Relation of rock mass characterization and damage

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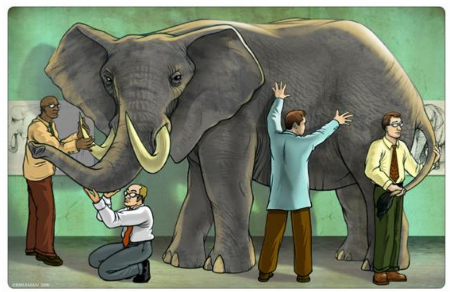
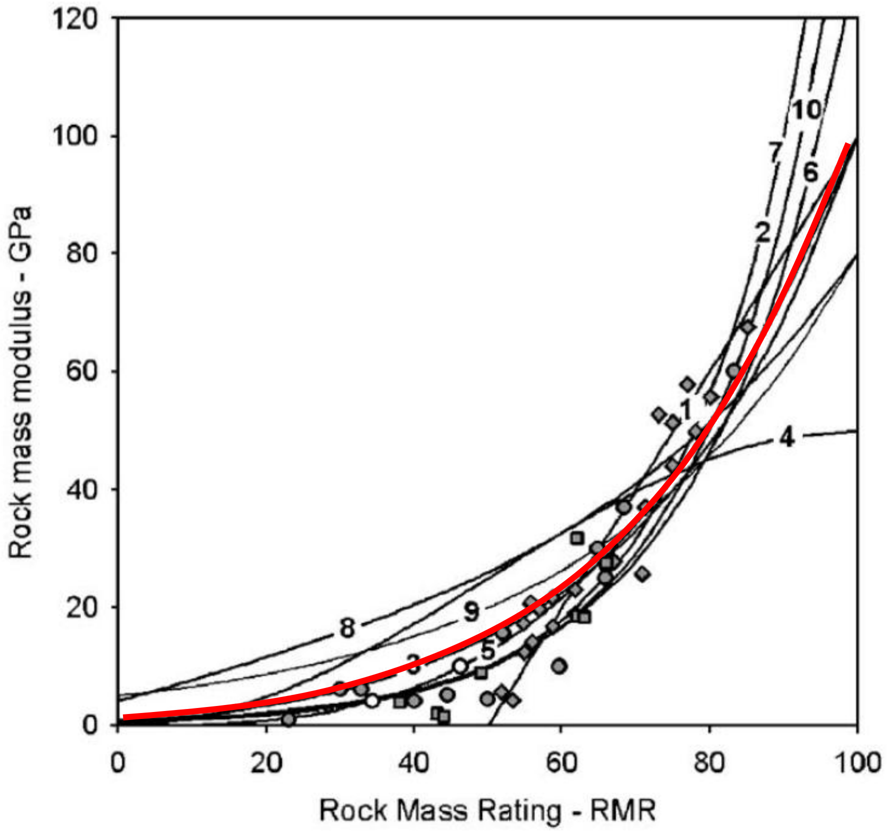
– Deformation moduli and strength of the rock mass
– Thermo-damage mechanics
– Summary, conclusions and outlook



Deformation modulus of the rock mass (E_{rm}) – formulas containing the elastic modulus of intact rock (E_i)



Hoek & Diederichs, 2006

3 – Nicholson and Bieniawski (1990)

$$\frac{E_{rm}}{E_i} = 0.0028\,RMR^2 + 0.9\,e^{\frac{RMR}{22.82}}$$

8 – Sonmez et. al. (2004)

$$\frac{E_{rm}}{E_i} = \left(s^a\right)^{0.4} \qquad a = 0.5 + \frac{1}{6}\left(e^{-\frac{RMR}{15}} - e^{-\frac{20}{3}}\right)$$

