Thermodynamics of relativistic fluids

P. Ván
HAS, Wigner RCP, Department of Theoretical Physics,
BME, Department of Energy Engineering,
Montavid Thermodynamic Research Group

1. Dissipative fluids
2. Basic questions
   causality, stability, frames, kinetic compatibility
   – Temperature of moving bodies.
3. Thermodynamics, flow and stability

Basic concepts:

$$T^{ab} = eu^a u^b + q^a u^b + q^b u^a + P^{ab},$$ energy-momentum density

$$N^a = nu^a + j^a.$$ particle number density

$$q^a u_a = j^a u_a = 0, \quad P^{ba} u_a = P^{ab} u_a = 0^b$$

$$T^{ab} = \begin{pmatrix} e & q^i \\ q^j & P^{ij} \end{pmatrix}, \qquad N^a = \begin{pmatrix} n \\ j^i \end{pmatrix}$$

$$a, b \in \{0,1,2,3\}; \quad i, j \in \{1,2,3\}; \quad diag(1,-1,-1,-1)$$

$$\dot{e} = u^a \partial_a e$$

- u^a – velocity field
- e – energy density
- q^a – momentum density or energy current??
- P^{ab} – pressure
- n – particle number density
- j^a – particle current

General, expressed by comoving splitting

$$u_a \partial_b T^{ab} = \dot{e} + e\, \partial_a u^a + \partial_a q^a + u_a \dot{q}^a + u_a \partial_b P^{ab} = 0^a$$ energy balance

$$\partial_b N^b = \dot{n} + n\, \partial_a u^a + \partial_a j^a = 0$$ particle number balance

Dissipative or ideal?

$$P^{ab} = -p\Delta^{ab} + \Pi^{ab} = (-p + \Pi)\Delta^{ab} + \pi^{ab}$$ pressure splitting

Constitutive theory:

Fields:

N^a	4
T^{ba}	10
u^a	3
	Σ17

j^a	3
q^a	3
Π^{ab}	6
	Σ 12

$$q^a u_a = j^a u_a = 0, \quad \Pi^{ba} u_a = \Pi^{ab} u_a = 0^b$$

Equations:

$\partial_a N^a = 0,$	1
$\partial_b T^{ab} = 0^a,$	4

N^a – *particle number density vector*
T^{ab} – energy-momentum tensor
u^a – velocity field

j^a – *particle current*
q^a – energy current??
Π^{ab} – viscous pressure

n, e, u^a – basic fields

Non-equilibrium thermodynamics, second law

Entropy inequality:

$$\partial_a S^a = \dot{s} + s\,\partial_a u^a + \partial_a J^a \geqslant 0$$

Eckart (1940):

$$S^a(T^{ab}, N^a) = s(e,\; n)u^a + \frac{q^a}{T}$$

(Müller)-Israel-Stewart (1969-72):

$$S^a(T^{ab}, N^a) = \left(s(e,\; n) - \frac{\beta_0}{2\mathrm{T}}\Pi^2 - \frac{\beta_1}{2\mathrm{T}} q_b q^b - \frac{\beta_2}{2\mathrm{T}} \pi^{bc}\pi_{bc} \right) u^a +$$

$$+ \frac{1}{T}\left(q^a + \alpha_0 \Pi q^a + \alpha_1 \pi^{ab} q_b \right)$$

The idea of Eckart:

$$u^a \partial_b T^{ab} = \dot{e} + e\partial_a u^a + \partial_a q^a + u_a \dot{q}^a + u_a \partial_b P^{ab} = 0^a$$

$$\partial_b N^b = \dot{n} + n\partial_a u^a + \partial_a j^a = 0$$

$$ds + \alpha\, dn = \beta\, de$$

$$\dot{e} = u^a \partial_a e$$

$$J^a = \beta q^a - \alpha\, j^a$$

$$\partial_a S^a = \dot{s}(e,n) + s\partial_a u^a + \partial_a J^a \geq 0$$

$$-\frac{1}{T}\left(P^{ij} - p\delta^{ij}\right)\partial_i v_j + q^i \partial_i \frac{1}{T} \geq 0$$

$$\sigma_s = -j^a \partial_a \alpha - \beta \underbrace{\left(P^{ab} - p\delta^{ab}\right)}_{\Pi^{ab}} \partial_b u_a + q^a\left(\partial_a \beta + \beta \dot{u}_a\right) \geq 0$$

Eckart term

Concept of dissipation

constitutive theory – closure by linear relations
thermodynamic fluxes and forces:

gradients?
basic state space – constitutive state space

$$\sigma_s = -j^a \partial_a \alpha - \beta \Pi^{ab} \partial_b u_a + q^a (\partial_a \beta + \beta \dot{u}_a) \geq 0$$

$$j^a = \chi \Delta^{ab} \partial_b \alpha ,$$
$$\Pi^{ab} = \nu \partial_c u^c \Delta^{ab} + \eta \Delta^{ac} \Delta^{bd} (\partial_c u_d + \partial_d u_c)/2,$$
$$q^a = \lambda \Delta^{ab} (\partial_b \beta + \beta \dot{u}_b)$$

+ balances

Concepts only in this conference:

P. Romatschke:

there is only viscosity

C. Lopez-Monsalvo:

dynamic equation of state
(Lagrangian: $L(T^{ab},S^{a})$)

A. Sandoval-Villalbazo:

heat conduction, instability

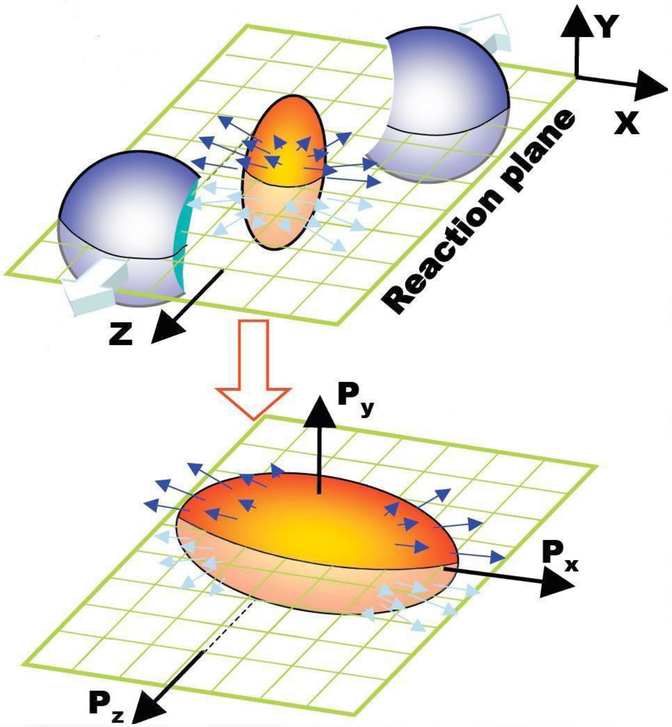
E. Calzetta:

kinetic theory

Dissipative relativistic fluids

heavy ion collisions

cosmology



– (quark-)gluon plasma

there is a minimal viscosity

What is viscous?

a need of theories

– *Causality*

hyperbolic or parabolic?

