One and two component weakly nonlocal fluids

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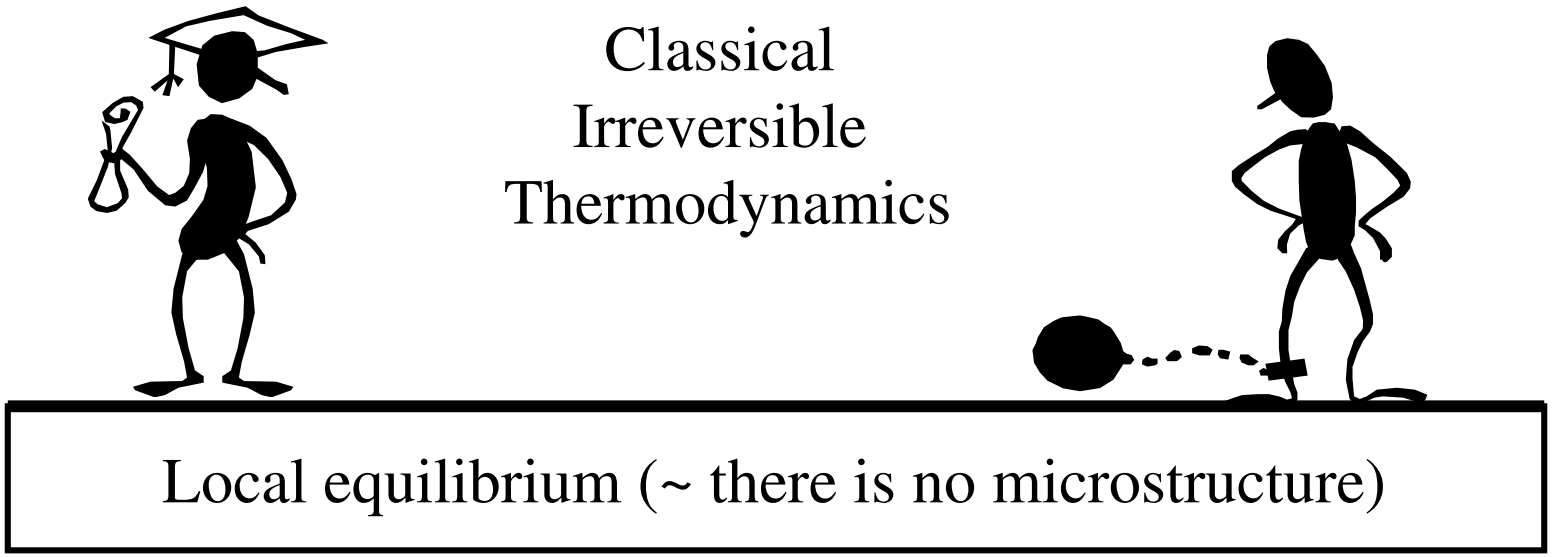
- Nonequilibrium thermodynamics - weakly nonlocal theories
- One component fluid mechanics - quantum (?) fluids
- Two component fluid mechanics - granular material
- Conclusions

- Thermodynamics = macrodynamics
- Weakly nonlocal = there are more gradients

- Examples:

  Guyer-Krumhans
  Ginzburg-Landau
  Cahn-Hilliard (- Frank)
  other phase field
  .
  .
  .



Beyond local equilibrium (nonlocality):

- in time (memory effects) → dynamic variables
- in space (structure effects) → ?

Nonlocalities:

	Space	Time
Strongly nonlocal	Space integrals	Memory functionals
Weakly nonlocal	Gradient dependent constitutive functions	Rate dependent constitutive functions
Relocalized	Current multipliers	Internal variables
	??	

Restrictions from the Second Law.

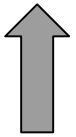
Nonequilibrium thermodynamics

$\dot{\mathbf{a}} + \nabla \cdot \mathbf{j}_a = \sigma_a$ basic balances $\mathbf{a} = (\rho, \rho\mathbf{v}, \ldots)$

– basic state: $\mathbf{a}$

– constitutive state: $C = (\mathbf{a}, \dot{\mathbf{a}}, \nabla\mathbf{a}, \ldots)$

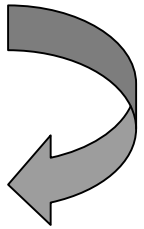
– constitutive functions: $\mathbf{j}_a(C)$



weakly nonlocal

<u>Second law:</u>

$\dot{s}(C) + \nabla \cdot \mathbf{j}_s(C) = \sigma_s \geq 0$



(universality)

Constitutive theory

Method: Liu procedure, Lagrange-Farkas multipliers

Special: irreversible thermodynamics

Example 1: One component weakly nonlocal fluid

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0$$
$$\rho \dot{\mathbf{v}} + \nabla \cdot \mathbf{P}(C) = \mathbf{0}$$
$$\rho \dot{s}(C) + \nabla \cdot \mathbf{j}_s(C) \geq 0$$