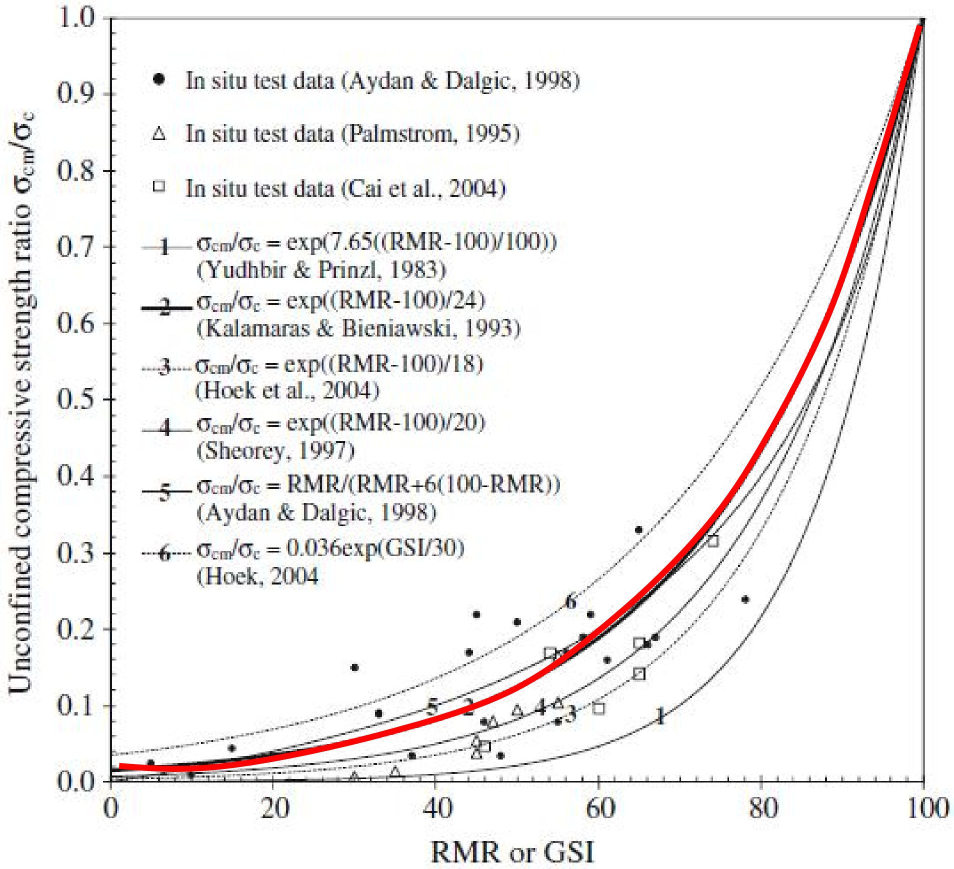
– Carvalcho (2004)

$$\frac{E_{rm}}{E_i} = s^{0.25} \qquad s = e^{\frac{RMR-100}{9}}$$

– Zhang and Einstein (2004)

$$\frac{E_{rm}}{E_i} = 10^{0.0186RQD-1.91}$$

Unconfined compressive strength of the rock mass (σ_{cm}) – formulas containing the strength of intact rock (σ_c)



Zhang, 2005

- Yudhbir et al. (1983))

$\sigma_{cm}/\sigma_c = \exp(7.65((RMR-100)/100)$

- Ramamurthy et al. (1985)

$\sigma_{cm}/\sigma_c = \exp((RMR-100)/18.5)$

- Kalamaras & Bieniawski (1993)

$\sigma_{cm}/\sigma_c = \exp((RMR-100)/25)$

- Hoek et al. (1995)

$\sigma_{cm}/\sigma_c = \exp((RMR-100)/18)$

- Sheorey (1997)