– *Stability* – second law

instable homogeneous equilibrium

– *Velocity* – *flow-frames*

Is there a freedom? (Eckart, Landau-Lifshitz, …) What is ideal?

– *Kinetic theory*

Do we need anything else?

Causality

- infinite speed of signal propagation
- second order time derivatives
- hyperbolic system of equations

  motivation of Extended Irreversible Thermodynamics (L. García-Colín)

Divergence type theories – finite speed is material

(Liu-Ruggeri-Müller, Geroch, Lindblom, Calzetta)

Physical:

Propagation speed of *continuum limit.*

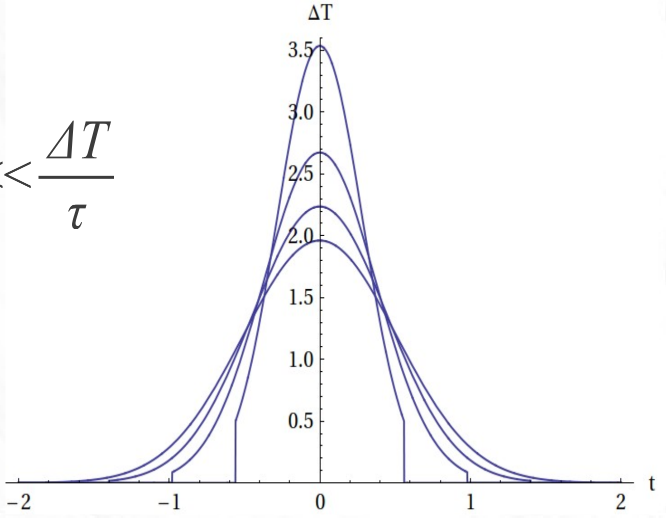
Propagation speed of observable signals.

Example:

$$\partial_t T = -\kappa \partial_x^2 T \qquad \partial_x T << \frac{\Delta T}{\xi}, \quad \partial_t T << \frac{\Delta T}{\tau}$$

water at room temperature:

$$v_{max} \approx \frac{\kappa}{\xi} = 14\, m/s$$



Stability

Generic stability: linear stability of homogeneous equilibrium

Instability of first order theories (Hiscock-Lindblom, 1985)
Stability of the (Müller)-Israel-Stewart theory (Hiscock-Lindblom, 1983)

Divergence type theories – built in stability, need of dissipation

Conceptual question:
Thermodynamics is related to stability.

There are conditions.

Stability conditions of the Israel-Stewart theory

(Hiscock-Lindblom 1983,87)

$$\Omega_1 = \frac{1}{e+p}\frac{\partial e}{\partial p}\Big|_{\frac{s}{n}} = \frac{T}{(e+p)\frac{\partial p}{\partial e}\Big|_n - n\frac{\partial p}{\partial n}\Big|_e} \geq 0,$$

$$\Omega_2 = \frac{1}{e+p}\frac{\partial e}{\partial (s/n)}\Bigg|_{\frac{\mu}{nT}}\frac{\partial p}{\partial (s/n)}\Bigg|_{\frac{\mu}{nT}} = \ldots \geq 0$$

Conditons for the

- EOS
- IS coefficients
- both

+ usual

Eckart frame

$$\Omega_5 = \beta_0 \geq 0, \quad \Omega_8 = \beta_2 \geq 0, \quad \Omega_7 = \beta_1 - \frac{\alpha_1^2}{2\beta_2} \geq 0,$$

$$\Omega_4 = e + p - \frac{2\beta_2 + \beta_1 + 2\alpha_1}{2\beta_1\beta_2 - \alpha_1^2} \geq 0, \quad \Omega_6 = \beta_1 - \frac{\alpha_0^2}{\beta_0} - \frac{2\alpha_1^2}{3\beta_2} - \frac{1}{n^2 T}\frac{\partial T}{\partial (s/n)}\Bigg|_n \geq 0,$$

$$\Omega_3 = (e+p)\left(1 - \frac{\partial p}{\partial e}\Big|_{\frac{s}{n}}\right) - \frac{1}{\beta_0} - \frac{2}{3\beta_2} - \frac{K^2}{\Omega_6} \geq 0, \quad K = 1 + \frac{\alpha_0}{\beta_0} + \frac{2\alpha_1}{3\beta_2} - \frac{n}{T}\frac{\partial T}{\partial n}\Bigg|_{s/n} \geq 0.$$

Velocity – flow-frames

What is a fluid? What is moving?

Eckart (material) frame:

$$u^a = \frac{N^a}{\sqrt{-N^b N_b}} \rightarrow N^a = n u^a$$

Landau-Lifshitz (energy) frame:

$$\hat{u}^a = \frac{E^a}{\sqrt{-E^b E_b}} \rightarrow T^{ab} = \hat{e}\,\hat{u}^a \hat{u}^b + \hat{P}^{ab}$$

Jüttner
(thermometer) frame:

$$\check{u}^a = \frac{\beta^a}{\sqrt{-\beta^b \beta_b}} \rightarrow \beta^a = \check{\beta}\,\check{u}^a$$

Non-relativistic (Brenner), multicomponent.

Do we have a choice?

Landau-Lifshitz:

$$N^a = \hat{n}\hat{u}^a + \hat{j}^a$$
$$T^{ab} = \hat{e}\hat{u}^b\hat{u}^a + \hat{P}^{ab} = \hat{e}\hat{u}^b\hat{u}^a - \hat{p}\hat{\Delta}^{ab} + \hat{\Pi}^{ab}$$

Eckart:

$$N^a = nu^a$$
$$T^{ab} = eu^b u^a + q^b u^a + q^a u^b - p\Delta^{ab} + \Pi^{ab}$$

Transformation:

$$u^a = \frac{\hat{u}^a + \hat{z}^a}{\zeta}$$

What is ideal?

$$N_0^a = nu^a$$
$$T_0^{ab} = eu^b u^a - p\Delta^{ab}$$

$$N_0^a = \hat{n}\hat{u}^a + j^a$$
$$T_0^{ab} = \hat{e}\hat{u}^b\hat{u}^a + q^b\hat{u}^a + q^a\hat{u}^b - p\hat{\Delta}^{ab} + \Pi^{ab}$$

$$\hat{n} = \frac{n}{\zeta}, \quad j^a = \frac{n\hat{z}^a}{\zeta}, \quad \hat{e} = \frac{e+p}{\zeta^2} - p, \quad q^a = (e+p)\hat{z}^a, \quad \Pi^{ab} = \frac{\hat{z}^a\hat{z}^b}{e+p}$$

Ideal fluid is a class of N^a, T^{ab}

Entropy production, Gibbs relation are flow-frame dependent.

