Test

Advanced Thermodynamics, 2012

(1) The thermic equation of state of a one dimensional Duhamel-Neuman body is the following

$$p(\varepsilon, T) = -E\varepsilon + a(T - T_0),$$

where E is the Young modulus, ε is the strain, a is the coefficient of thermal expansion, T is the temperature and p is the pressure, a and E are constants.

(a) Prove that

$$\left.\frac{\partial e}{\partial \varepsilon}\right|_T = T \left.\frac{\partial p}{\partial T}\right|_\varepsilon - p.$$

(b) Calculate the caloric equation of state $e(T, \varepsilon)$ of a one dimensional Duhamel-Neumann body, if $\partial e/\partial T|_\varepsilon = c = const.$.

(c) Calculate the specific entropy $s(e, \varepsilon)$ of a one dimensional Duhamel-Neumann body.

(d) What are the conditions of thermodynamic stability of a one dimensional Duhamel-Neumann body?

(2) In a system of thre ordinary thermodynamic bodies every body is in a thermal contact with the other two bodies (see figure 1). The governing differential equations are

$$\begin{aligned}
\frac{\mathrm{d}E_1}{\mathrm{d}t} &= \dot{Q}_{12} + \dot{Q}_{13}, \\
\frac{\mathrm{d}E_2}{\mathrm{d}t} &= \dot{Q}_{21} + \dot{Q}_{23}, \\
\frac{\mathrm{d}E_3}{\mathrm{d}t} &= \dot{Q}_{31} + \dot{Q}_{32},
\end{aligned}$$

where E_1, E_2, E_3 are the internal energies of the bodies, respectively.

(a) The total energy of the three bodies is conserved. Prove, that in case of equal temperatures ($T_1 = T_2 = T_3$) there is no dissipation in the system, independently of the specific form of the thermal interaction. (Hint: $S^{\square}_{tot}$=0).

(b) Prove, that the total entropy is concave, independently of the energy conservation if the specific heats of the bodies are constant.

(c) Calculate the derivative of the total entropy of the system along the differential equation in the following cases:
– the total energy of the three bodies is conserved,
– there is a *detailed balance* of the interactions: $\dot{Q}_{ij} + \dot{Q}_{ji} = 0,$, $i, j = 1, 2, 3, i \neq j$.

(d) Let be the heat exchange functions linear in the temperature difference: $\dot{Q}_{ij} = a_{ij}(T_j - T_i)$, $i, j = 1, 2, 3, i \neq j$. Give conditions for the constant a_{ij} coefficients to fulfill both detailed balance and the entropy inequality.

(e) Is there an equilibrium where the temperature of the bodies is not uniform if $a = a_{21} = a_{23} = a_{31} = a_{32} = 2a_{12} = 2a_{13}$?

(3) Let us consider a process of an ideal gas, when the specific internal energy e and the specific volume v are determined by the following differential equations

$$\begin{aligned}\frac{\mathrm{d}e}{\mathrm{d}t} &= -\alpha(T-T_a) - p\beta(p-p_a(t)),\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= \beta(p-p_a(t)).\end{aligned}$$

The volumetric rate factor $\beta = 5.2\cdot 10^{-10}\frac{m^4 s}{kg^2}$ and the heat exchange coefficient $\alpha = 7.05\frac{W}{kgK}$, the mass of the gas is $m = 0.002kg$. At the beginning of the process the temperature and the specific volume of the gas are $T_i = 600K$ and $v_i = 1.692m^3/kg$. The environmental temperature $T_a = 300K$. The specific gas constant $R = 282\frac{J}{kgK}$ and the specific heat of the gas $c = 2.5R$. The process is isochoric and stops when $p = 3p_i/4$.

(a) Calculate the temperature of the gas as the function of time.
(b) How should we change the external pressure $p_a(t)$ to ensure the constant specific volume?
(c) How long is the process?
(d) Calculate the total emitted heat.

(4) (a)

$$\partial^{ij}\left(x^i x^j x^k\right) =?$$

(b)

$$\partial^{ij}\left(\frac{x^i x^j x^k}{r}\right) =?$$

(c) The Levi-Civita symbol is defined as

$$\epsilon^{ijk} = \begin{cases} +1, & \text{if (i,j,k) is (1,2,3), (2,3,1) or (3,1,2),}\\ -1, & \text{if (i,j,k) is (3,2,1), (2,1,3) or (1,3,2),}\\ 0, & \text{otherwise: i=j or j=k, or k=i.}\end{cases}$$

Calculate the components of the following expressions:

$$\epsilon^{ijk}\delta^{jk} =?$$

(d)

$$\epsilon^{ijk}A^{jk} =?$$