$\sigma_{cm}/\sigma_c = \exp((RMR-100)/20)$

Using a damage variable - *D*

Intact rock: $D=0$
Fractured rock at the edge of failure: $D= D_{cr}$

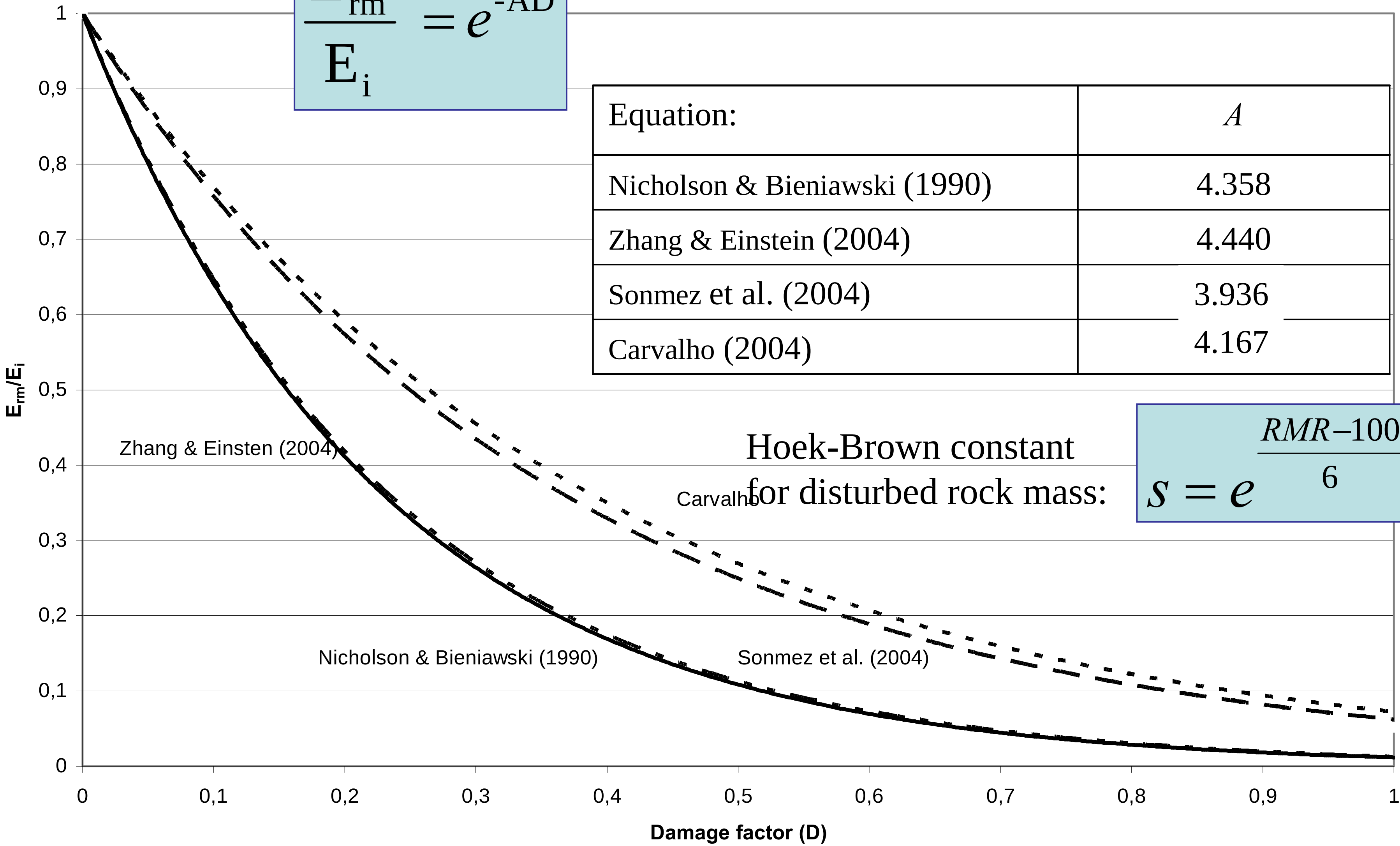
rock mass quality measure = damage measure

	Intact rock	Fractured rock
RMR scales	100	0
Damage scales	0	D_{cr}

$$RMR = 100\left(1 - \frac{D}{D_{cr}}\right) \quad \Longleftrightarrow \quad D = D_{cr}\left(1 - \frac{RMR}{100}\right)$$

Deformation modulus: exponential form

$$\frac{E_{rm}}{E_i} = e^{-AD}$$



Equation:	*A*
Nicholson & Bieniawski (1990)	4.358
Zhang & Einstein (2004)	4.440
Sonmez et al. (2004)	3.936
Carvalho (2004)	4.167