Remarks

– The Israel-Stewart theory is *not* proved to be symmetric hyperbolic.

– In Israel-Stewart theory the symmetric hyperbolicity *conditions* of the *perturbation* equations follow from the stability conditions.

– Relaxation to the (unstable) first order theory?
(Geroch 1995, Lindblom 1995)

– Stable theories can be transformed to hyperbolic, if they are based on consistent thermodynamics.

– Do we need a choice of flow-frames?? Landau-Lifshitz is the best?
(Tsumura-Kunihiro 2012, Romatschke)

– Kinetic theory – thermodynamics is built in – the problems are inherited.

Kinetic theory → thermodynamics

$$p^a \partial_a f = C(f)$$

Boltzmann equation

$$p^a p_a = m^2$$

Boltzmann gas

Thermodynamic equilibrium = no dissipation:

$$\sigma = \partial_a S^a = \partial_a \left(-\int \frac{d^3 p}{p^0} p^a f (\ln f - 1) \right) =$$

$$\frac{1}{4} \sum_{i,j,k,l} \int \frac{d^3 p_i}{p_i^0} \frac{d^3 p_j}{p_j^0} \frac{d^3 p_k}{p_k^0} \frac{d^3 p_l}{p_l^0} \left(\frac{f_k f_l}{f_i f_j} - \ln \frac{f_k f_l}{f_i f_j} - 1 \right) f_i f_j W_{ij|kl} = 0$$

$$\Leftrightarrow f f_1 = f' f_1' \Leftrightarrow$$

$$f_0(x,k) = e^{\alpha(x) - \beta_b(x) p^b}$$

(local) equilibrium distribution

Thermodynamic relations – normalization

$$f_0(x,p)=e^{\alpha(x)-\beta_b(x)p^b}$$

$$N_0^a=\int p^a f_0$$
$$T_0^{ab}=\int p^b p^a f_0$$

Jüttner distribution AND Jüttner flow-frame

$$\alpha=\frac{\check{\mu}}{\check{T}},\ \beta_a=\frac{\check{u}_a}{\check{T}}$$

$$f_0(x,p)=e^{\frac{\check{\mu}-\check{u}_b p^b}{\check{T}}}$$

When calculated frame independently, one obtains:

$$\beta^a=\beta(u^a+w^a)$$

$$\check{u}^a=\frac{u^a+w^a}{\sqrt{1-w^2}}$$

$$\check{T}=\frac{T}{\sqrt{1-w^2}},\ \check{\mu}=\frac{\mu}{\sqrt{1-w^2}},$$

$$f_0(x,p)=e^{\alpha-\beta_b p^b}=e^{\frac{\mu-(u_b+w_b)p^b}{T}}=e^{\frac{\check{\mu}-\check{u}_b p^b}{\check{T}}}$$

Energy-momentum density:

$$N_0^a = \int p^a f_0 = \check{n}\check{u}^a = nu^a + nw^a$$

$$\check{n} = n\sqrt{1-w^2} = 4\pi \mathrm{m}^2 \check{T} K_2\left(\frac{m}{\check{T}}\right) e^{\frac{\check{\mu}}{\check{T}}}$$

$$T_0^{ab} = \int p^a p^b f_0 = \check{e}\check{u}^b\check{u}^a + \check{p}\check{\Delta}^{ab}$$

$$\check{e} = 3\check{n}\check{T} + m\check{n}\frac{K_1\left(\frac{m}{\check{T}}\right)}{K_2\left(\frac{m}{\check{T}}\right)}$$

$$T_0^{ab} = eu^b u^a + q^b u^a + u^b q^a - p\Delta^{ab} + \frac{q^b q^a}{e+p}$$

$$\check{p} = p, \quad e = \frac{\check{e} + p w^2}{1-w^2}, \quad q^a = (e+p) w^a$$

Heat flux:

$$I^a = q^a - \frac{e+p}{n} j^a = 0$$

What is a fluid (continuum)?

Flow of extensives? (Eckart, Landau-Lifshitz, …)
Velocity field?
Thermodynamics?

What is non-equilibrium thermodynamics?

Is that something statistical? (Is that an explanation?)

Thermodynamics is a stability theory.
(stability – observability, generality – universality)

Requirements:

– Generic stability (hyperbolicity can be added)

– Kinetic compatibility: $q^a = (e + p) w^a$

Freedom:

– Flow-frames: arbitrary or fixed?
– *Thermodynamics*

Remark: thermo is flow frame dependent

Gibbs: $ds + \alpha\, dn = \beta\, de$

entropy production

Digression and connection:

to the

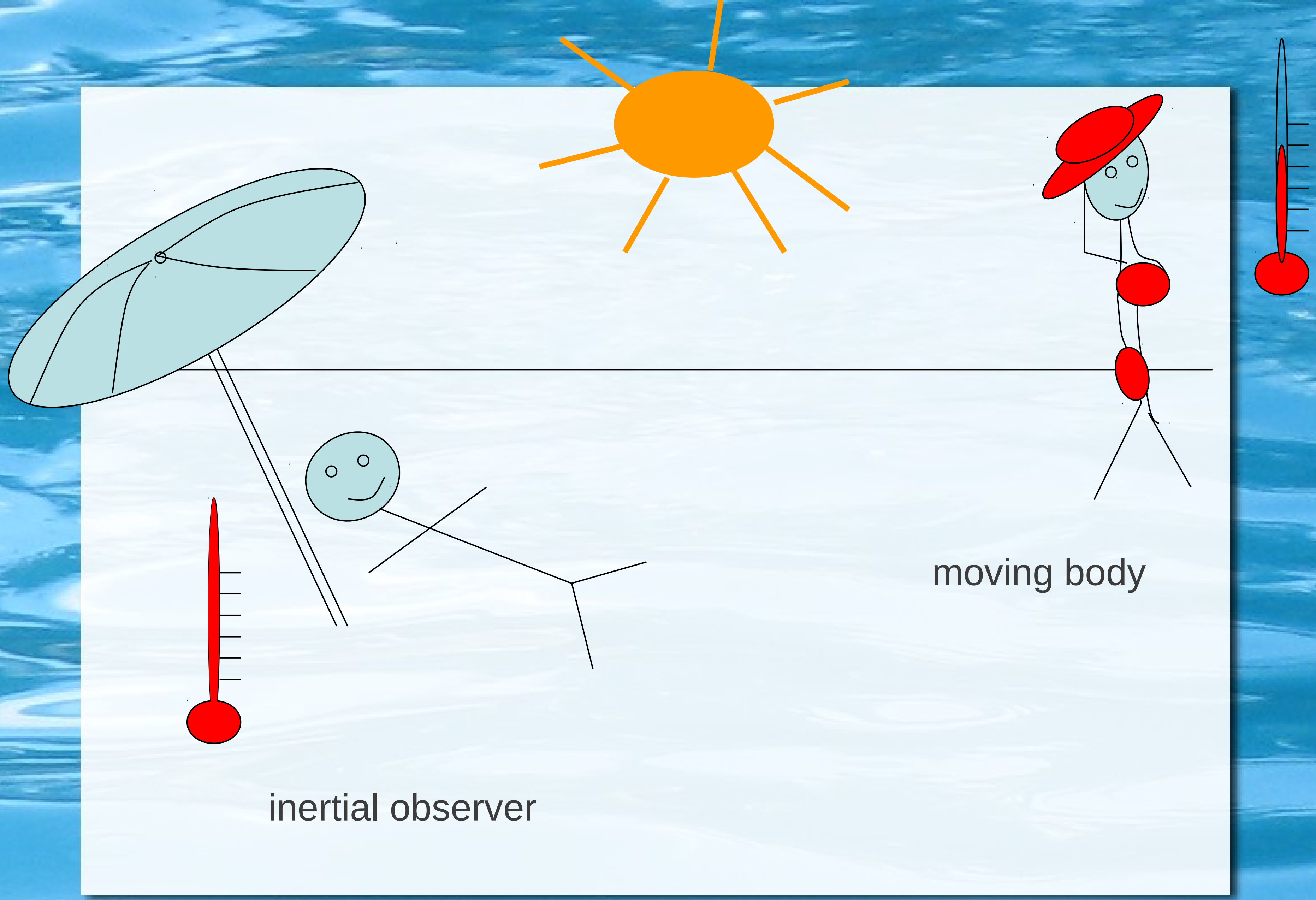
ancient

and

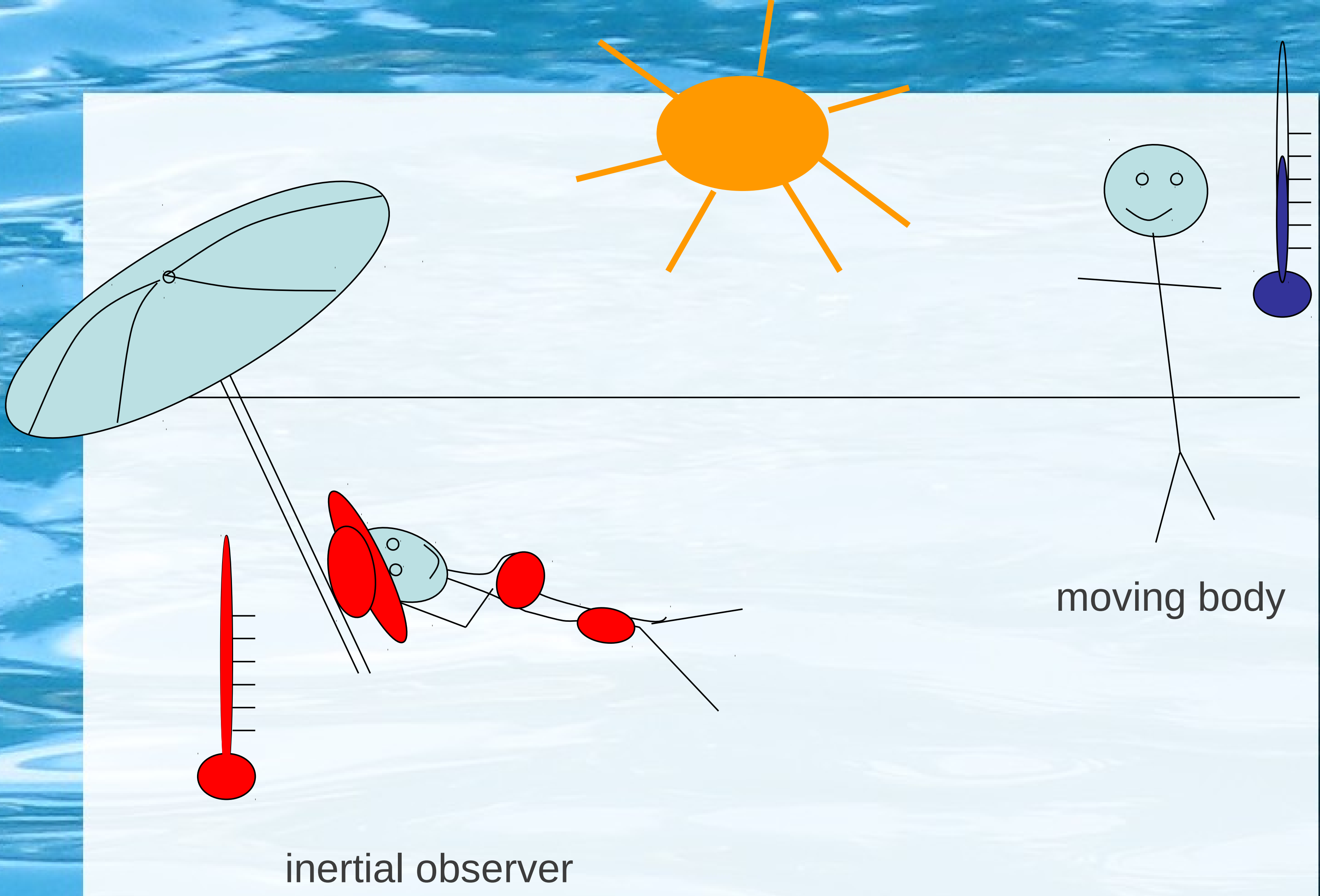
important

problem of

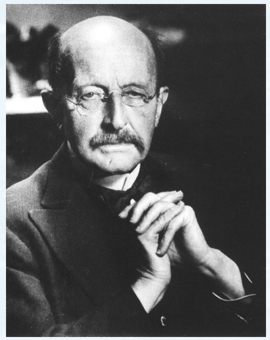
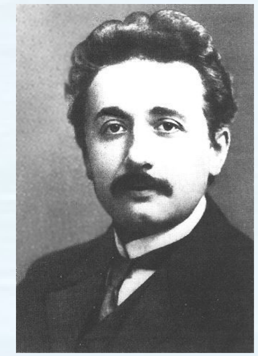
Temperature of moving bodies

