Examination questions

Advanced Thermodynamics 2012

(1) Thermodynamic models and theories, classification. Concepts of irreversibility.

(2) Classical thermodynamics. Homogeneity, partial derivatives, Legendre transformation.

(3) Thermodynamic stability. Ideal gas, van der Waals gas.

(4) Laws of thermodynamics. Classical versus dynamic formulation.

(5) Ordinary thermodynamics I. Single reversible body in an environment. Asymptotic stability of equilibrium,

(6) Heat engines. Carnot cycle, Curzon-Ahlborn machine.

(7) Ordinary thermodynamics II. Single extended body in an environment. Asymptotic stability of equilibrium.

(8) System of thermodynamic bodies. Gouy-Stodola theorem.

(9) General balances. Global and differential, local and substantial forms.

(10) Mass, momentum and energy balances.

(11) Local equilibrium. Entropy balance of simple fluids. Thermodynamic fluxes and forces.

(12) Isotropy. Representation theorem.

(13) Fluids. Fourier and Navier-Stokes equations.

(14) Beyond local equilibrium. Extended heat conduction.

Problems

Advanced Thermodynamics 2012

([C] = there is a related problem with Maple)

1 Entropy, existence of entropy

(1) [C] The thermic and caloric state functions of a Clausius gas are:

$$p(T,v) = \frac{RT}{v-b} - \frac{a}{Tv}, \qquad T(e,v) = \frac{2a}{e} \ln \frac{v}{v_0}.$$

where p, T, e, v are the pressure, temperature, specific internal energy and specific volume of the gas R, a, b, v_o are constants. Prove that the gas is entropic.

(2) [C] Calculate the entropy function of the Clausius gas.

(3) [C] The thermic, caloric and material state functions of a Van der Waals gas are:

$$\begin{aligned} p(T,V,m) &= \frac{mRT}{V-mb} - \frac{m^2 a}{V^2}, \\ T(E,V,m) &= \frac{E}{mc} + \frac{am}{cV}, \\ \mu(T,V,m) &= R\left(\frac{V}{m} - b\right) + c\ln(cT) - c + \frac{2amV}{3EV + am^2}, \end{aligned}$$

where p, T, μ, E, V, m are the pressure, temperature, chemical potential, internal energy, volume and mass of the gas and R, a, b are constants. Prove that the gas is entropic.

(4) [C] Calculate the entropy of the following gas:

$$p(T,v) = \frac{RT}{v}, \qquad T(e,v) = ae^2.$$

(5) [C] Is the gas below entropic?

$$\begin{aligned} p(T,V,m) &= \frac{mRT}{V}, \\ T(E,V,m) &= \frac{aE^2}{m^2}, \\ \mu(E,V,m) &= 2\frac{E}{m} - RT\ln\left(\frac{V}{m}\right). \end{aligned}$$

(6) Prove that

$$\left.\frac{\partial e}{\partial v}\right|_T = T \left.\frac{\partial p}{\partial T}\right|_v - p.$$

(7) The thermic equation of state of a Callendar gas is the following

$$p(v,T) = \frac{RT}{v - b + a/T},$$

where R is the specific gas constant, T is the temperature, p is the pressure, v is the specific volume and a and b are constant parameters. Calculate the caloric equation of state $e(T,v)$ of a Callendar gas.

(8) [C] Calculate the specific entropy $s(e, v)$ of a Callendar gas.

2 Thermodynamic stability

(9) [C] What are the conditions of thermodynamic stability of a Clausius gas?

(10) [C] What are the conditions of thermodynamic stability of a Van der Waals gas?

(11) What are the conditions of thermodynamic stability of a Callendar gas?

(12) What are the conditions of thermodynamic stability of the gas in (**??**)?

3 Liapunov stability

(13) [C] Let us assume that the change of the specific internal energy e of a thermodynamic body with temperature T in an environment with temperature T_a is given by the differential equation

$$\dot{e} = -\alpha(T^4 - T_a^4),$$

where α is constant. How does the logarithm of the specific entropy changes by the processes determined by the above differential equation. (Calculate the derivative of $\ln s$ along the differential equation)

(14) [C] The differential equation of an equilibrium thermodynamic body in an environment with constant T_a temperature and p_a pressure is given as:

$$\dot{e} = (aT + b)(T - T_a) - \beta p(p - p_a), \qquad \dot{v} = \beta(p - p_a).$$

where e is the specific internal energy, v is the specific volume, T, p are the temperature and the pressure of the body and a, b, β are material constant parameters. Calculate the derivative of the entropy along the differential equation.

(15) The governing differential equations of a thermodynamic system of two reversible bodies and a reservoir are given as

$$\begin{aligned}
\frac{\mathrm{d}e_1}{\mathrm{d}t} &= \dot{q}_1 - p_1 f_1, \\
\frac{\mathrm{d}v_1}{\mathrm{d}t} &= f_1, \\
\frac{\mathrm{d}e_2}{\mathrm{d}t} &= \dot{q}_2 + \dot{q}_0 - p_2 f_2, \\
\frac{\mathrm{d}v_2}{\mathrm{d}t} &= f_2, \\
\frac{\mathrm{d}e_0}{\mathrm{d}t} &= -\dot{q}_0.
\end{aligned}$$

The bodies 1 and 2 are in a thermal and mechanical interaction, and the second body can exchange heat to an environment with constant temperature T_0. The energy of the two bodies and the environment is conserved as well as the volume of the two bodies.

Prove that the derivative of s_{tot} is zero if $p_1 = p_2$ and $T_1 = T_2 = T_0$.

(16) Prove that the second derivative of s_{tot} of the system of thermodynamic bodies in (**??**) is negative definite, as a consequence of the concavity of the entropies of bodies 1 and 2.

(17) Calculate the derivative of s_{tot} along the differential equation of (**??**) (considering the conservation of energy and volume).

4 Calculations with indices

Simplify the following expressions

(18) $[C]$

$$\partial^k(\delta^{ij}x^ix^j),$$

(19) $[C]$

$$\partial^k\sqrt{1-x^ix^i},$$

(20)

$$\left(A^{ij}-\frac{1}{3}A^{kk}\delta^{ij}\right)\frac{1}{3}A^{kk}\delta^{ij},$$

(21)

$$\left(A^{ij}-\frac{A^{ij}-A^{ji}}{2}-\frac{1}{3}A^{kk}\delta^{ij}\right)\delta^{ij},$$

(22)

$$\partial^{ij}\left(\frac{x^ix^j}{r}\right),$$

where $r=\sqrt{x^ix^i}$.

(23)

$$\partial_i\left(\frac{x^j}{r}\right)-\partial_j\left(\frac{x^i}{r}\right).$$

(24) Expand the expression $\partial^{ij}\left(a^ib^j\right)$, where a^j and b^j are vector fields.

5 Entropy balance calculations

(25) Let us consider a balance of the internal energy with a special source term (Joule heat):

$$\rho\dot{e}+\partial^iq^i=j^iE^i,$$

where j^i is the conductive electric current density and E^i is the electric field strength. Calculate the entropy production if the entropy current is the classical $J^i=q^i/T$ and the density ρ is constant.

(26) Calculate the entropy production, if the internal energy balance has the following form

$$\rho\dot{e}+\partial^iq^i=0.$$

The entropy current is the classical $J^i=q^i/T$ and the density is constant.