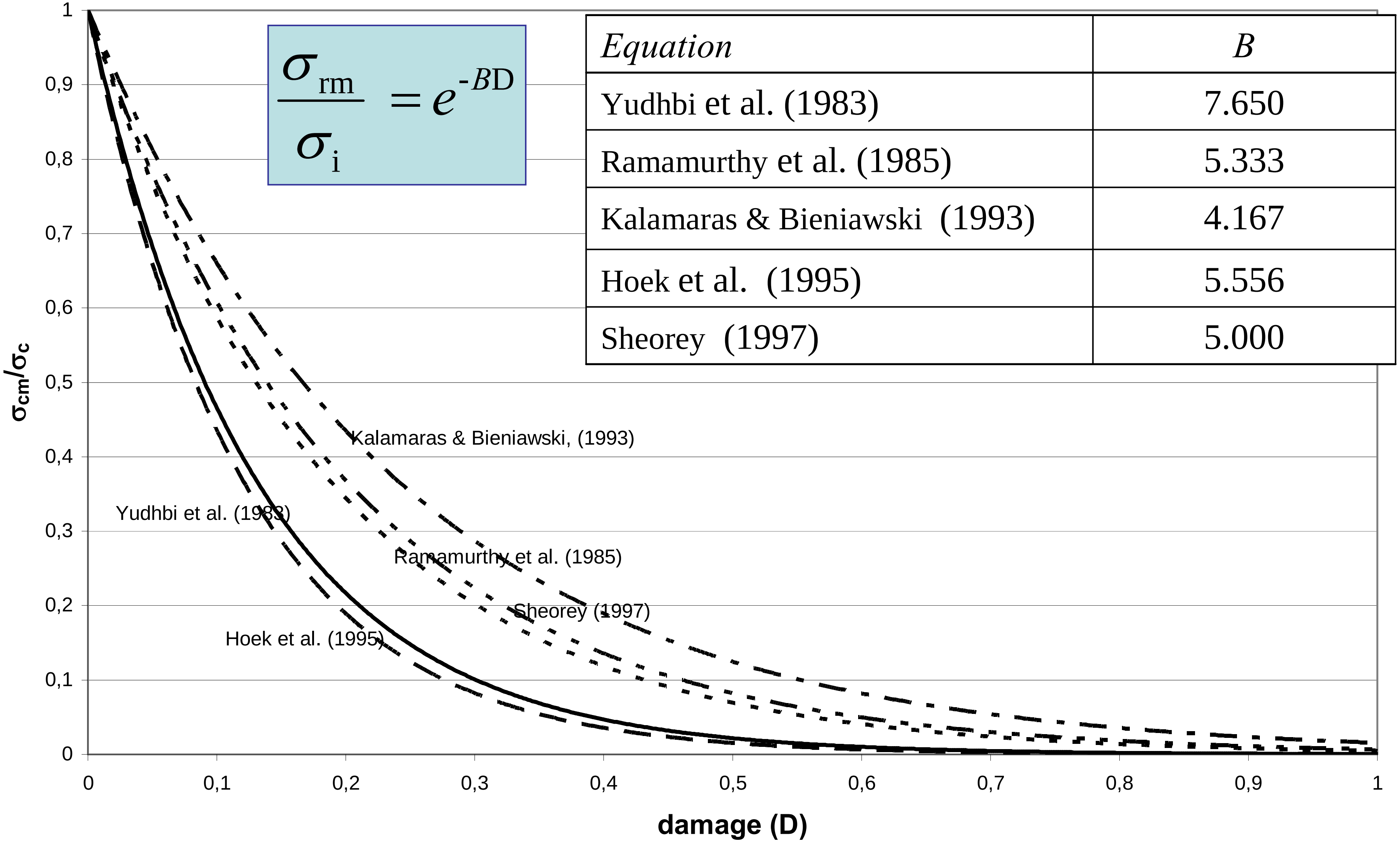
Hoek-Brown constant for disturbed rock mass:

$$s = e^{\frac{RMR-100}{6}}$$

Uniaxial strength: exponential form

$$\frac{\sigma_{rm}}{\sigma_i} = e^{-BD}$$

Equation	*B*
Yudhbi et al. (1983)	7.650
Ramamurthy et al. (1985)	5.333
Kalamaras & Bieniawski (1993)	4.167
Hoek et al. (1995)	5.556
Sheorey (1997)	5.000



Thermo-damage mechanics I.

Thermostatic potential: Helmholtz free energy

linear elasticity: $$F(\varepsilon) = \frac{E\varepsilon^2}{2} \Rightarrow \sigma = \frac{\partial F}{\partial \varepsilon} = E\varepsilon$$

damaged rock: $$F(\varepsilon, D) = ? \quad \Rightarrow \sigma = \left.\frac{\partial F}{\partial \varepsilon}\right|_D = E(D)\varepsilon$$

$$E(0) = E_i$$

Energetic damage: $$\left.\frac{\partial F}{\partial D}\right|_\varepsilon = -\alpha\, F(\varepsilon, D)$$

The energy content of more deformed rock mass is more reduced by damage.