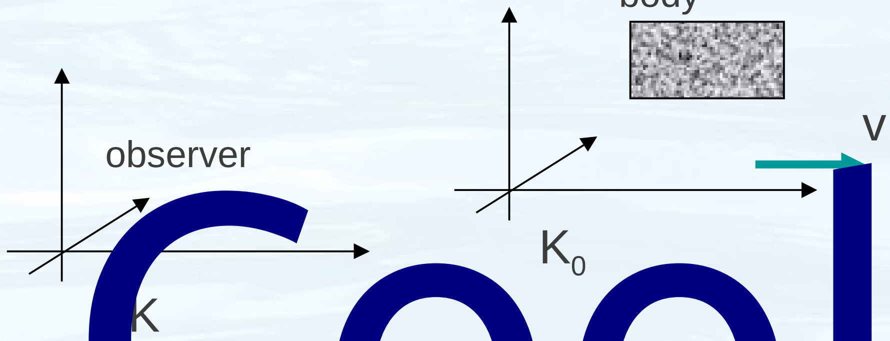


moving body
inertial observer


moving body
inertial observer

About the temperature of moving bodies

Planck  and Einstein 

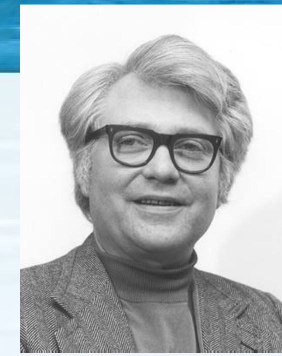


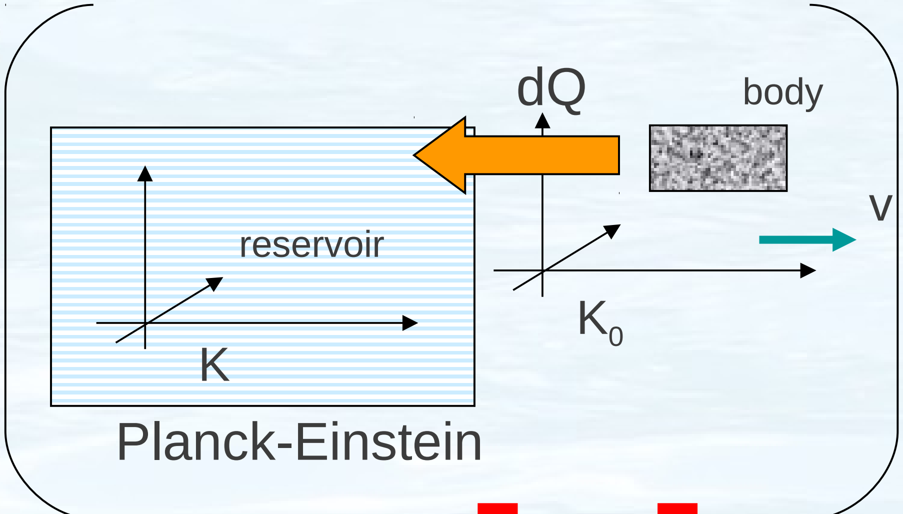
$$dE = TdS - pdV$$

Cool

Relativistic thermodynamics?

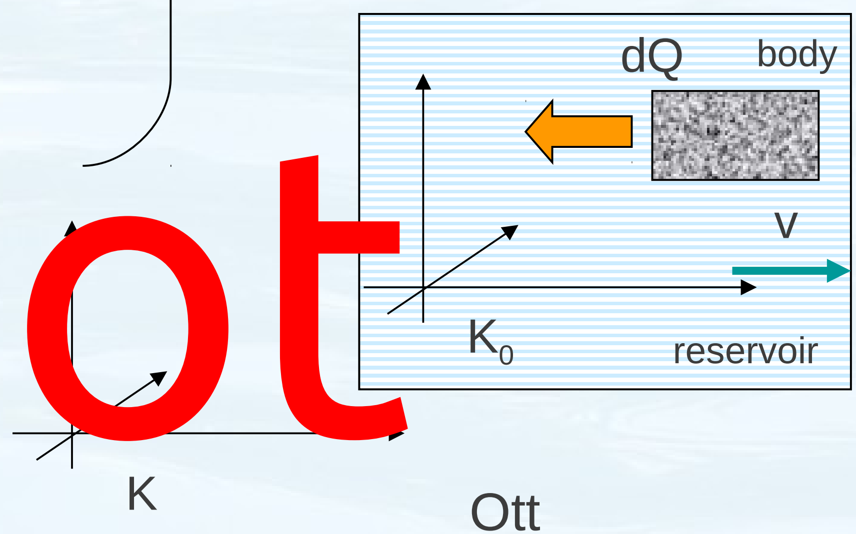
Rest frame arguments:
Ott (1963)





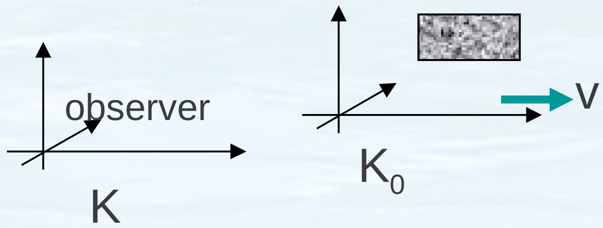
$$dE = TdS - pdV + vdG$$

Hot



$$dE = TdS - pdV + \cancel{vdG}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$



Permanent discussion

~1963-70, Møller, von Treder, Israel, ter Haar, Callen, …, renewed Dunkel-Talkner-Hänggi 2007, …:

– new (?) arguments, no (re)solution.

- Planck-Einstein (1907): cooler $T = \frac{T_0}{\gamma}$
- Ott (1963) [Blanusa (1947)] : hotter $T = \gamma T_0$
- Landsberg (1966-67): equal $T = T_0$
- Costa-Matsas-Landsberg (1995): direction dependent (Doppler) $T = \frac{T_0}{\gamma(1 - v\cos\alpha)}$

?????

Why is the instability?

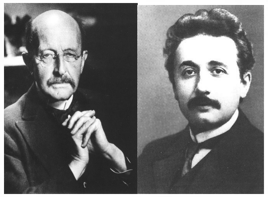
$$\sigma_s = -j^a \partial_a \alpha - \beta \underbrace{\left(P^{ab} - p\delta^{ab}\right)}_{\Pi^{ab}} \partial_b u_a + q^a \left(\partial_a \beta + \beta \dot{u}_a\right) \geq 0$$

Acceleration

$$q^a = \lambda \Delta^a_b \left(\partial_b \beta + \beta \dot{u}^a\right)$$

$$\Delta^a_c \partial_b T^{cb} = h \dot{u}^a + \Delta^a_c \dot{q}^c + q^a \partial_b u^b + q^b \partial_b u^a - \Delta^{ab} \partial_b p + \Delta^a_c \partial_b \Pi^{cb} = 0^a$$

$$T^{ab} = \begin{pmatrix} e & q^i \\ q^j & P^{ij} \end{pmatrix}$$



momentum or heat?

Total or internal energy?

Killing the Eckart term is not enough.

