression in matrix form

$$\partial^i v^i - \partial^j v^j$$

- Simplify the following expression

$$(A^{ij} - A^{ji})(A^{ij} + A^{ji}),$$

- Give an  $A^{ij}$  tensor with index notation as a sum of a symmetric traceless, antisymmetric and a scalar multiplied by the unit tensor.

- (10) BIG computer homework: Diesel cycle modelling.

An ideal Diesel cycle (adiabatic-isobaric-adiabatic-isochoric processes) is characterized by the following parameters: the compression ratio,  $r_v = 20$ , the cut-off ratio  $a_v = 3$ , the ratio of specific heats  $\gamma = 1.4$ , initial temperature  $T_1 = 20C$ , initial pressure  $p = 1bar$ , the mass  $m = 2.408g$ , the specific gas constant  $R = 287J/kg/K$ . Construct a time dependent simulation with a reversible ordinary thermodynamic model in Maple or Mathematica that runs close to the ideal one with a suitable specification of the heat exchange coefficient  $\alpha = 10^2 J/s/K$  and the volume regulation coefficient  $\beta = 10^{-8} m^4/s/kg$ . The total heating/cooling at the isobaric/isochoric parts let be approximately identical to the ideal case. A convenient way is to prescribe and external linear source in the differential equation.