Consequence: $$F(\varepsilon, D) = e^{-\alpha D}\left(E_i \frac{\varepsilon^2}{2} + F_0\right)$$

Thermo-damage mechanics II.

$$F(\varepsilon, D) = e^{-\alpha D}\left(E_i \frac{\varepsilon^2}{2} + F_0\right)$$

Deformation modulus

$$E_{RM} = \frac{\sigma}{\varepsilon} = \frac{1}{\varepsilon}\frac{\partial F}{\partial \varepsilon}\bigg|_D = E_i e^{-\alpha D}$$

…exponential, like the empirical data.

Strength – thermodynamic stability (Ván and Vásárhelyi, 2001)
(convex free energy, positive definite second derivative)

$$\partial^2 F = e^{-\alpha D}\begin{pmatrix} E_i & -\alpha\varepsilon E_i \\ -\alpha\varepsilon E_i & \alpha^2\left(E_i \frac{\varepsilon^2}{2} + F_0\right)\end{pmatrix} \Rightarrow \det(\partial^2 F) \geq 0$$

$$0 \leq \det(\partial^2 F) = \alpha^2 e^{-2\alpha D}\left(F_0 - E_i \frac{\varepsilon^2}{2}\right) \Rightarrow$$

$$\frac{\sigma_{cm}}{\sigma_c} = e^{-\alpha D}$$

…exponential, like the empirical data.

Summary

Equation:	*a*
Nicholson & Bieniawski (1990)	22.95
Zhang & Einstein (2004)	22.52
Sonmez et al. (2004)	38.11 (25.41)
Carvalho (2004)	36.00 (24.00)

a = 22.52…(25.41)…38.11

average: 30 (or 23.7, with disturbed)

$$\frac{E_{rm}}{E_i} = e^{\frac{RMR-100}{a}}$$

$$a = b\ ?$$

Equation:	*b*
Yudhbi et al. (1983)	13.07
Ramamurthy et al. (1985)	18.75
Kalamaras & Bieniawski (1993)	24.00
Hoek et al. (1995)	18.00
Sheorey (1997)	20.00

$$\frac{\sigma_{cm}}{\sigma_c} = e^{\frac{RMR-100}{b}}$$

b = 13.07…24.00

average: 18.76 (or 20.19, without Yudhbi)

Conclusions

Damage model:

$$a = b \Rightarrow \frac{\sigma_{cm}}{\sigma_c} = \frac{E_{rm}}{E_i} \Rightarrow \frac{E_{rm}}{\sigma_{cm}} = \frac{E_i}{\sigma_c} = MR$$

MR=Modification Ratio

Empirical relations:

$$\left.\begin{aligned} \frac{\sigma_{rm}}{\sigma_i} &= e^{-BD} \\ \frac{E_{rm}}{E_i} &= e^{-AD} \end{aligned}\right\} \Rightarrow \frac{E_{rm}}{\sigma_{cm}} = \frac{E_i}{\sigma_c} e^{\frac{(B-A)(RMR-100)}{100}} = MR\, e^{\frac{2(RMR-100)}{100}}$$