Requirements:

– Generic stability

– Kinetic compatibility: $q^a = (e + p) w^a$

Freedom:

– Arbitrary flow-frames

– Covariant *thermodynamics?*

Thermodynamics in arbitrary frames:

$$S^a + \alpha N^a - \beta_b T^{ab} = \Phi^a$$ objective/covariant starting point

$$\beta^a = \beta(u^a + w^a)$$ temperature vector

Constrained entropy inequality:

$$0 \leq \partial_a S^a + \alpha \partial_a N^a - \beta_b \partial_a T^{ab} = -N^a \partial_a \alpha - T^{ab} \partial_a \beta_b + \partial_a \Phi^a =$$

$$-n\dot{\alpha} + h\dot{\beta} + q^a (\dot{\beta w_a}) + \beta \dot{p} + \Pi^{ab} \partial_a u_b - j^a \partial_a \alpha + \ldots = \Sigma \geq 0$$

Thermodynamics:

a) $\Phi^a = p\beta^a$ matching $S_0^a + \alpha N_0^a - \beta_b T_0^{ab} = \beta^a p_0$

b) $ds + \alpha\, dn = \beta(de + w_a\, dq^a)$ $w^\mu = 0 \Rightarrow ds + \alpha dn = \beta de$

Entropy production:

$$0 \leq \Sigma = \left(n w^a - j^a\right)\partial_\mu \alpha + \left(q^a - h w^a\right)\left(\partial_a \beta + \beta \dot{u}_a\right) +$$
$$\left(\Pi^{ab} - w^{(a} q^{b)}\right)\partial_a \beta_b + w^{(a} q^{b)} \partial_b \left(\beta\left(u_a - w_a\right)\right)$$

dynamic EOS: $\beta^a = \beta\left(u^a + w^a\right)$

thermometer dEOS/Jüttner: $w^a = 0$

$$ds + \alpha\, dn = \beta\, de$$

natural dEOS/Landau-Lifshitz: $w^a = q^a / e$

$$ds + \alpha\, dn = \beta_a dE^a = \beta\left(de + \frac{q_a}{e} dq^a\right) \quad s\left(E^a, n\right) = \hat{s}\left(E = -\sqrt{-E_a E^a}\right)$$

kinetic dEOS: $w^a = q^a / h$

$$ds + \alpha\, dn = \beta\left(de + \frac{q_a}{h} dq^a\right) \qquad s\left(E^a, u^a, n\right)$$

Jüttner dEOS: generic instable

arbitrary frame, (Hiscock-Lindblom, 1985)

Natural dEOS: generic stable, not kinetic compatible

arbitrary frame, only thermodynamic conditions
(PV- TS Biró, 2008, PV, 2009)

Kinetic dEOS: generic stable, kinetic compatible

Special case, only thermodynamic conditions
(PV-TS Biró, 2012)

General dEOS?? $s\left(E^a = e u^a + q^a, n\right) = \hat{s}\left(e, q^a, u^a\right)$

Cannot be stable.

Thermodynamics defines local equilibrium and the flow frame.

Entropy production with kinetic dEOS:

$$\partial_a S^a = \Pi^{ab}\partial_b \beta_a + q^a\left(\partial_a \beta + \frac{\beta}{h}\left(h\dot{u}^a + \dot{q}^a + q^a \partial_b u^b + \partial_b \Pi^{ab}\right)\right) =$$

$$= \left(\Pi^{ab} - \frac{q^a q^b}{h}\right)\partial_a \beta_b \geqslant 0$$

$$\beta^a = \beta\left(u^a + w^a\right) = \tilde{\beta}\,\tilde{u}^a$$

$$\tilde{u}^a = \frac{\left(h u^a + q^a\right)}{\sqrt{h^2 - q^2}}$$

Prediction: heat conduction and viscosity are interdependent.

Conclusions

Thermodynamics fixes the flow with dEOS.

- Dissipation is spacelike. Can we fix a flow-frame twice?

Kinetic dEOS is preferred.

- Generic stability:
  - thermodynamic stability+ nonnegative transport coefficients
- Kinetic compatibility.

Momentum balance is a constraint,
    momentum may be a state variable.
    Non-relativistic! Total energy, internal energy.

Temperature!

VP., Biró, TS., EPJ–ST, 155:201–212, 2008, (arXiv:0704.2039v2).
VP., MMS, 3(6):1161–1169, 2008, (arXiv:07121437).
Biró, TS., VP. EPL, 89:30001, 2010.
VP., EPJ WoC, 13:07004, 2011, (arXiv:1102.0323).
VP., Biró, TS., PLB, 709(1-2):106–110, 2012, (arXiv:1109.0985).
VP. , Biró TS., in Proc. of JETC13, (2013), arXiv: 1305.3190

Thermodynamics

is

~~amazing.~~

stability?

Thermodynamic relations - normalization

$$f_0(x,k) = e^{\alpha(x) - \beta_\nu(x) k^\nu}$$

$$N_0^\mu = \int k^\mu f_0$$

$$T_0^{\mu\nu} = \int k^\nu k^\mu f_0$$

Jüttner distribution? $\alpha = \frac{\mu}{T}, \beta_\nu = \frac{u_\nu}{T}$ $f_0(x,k) = e^{\frac{\mu - u_\mu k^\nu}{T}}$

$$\partial_\mu N_0^\mu = \partial_\mu \int k^\mu f_0 = \int \left(f_0 k^\mu \partial_\mu \alpha - f_0 k^\nu k^\mu \partial_\mu \beta_\nu \right) =$$

$$\partial_\mu N_0^\mu = N_0^\mu \partial_\mu \alpha - T_0^{\mu\nu} \partial_\mu \beta_\nu$$

$$S_0^\mu := (1-\alpha) N_0^\mu + \beta_\nu T_0^{\mu\nu}$$ Legendre transformation

$$\partial_\mu S_0^\mu = -\alpha \partial_\mu N_0^\mu + \beta_\nu \partial_\mu T_0^{\mu\nu}$$

$$\partial_\mu S_0^\mu + \alpha \partial_\mu N_0^\mu - \beta_\nu \partial_\mu T_0^{\mu\nu} = 0$$

covariant Gibbs relation (Israel,1963)

Remark: $$\partial_\mu S^\mu + \alpha \partial_\mu N^\mu - \beta_\nu \partial_\mu T^{\mu\nu} = \sigma \geq 0$$

Lagrange multipliers – non-equilibrium

Rest frame quantities:

$$S^\mu = s u^\mu + J^\mu$$

$$N^\mu = n u^\mu + j^\mu$$

$$T^{\mu\nu} = u^\mu E^\nu + q^\mu u^\nu + P^{\mu\nu}$$

$$\boxed{E^\nu = e u^\nu + q^\nu}$$

$$u_\mu u^\mu = 1,\ \Delta^{\mu\nu} = \delta^{\mu\nu} - u^\mu u^\nu;$$

$$u_\mu J^\mu = 0, u_\mu j^\mu = 0;$$

$$u_\mu q^\mu = 0, u_\mu P^{\mu\nu} = P^{\mu\nu} u_\nu = 0.$$

$$\partial_\mu S^\mu + \alpha \partial_\mu N_0^\mu - \beta_\nu \partial_\mu T_0^{\mu\nu} =$$
$$\boxed{\dot{s} + \alpha \dot{n} - \beta_\mu \dot{E}^\mu} + \left(s + \alpha n - \beta_\mu E^\mu\right) \partial_\nu u^\nu -$$
$$\alpha \partial_\mu j^\mu + \beta_\nu \partial_\mu (q^\mu u^\nu + P^{\mu\nu}) = 0$$

$$\dot{s} + \alpha\dot{n} - \beta_\mu \dot{E}^\mu + \left(s + \alpha n - \beta_\mu E^\mu\right)\partial_\nu u^\nu - j^\mu \partial_\mu \alpha + (q^\mu u^\nu + P^{\mu\nu})\partial_\mu \beta_\nu = 0$$

