Test

Advanced Thermodynamics, 2011

(1) The thermic equation of state of a Clausius gas is the following

$$p(v,T) = \frac{RT}{v-b} - \frac{a}{Tv},$$

where R is the specific gas constant, T is the temperature, p is the pressure, v is the specific volume and a and b are constant parameters.

(a) Prove that

$$\left.\frac{\partial e}{\partial v}\right|_T = T\left.\frac{\partial p}{\partial T}\right|_v - p.$$

(b) Calculate the caloric equation of state $e(T,v)$ of a Clausius gas.

(c) Calculate the specific entropy $s(e,v)$ of a Clausius gas.

(d) Prove that

$$c_p - c_v = -T\left(\left.\frac{\partial p}{\partial T}\right|_v\right)^2 \left.\frac{\partial v}{\partial p}\right|_T.$$

(e) What is the difference of the isobaric and isochoric specific heats of a Clausius gas?

(2) In a system of three ordinary thermodynamic bodies the first body is in a mechanical contact with the second body and in a thermal contact with the third body (see figure 1). The governing differential equations are

$$\begin{aligned}
\frac{\mathrm{d}e_1}{\mathrm{d}t} &= q_1 - p_1 f_1,\\
\frac{\mathrm{d}v_1}{\mathrm{d}t} &= f_1,\\
\frac{\mathrm{d}e_2}{\mathrm{d}t} &= -p_2 f_2,\\
\frac{\mathrm{d}v_2}{\mathrm{d}t} &= f_2,\\
\frac{\mathrm{d}e_3}{\mathrm{d}t} &= q_3.
\end{aligned}$$

(a) The overall energy of the three bodies is conserved as well as the total volume of the first two bodies. Calculate the net specific heat exchange $(q_1 + q_2)$.

(b) Calculate the derivative of $s_{tot} = s_1 + s_2 + s_3$ along the differential equation. (Hint: use the constraints.)

(c) The net heat exchange of the first and the third body is zero and the total volume of the first two bodies is conserved. Calculate the time derivative of the overall energy of the three bodies.

(d) Calculate the derivative of s_{tot} along the differential equation.

(3) Let us consider and isothermal process of an ideal gas, where the temperature of the gas is different of the temperature of the environment ($T = const. \neq T_a = const.$). The specific internal energy e and the specific volume v of the ideal gas are determined by the following differential

equations

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\alpha(T - T_a) - p\beta(p - p_a(t)),$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = \beta(p - p_a(t)).$$

The volumetric rate factor $\beta = 5.2 \cdot 10^{-10} \frac{m^4 s}{kg^2}$ and the heat exchange coefficient $\alpha = 8262.6 \frac{m^3}{s^3 K}$. At the beginning of the process the temperature and the volume of the gas are $T_i = 393K$ and $V_i = 1.21m^3$. The environmental temperature is $T_a = 293K$. The specific gas constant $R = 282 \frac{J}{kgK}$ and the mass of the gas is $m = 12.1g$. The process stops when the volume of the body $V = 0.4451m^3$.

(a) Calculate the pressure of the gas as the function of time (Hint: use the time derivative of the thermal eos $pv = RT$ and the differential equations above).
(b) How should we regulate the external pressure $p_a(t)$ to ensure the isothermal conditions?
(c) How long is the process?
(d) Calculate the total emitted heat.

(4) (a)

$$\partial^{ij}\left(a^i b^j\right) = ?,$$

Here a^j and b^j are vector fields.

(b) The Levi-Civita symbol is defined as

$$\epsilon^{ijk} = \begin{cases} +1, & \text{if (i,j,k) is (1,2,3), (2,3,1) or (3,1,2),} \\ -1, & \text{if (i,j,k) is (1,2,3), (2,3,1) or (3,1,2),} \\ 0, & \text{otherwise: i=j or j=k, or k=i.} \end{cases}$$

Calculate the components of the following expressions:

$$\epsilon^{ijk}\partial^j a^k = ?$$

(c)

$$\partial^i(\epsilon^{ijk}\partial^j a^k) = ?$$