average: $B - A \approx 1,\ldots,0$

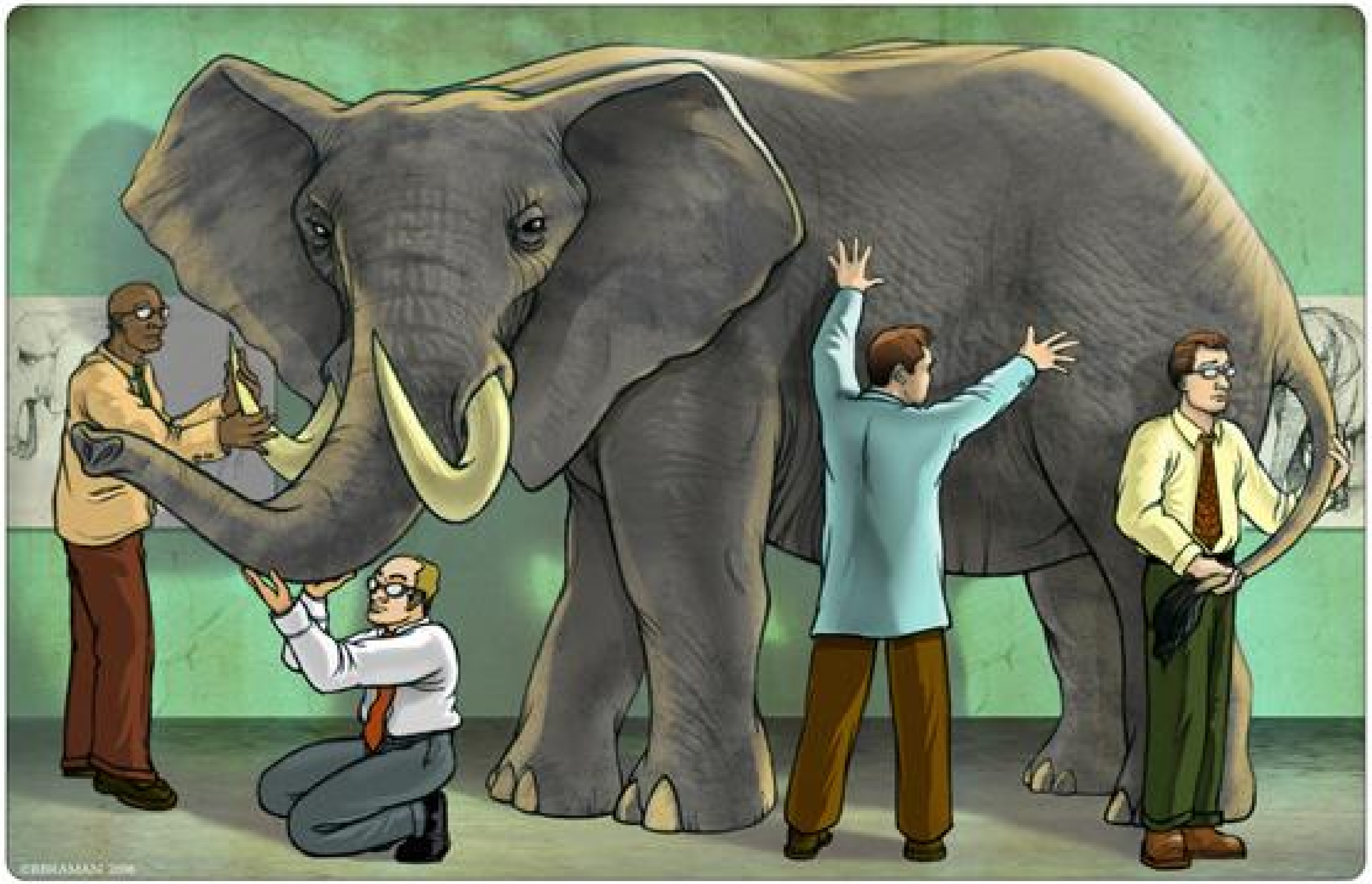
Outlook

- Linear elasticity: $F(\boldsymbol{\varepsilon}) = \frac{\lambda}{2}\left(tr\boldsymbol{\varepsilon}\right)^2 + \mu\boldsymbol{\varepsilon} : \boldsymbol{\varepsilon}$