A) $\dot{s} + \alpha\dot{n} - \beta_\mu \dot{E}^\mu = 0$

$$s(n, E^\mu)$$

$$\frac{\partial s}{\partial n} = \alpha = \frac{\mu}{T}$$

deviation from Jüttner

$$\frac{\partial s}{\partial E^\mu} = \beta_\mu = \frac{g_\mu}{T} = \frac{u_\mu + w_\mu}{T} \qquad u^\mu w_\mu = 0$$

Velocity dependence?

$$Tds + \mu dn = \beta_\mu dE^\mu = (u_\mu + w_\mu) d(eu^\mu + q^\mu) = de + w_\mu dq^\mu + (ew_\mu - q_\mu) du^\mu$$

B) $\left(s + \alpha n - \beta_\mu E^\mu\right)\partial_\nu u^\nu - j^\mu \partial_\mu \alpha + (q^\mu u^\nu + P^{\mu\nu})\partial_\mu \beta_\nu = 0$

$$p = Ts + \mu n - \beta_\mu E^\mu = nT$$

$$\nabla_\nu \mu = 0,\ \nabla_\nu T = 0,\ \nabla_\nu w_\mu = 0$$

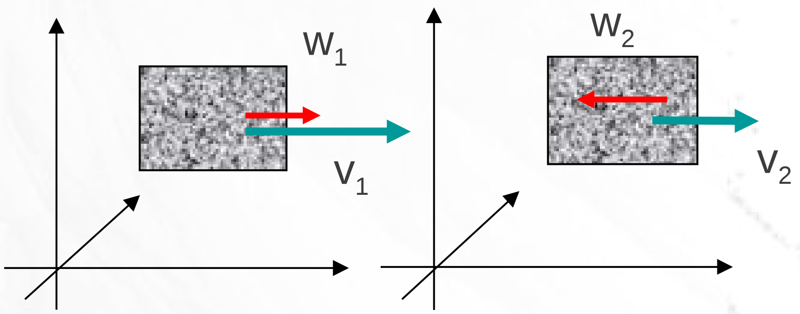
ideal gas

rest frame/uniform intensives

$$TdS = (u_a + w_a)\, dE^a \;\Rightarrow\; \frac{(u_1^a + w_1^a)}{T_1} = \frac{(u_2^a + w_2^a)}{T_2}$$

1+1 dimensions:

$$u^a = (\gamma, \gamma v), \quad w^a = (\gamma v w, \gamma w)$$



$$\frac{\gamma_1 (1 + v_1 w_1)}{T_1} = \frac{\gamma_2 (1 + v_2 w_2)}{T_2}$$

$$\frac{\gamma_1 (v_1 + w_1)}{T_1} = \frac{\gamma_2 (v_2 + w_2)}{T_2}$$

$\Rightarrow$

$$\frac{v_1 + w_1}{1 + v_1 w_1} = \frac{v_2 + w_2}{1 + v_2 w_2}$$

$$\frac{\sqrt{1 - w_1^2}}{T_1} = \frac{\sqrt{1 - w_2^2}}{T_2}$$

Transformation of temperatures

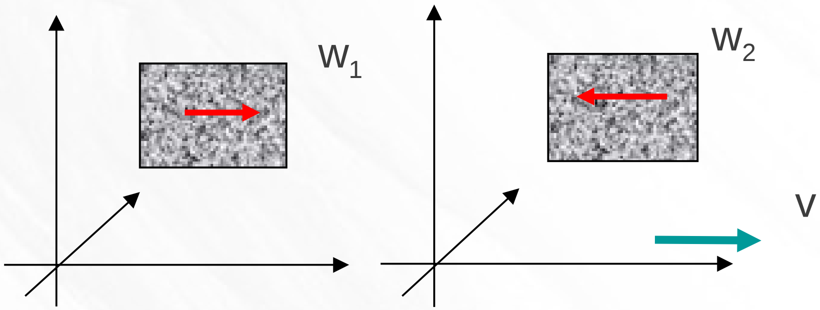
$$\frac{v_1+w_1}{1+v_1w_1}=\frac{v_2+w_2}{1+v_2w_2},\quad \frac{\sqrt{1-w_1^2}}{T_1}=\frac{\sqrt{1-w_2^2}}{T_2}$$

Four velocities: v_1, v_2, w_1, w_2

Relative velocity

(Lorentz transformation)

$$v=\frac{v_2-v_1}{1-v_1v_2}$$

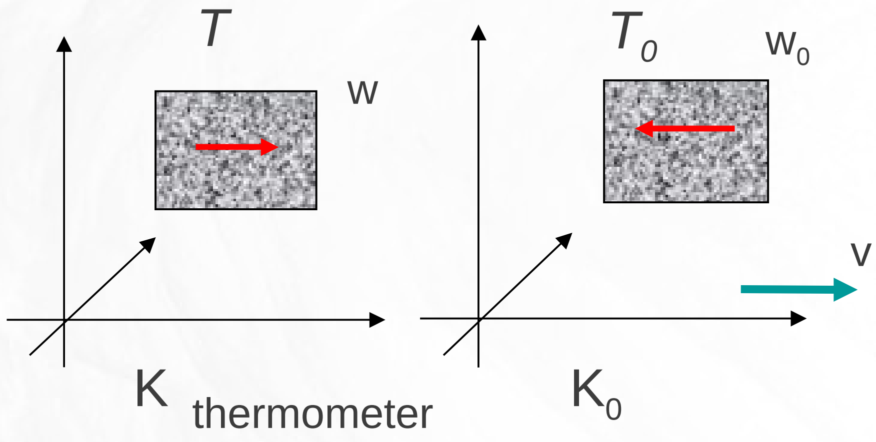


$$w_1=\frac{v+w_2}{1+vw_2}$$

$$\frac{T_1}{T_2}=\frac{\sqrt{1-v^2}}{1+vw_2}$$

general Doppler-like form!