$(\rho, \mathbf{v})$ basic state

$C_{wnl} = (\rho, \nabla\rho, \nabla\nabla\rho, \mathbf{v}, \nabla\mathbf{v})$

constitutive state

$s(C), \mathbf{j}_s(C), \mathbf{P}(C)$

⇧

constitutive functions

Liu procedure (Farkas's lemma):

$$s(\rho, \mathbf{v}, \nabla\rho) = s_e(\rho, \nabla\rho) - \frac{\mathbf{v}^2}{2} \qquad \mathbf{j}_s = -\mathbf{v} \cdot \mathbf{P} + \ldots$$

$$\sigma_s = -\Big(\underbrace{\mathbf{P} - \rho^2 \left[\frac{1}{2} \nabla \cdot \partial_{\nabla\rho} s - \partial_\rho s \right] \mathbf{I} - \frac{\rho^2}{2} \nabla \partial_{\nabla\rho} s}_{\mathbf{P}^v} \Big) : \nabla \mathbf{v} \geq 0$$

<u>Schrödinger-Madelung fluid</u>

(Fisher entropy)

$$s_{SchM}(\rho, \mathbf{v}, \nabla\rho) = -\frac{1}{2} \left(\frac{\nabla\rho}{2\rho} \right)^2 - \frac{\mathbf{v}^2}{2}$$

$$\mathbf{P}^r_{SchM} = -\frac{1}{8} \left(\Delta\rho \mathbf{I} + \nabla^2 \rho - \frac{2 \nabla\rho \otimes \nabla\rho}{\rho} \right)$$

Potential form: $\nabla \cdot \mathbf{P}^{r} = \rho \nabla U_{Q}$

$\Psi = \sqrt{\rho} e^{-i\nabla \mathbf{v}}$

Bernoulli equation → Schrödinger equation

$$U_Q = -\partial_\rho(\rho s_e) + \nabla \cdot (\rho \partial_{\nabla\rho} s_e)$$

Euler-Lagrange form

Remark: Not only quantum mechanics

- more nonlocal fluids
- structures (cosmic)
- stability (strange)

Example 2: Two component weakly nonlocal fluid

$$\rho = \nu_1\gamma_1 + \nu_2\gamma_2 = \nu\gamma$$

γ density of the solid component
ν volume distribution function

$$\dot{\gamma} = 0$$
$$\dot{\nu} + \nu\nabla\cdot\mathbf{v} = 0$$
$$\nu\gamma\dot{\mathbf{v}} + \nabla\cdot\mathbf{P}(\mathrm{C}) = \mathbf{0}$$
$$\dot{\mathrm{s}}(\mathrm{C}) + \nabla\cdot\mathbf{j}_{\mathrm{s}}(\mathrm{C}) \geq 0$$

$(\gamma, \nu, \mathbf{v})$ basic state

$$C = (\gamma, \nabla\gamma, \nu, \nabla\nu, \mathbf{v}, \nabla\mathbf{v})$$

constitutive state

$\mathrm{s}(\mathrm{C}), \mathbf{j}_{\mathrm{s}}(\mathrm{C}), \mathbf{P}(\mathrm{C})$

constitutive functions

Constraints: $(1), \nabla(1), (2), \nabla(2), (3)$

Liu equations

$$\rho\partial_{\gamma}s = \Gamma_1,$$
$$\rho\partial_{\nabla\gamma}s = \Gamma_2,$$
$$\rho\partial_{\nu}s = \Gamma_3,$$
$$\rho\partial_{\nabla\nu}s = \Gamma_4,$$
$$\rho\partial_{\mathbf{v}}s = \Gamma_5,$$
$$\rho\partial_{\nabla\mathbf{v}}s = \mathbf{0},$$
$$(\partial_{\nabla\nu}\mathbf{j}_s - \Gamma_5\partial_{\nabla\nu}\mathbf{P})^s = \mathbf{0},$$
$$(\partial_{\nabla\lambda}\mathbf{j}_s - \Gamma_5\cdot\partial_{\nabla\gamma}\mathbf{P})^s = \mathbf{0},$$
$$(\partial_{\nabla\mathbf{v}}\mathbf{j}_s - \Gamma_4\nu\mathbf{I} - \Gamma_5\cdot\partial_{\nabla\mathbf{v}}\mathbf{P})^s = \mathbf{0}.$$

$$\partial_{\nu\nabla\nu}s = \mathbf{0},$$
$$\partial_{\nu\nabla\gamma}s = \mathbf{0},$$
$$\partial_{\nabla\nu\nabla\nu}s = \mathbf{0},$$
$$\partial_{\nabla\nu\nabla\gamma}s = \mathbf{0}.$$

isotropic, second order

Solution:

$$\boxed{s(\nu, \nabla\nu, \lambda, \nabla\gamma, \mathbf{v}) = s_e(\nu, \lambda) - m(\nu, \lambda)\,\frac{\mathbf{v}^2}{2} - \alpha(\nu, \lambda)\,\frac{(\nabla\gamma)^2}{2}}$$

$$\mathbf{j}_s(C) = -m(\nu, \lambda)\ \mathbf{v} \cdot \mathbf{P}(C) - \mathbf{j}_1(\nu, \lambda, \mathbf{v}).$$

$$-\nabla(m\mathbf{v}) : \mathbf{P} + \rho(\alpha\nabla\gamma \circ \nabla\gamma - \nu\partial_\nu s\mathbf{I}) : \nabla\mathbf{v} \geq 0$$

