Test

Advanced Thermodynamics, 2010

(1) The thermic equation of state of a Callendar gas is the following

$$p(v,T) = \frac{RT}{v - b + a/T},$$

where R is the specific gas constant, T is the temperature, p is the pressure, v is the specific volume and a and b are constant parameters.

(a) Prove that

$$\left.\frac{\partial e}{\partial v}\right|_T = T \left.\frac{\partial p}{\partial T}\right|_v - p.$$

(b) Calculate the caloric equation of state $e(T,v)$ of a Callendar gas.

(c) Calculate the specific entropy $s(e,v)$ of a Callendar gas.

(d) What is the difference of the isobaric and isochor specific heats of the gas? (Hint: prove that $c_p - c_v = -T \left(\left.\frac{\partial p}{\partial T}\right|_v \right)^2 \left.\frac{\partial v}{\partial p}\right|_T \quad \Big)$.

(2) The governing differential equations of a thermodynamic system of two reversible bodies and a reservoir are given as

$$\begin{aligned}
\frac{\mathrm{d}e_1}{\mathrm{d}t} &= \dot{q}_1 - p_1 f_1, \\
\frac{\mathrm{d}v_1}{\mathrm{d}t} &= f_1, \\
\frac{\mathrm{d}e_2}{\mathrm{d}t} &= \dot{q}_2 + \dot{q}_0 - p_2 f_2, \\
\frac{\mathrm{d}v_2}{\mathrm{d}t} &= f_2, \\
\frac{\mathrm{d}e_0}{\mathrm{d}t} &= -\dot{q}_0.
\end{aligned}$$

The bodies 1 and 2 are in a thermal and mechanical interaction, and the second body can exchange heat to an environment with constant temperature T_0 (see figure 1). The energy of the two bodies and the environment is conserved as well as the volume of the two bodies.

(a) Give the total specific entropy of the system s_{tot} as the function of e_1, e_2, and v_1.

(b) Prove that the derivative of s_{tot} is zero if $p_1 = p_2$ and $T_1 = T_2 = T_0$.

(c) Prove that the second derivative of s_{tot} is negative definite, as a consequence of the concavity of the entropies of bodies 1 and 2.

(d) Calculate the derivative of s_{tot} along the differential equation above (considering the conservation of energy and volume).

(e) Formulate the conditions of the asymptotic stability of the equilibrium of the above system.

(3) An ideal gas expands at a constant $p_0 = 200kPa$ pressure. The internal energy E and the volume V of the gas are determined by the following

differential equations

$$\begin{aligned}\frac{\mathrm{d}E}{\mathrm{d}t} &= -\alpha(T - T_0) - p_0\beta(p_0 - p_a(t)),\\ \frac{\mathrm{d}V}{\mathrm{d}t} &= \beta(p_0 - p_a(t)).\end{aligned}$$

The volumetric rate factor $\beta = 5.2 \cdot 10^{-10}\frac{m^4 s}{kg}$ and the heat exchange coefficients $\alpha = 0.01645\frac{J}{sK}$. The temperature of the gas at the beginning of the process is $T_i = 393K$ and the temperature of the environment is $T_0 = 293K$. The specific gas constant $R = 282\frac{J}{kgK}$, the mass of the gas is $m = 1g$ and the heat capacity ratio is $\kappa = 1.4$. The process finishes when the volume is $V_f = 0.4653dm^3$.

(a) Calculate the temperature of the gas as the function of time.
(b) How long is the process?
(c) Calculate the total emitted heat $Q = \int_0^{t_f} \dot{Q}(t)dt$.
(d) Calculate the external pressure p_a at the end of the process. (Find the time dependence of the external pressure $p_a(t)$)

(4) (a)

$$\partial_{ij}\left(\frac{x^i x^j}{r}\right) =?$$

(b)

$$\partial_i\left(\frac{x^j}{r}\right) - \partial_j\left(\frac{x^i}{r}\right) =?$$