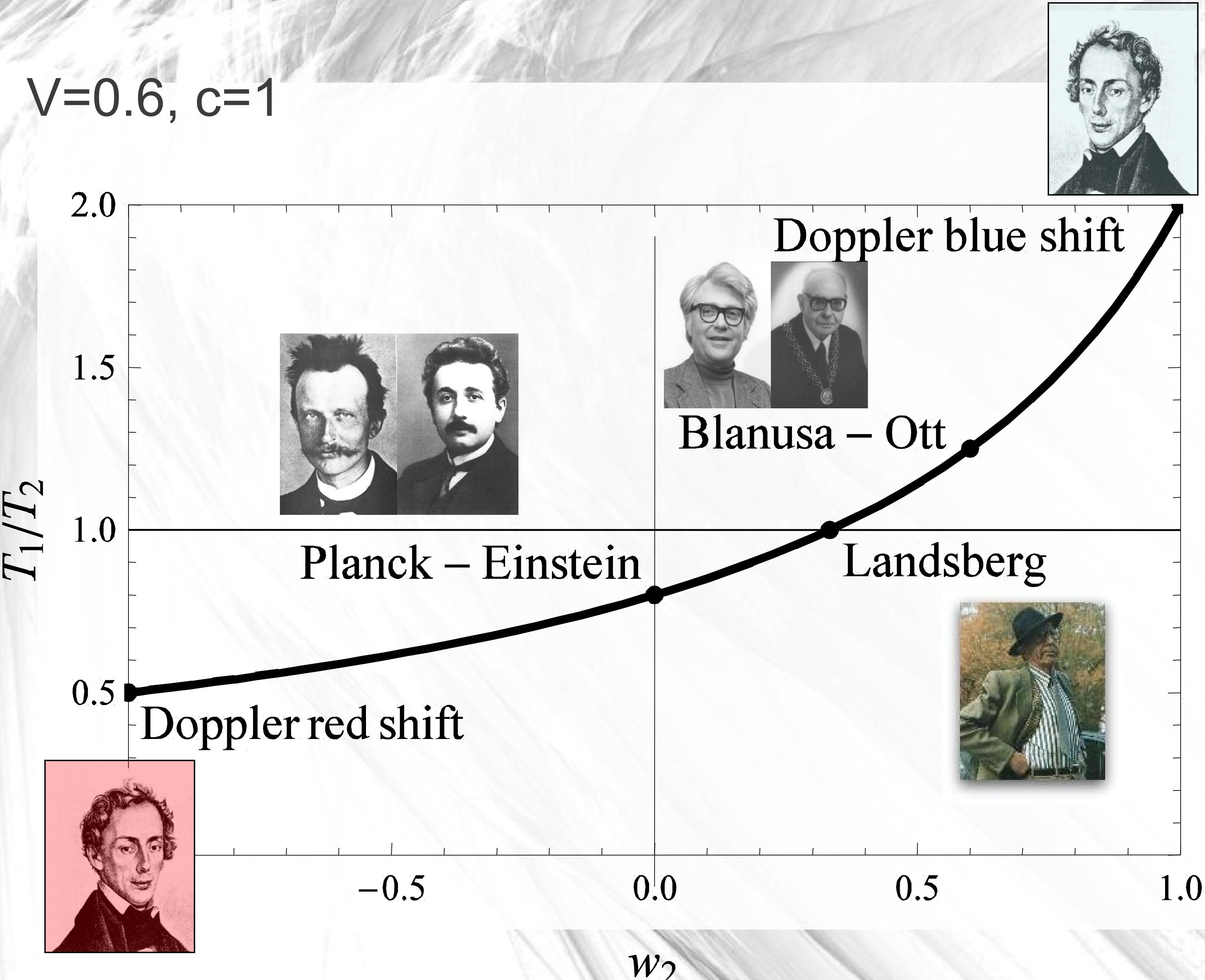
$$\frac{T}{T_0} = \frac{\sqrt{1-v^2}}{1+vw_0}$$



Special:

$w_0 = 0$	$T = T_0/\gamma$	*Planck-Einstein*
$w = 0$	$T = \gamma\, T_0$	*Ott*
$w_0 = 1,\ v > 0$	$T = T_0 \bullet$ *red*	*Doppler*
$w_0 = 1,\ v < 0$	$T = T_0 \bullet$ *blue*	*Doppler*
$w_0 + w = 0$	$T = T_0$	*Landsberg*

V=0.6, c=1


Doppler blue shift
Blanusa – Ott
Planck – Einstein
Landsberg
Doppler red shift
2.0
1.5
1.0
0.5
−0.5
0.0
0.5
1.0
T_1/T_2
w_2

Universal?

Simple, sound and exact formulation.

THE
Second Law

Thermodynamic equilibrium is stable.

This is a <u>discipline related</u> exact statement.

- There is entropy: statics.
- (It is concave: statics.)
- Entropy is not decreasing: dynamics.

? → Stability

Necessary condition: <u>generic stability</u> – linear stability of homogeneous thermodynamic equilibrium..

Objectivity is easier!

V.P. Thermodynamics of continua: the challenge of universality, in Proc. Of JETC13, arXiv: 1305.3582

Dissipative hydrodynamics

$$\partial_a N^a = \dot{n} + n\partial_a u^a + \partial_a j^a = 0,$$

$$u^a \partial_b T^{ab} = \dot{e} + (e+p)\partial_a u^a + \partial_a q^a + q^a \dot{u}_a - \Pi^{ab}\partial_b u_a = 0,$$

$$\Delta^a_c \partial_b T^{cb} = (e+p)\dot{u}^a + q^a \partial_b u^b + q^b \partial_b u^a + \Delta^a_c(\dot{q}^c + \partial_b \Pi^{cb}) = 0^a,$$

$$q^a = -\lambda \Delta^{ac}\left(\partial_c T + T\dot{u}_c + T\frac{\dot{q}_c}{e}\right),$$

$$\nu^a = -\varsigma \Delta^{ac}\partial_c \frac{\mu}{T},$$

$$\Pi^a_{\ a} = P^a_a - p = -\xi \partial_c u^c,$$

$$\Pi^a_{\ b} = -2\eta < \partial_b u^a > .$$

$< >$ symmetric, traceless, spacelike

⇒ Generic stability.

CONDITION: thermodynamic stability

(Ván: J. Stat. Mech. (2009), 02054)

Fluid families:

	Nonrelativistic	Relativistic
Local equilibrium (1st order)	Fourier+Navier-Stokes	~~Eckart~~ (1940), Tsumura-Kunihiro (2008)
Beyond local equilibrium (2nd order)	Cattaneo-Vernotte, generalized Navier-Stokes, rheology, etc…	Israel-Stewart (1969-72), Pavón-Jou-Casas-V. (1982), Liu-Müller-Ruggieri (1982), Geroch, Öttinger, Carter,… conformal (2007-08), our (2008), Betz et. al. (2009)

Eckart – Israel–Stewart – Pavón–Jou–Casas-Vázquez:

$$S^a(T^{ab}, N^a) = \left(s(e,\ n) - \frac{\beta_0}{2T}\Pi^2 - \frac{\beta_1}{2T} q_b q^b - \frac{\beta_2}{2T}\pi^{bc}\pi_{bc} \right) u^a + \frac{1}{T}\left(q^a + \alpha_0 \Pi q^a + \alpha_1 \pi^{ab} q_b \right)$$

(+ order estimates)