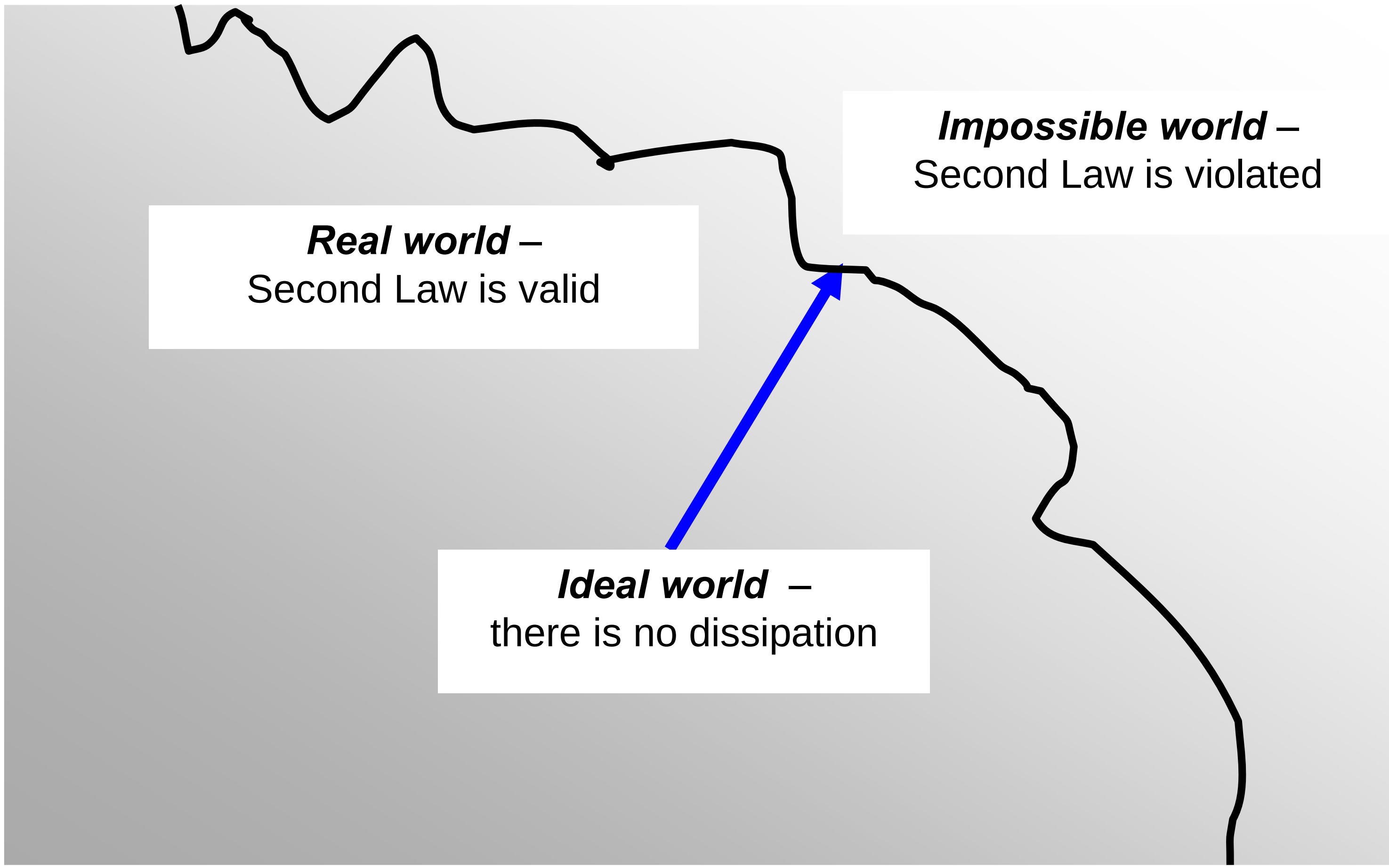
  damage: scalar, vector, tensor, …

- Nonideal damage and thermodynamics:
  damage evolution
  damage gradients

Thank you for your attention!



Thermodinamics - Mechanics



Conclusions

Damage model:

$$\mathrm{a} = b \Rightarrow \frac{\sigma_{\mathrm{cm}}}{\sigma_{\mathrm{c}}} = \frac{\mathrm{E_{rm}}}{\mathrm{E_i}} \Rightarrow \frac{\mathrm{E_{rm}}}{\sigma_{\mathrm{cm}}} = \frac{\mathrm{E_i}}{\sigma_{\mathrm{c}}} = MR$$

MR=Modification Ratio

Empirical relations:

$$\left.\begin{aligned} \frac{\sigma_{\mathrm{rm}}}{\sigma_{\mathrm{i}}} &= e^{\text{-BD}} \\ \frac{\mathrm{E_{rm}}}{\mathrm{E_i}} &= e^{\text{-AD}} \end{aligned}\right\} \Rightarrow \frac{\mathrm{E_{rm}}}{\sigma_{\mathrm{cm}}} = \frac{\mathrm{E_i}}{\sigma_{\mathrm{c}}} e^{\frac{(B-A)(RMR-100)}{100}} = MR\, e^{\frac{2(RMR-100)}{100}}$$

average: $B - A \approx 1,...,0$

$$\frac{\sigma_{\mathrm{cm}}}{\sigma_{\mathrm{c}}} = e^{-BD} = \left(e^{-AD}\right)^{\frac{B}{A}} = \left(\frac{\mathrm{E_{rm}}}{\mathrm{E_i}}\right)^{\mathrm{q}}$$

$$q = \frac{B}{A}$$

Zhang, 2009