Simplification:

$$m = 1, \quad \mathbf{j}_1(\nu, \lambda, \mathbf{v}) = \mathbf{0}, \quad \partial_\nu s_e = -\frac{p}{\rho^2}.$$

Entropy inequality:

$$-\left(\mathbf{P}-\underbrace{\left(p+\rho\partial_{\nu}\alpha\frac{(\nabla\gamma)^2}{2}\right)\mathbf{I}-\rho\alpha\nabla\gamma\circ\nabla\gamma}_{\mathbf{P}^{r}}\right):\nabla\mathbf{v}\geq 0$$

Coulomb-Mohr

$$\mathbf{P}-\mathbf{P}^{r}=\mathbf{P}^{v}=-\mathbf{L}\nabla\mathbf{v}$$

isotropy: Navier-Stokes like + ...

Properties

1 Other models:

a) Goodman-Cowin

$$\mathbf{P}^{r} = (p + \alpha(\nabla v)^2 + 2\alpha v \Delta v)\mathbf{I} - 2\alpha \nabla v \circ \nabla v$$

$$\gamma \ddot{v} - \nabla \cdot \mathbf{h} = \sigma_{\dot{v}}$$ configurational force balance

b) Navier-Stokes type: $+\nabla\rho$ somewhere

2 Coulomb-Mohr

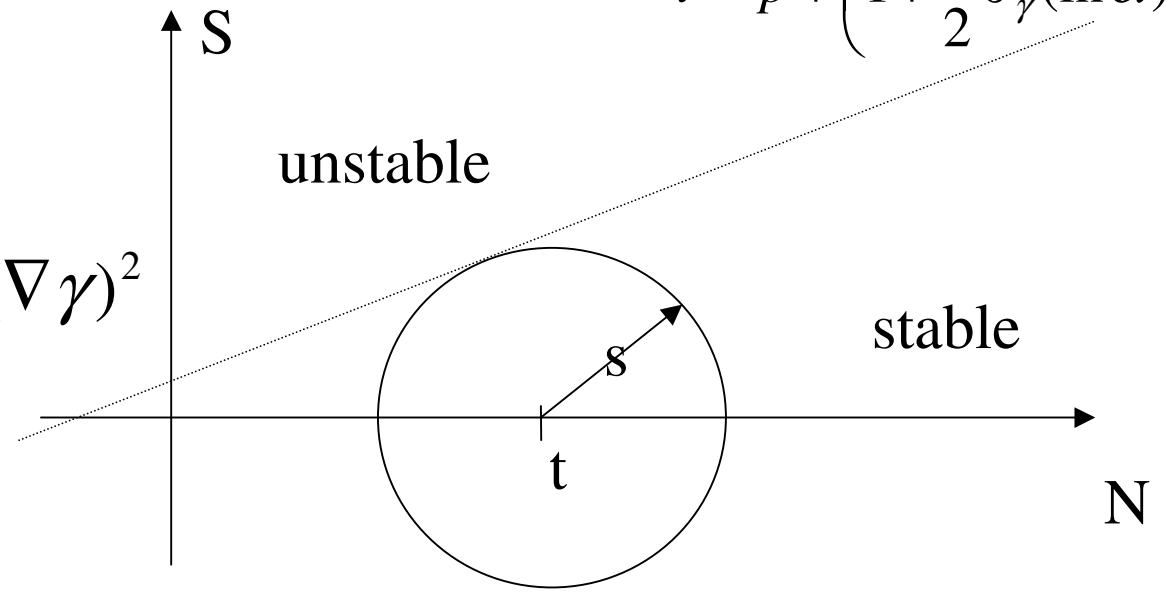
$$\mathbf{N} := \mathbf{n} \cdot \mathbf{P}^{\mathrm{r}} \cdot \mathbf{n} \qquad \mathbf{S} := \mathbf{P}^{\mathrm{r}} - \mathbf{N}$$

$$\mathbf{S}^2 + (\mathbf{N} - \mathbf{t})^2 = \mathbf{s}^2$$

$$s = \rho\alpha(\nabla\gamma)^2$$

$$t = p + \rho\left(\frac{\partial_\nu \alpha}{2} - \alpha\right)(\nabla\gamma)^2$$

$$t = p + \left(1 + \frac{1}{2}\partial_\gamma(\ln\alpha)\right)s$$



3 solid-fluid(gas) transition

$$(\nabla \dot{\gamma}) = -\nabla \mathbf{v} \cdot \nabla \gamma$$

relaxation (1D)

$$\Downarrow$$

$$\mathbf{P}^{r} \rightarrow p\mathbf{I}$$

4 internal spin: no corrections

Conclusions

-- Phenomenological background

- for any statistical-kinetic theory
- Kaniadakis (kinetic), Plastino (maxent)

-- Nontrivial material (in)stability

- not a Ginzburg-Landau
- phase ‘loss’