Rock rheology – time dependence of dilation and stress around a tunnel

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– Poynting-Thomson: not one of the many
– Unloading – rod
– Unloading – circular tunnel
– Conclusions

Rock rheology – the Poynting-Thomson modell

$$\rho\dot{\mathbf{v}} + \operatorname{div}\mathbf{F} = \rho\mathbf{f}$$

balance of momentum

$$f(\mathbf{D},\boldsymbol{\xi}) = \hat{f}(\mathbf{D}) + \boldsymbol{\xi} : \boldsymbol{\xi}$$

Helmholtz free energy

$$T\sigma_s = \left(\mathbf{F} + \underbrace{\rho\mathbf{D}\frac{\partial f}{\partial \mathbf{D}}}_{\mathbf{F}^E}\right) : \dot{\mathbf{D}}\mathbf{D}^{-1} - \boldsymbol{\xi} : \dot{\boldsymbol{\xi}} \geq 0.$$

Universal – independent of microstructure

$$\mathbf{F}^E = \mathbf{T} + \sigma_0 \mathbf{I} = 2G\mathbf{E} + 3K\varepsilon_0 \mathbf{I}$$

Ideal elastic

$$\mathbf{F} = \mathbf{T} + \sigma_0 \mathbf{I} \qquad (\mathrm{tr}\,\mathbf{T} = 0)$$

Small deformation

$$\mathbf{D} = \mathbf{E} + \varepsilon_0 \mathbf{I} \qquad (\mathrm{tr}\,\mathbf{E} = 0)$$

Isotropic

$$\Big(\mathbf{T} + \sigma_0 \mathbf{I} - (2G\mathbf{E} + 3K\varepsilon_0 \mathbf{I})\Big) : \dot{\mathbf{D}} - \boldsymbol{\xi} : \dot{\boldsymbol{\xi}} \geq 0.$$

Entropy production

$$\boxed{\begin{aligned} \mathbf{T} - 2G\mathbf{E} &= L_{11}\dot{\mathbf{E}} - L_{12}\boldsymbol{\xi}, \\ \dot{\boldsymbol{\xi}} &= L_{12}\dot{\mathbf{E}} - L_{22}\boldsymbol{\xi}, \\ \sigma_o - 3K\varepsilon_o &= L\dot{\varepsilon}_o, \end{aligned}}$$

$$
\begin{aligned}
\mathbf{T} - 2G\mathbf{E} &= L_{11}\dot{\mathbf{E}} - L_{12}\boldsymbol{\xi}, \\
\dot{\boldsymbol{\xi}} &= L_{12}\dot{\mathbf{E}} - L_{22}\boldsymbol{\xi}, \\
\sigma_o - 3K\varepsilon_o &= L\dot{\varepsilon}_o,
\end{aligned}
$$

$$
L > 0,\ L_{11} > 0,\ L_{22} > 0,
$$

$$
L_{11}L_{22} - L_{12}^2 > 0
$$

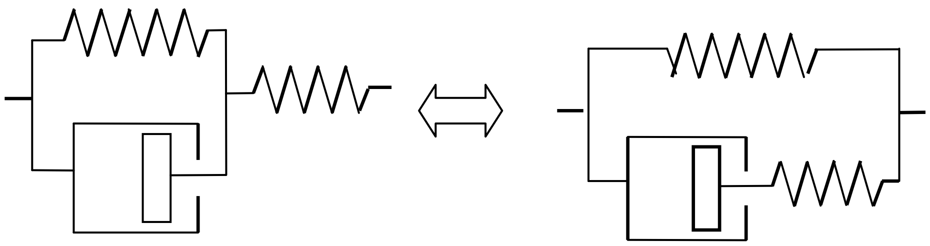
$$
\tau = L_{22}^{-1}, \quad \tau_d = L_{11}L_{22}^{-1},
$$

$$
2\eta = L_{11} - L_{12}^2 L_{22}^{-1} + 2GL_{22}^{-1},
$$

$$
\tau > 0, \quad \tau_d > 0,
$$

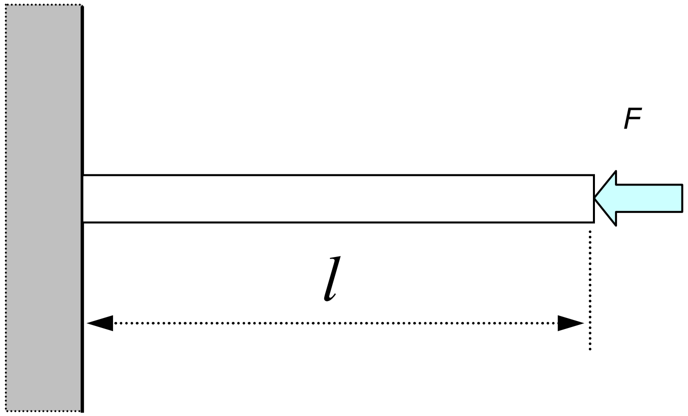
$$
\eta > G\tau,
$$

$$
\boxed{
\begin{gathered}
\mathbf{T} + \tau\dot{\mathbf{T}} = 2G\mathbf{E} + 2\eta\dot{\mathbf{E}} + \cancel{\tau_d\ddot{\mathbf{E}}}, \\
\sigma_o = 3K\varepsilon_o + \cancel{L\dot{\varepsilon}_o}.
\end{gathered}
}
$$



Poynting-Thomson =
Hooke+Kelvin-Voigt

Opening of 1D tunnel – released rod



$$\mathbf{T} + \tau\,\dot{\mathbf{T}} = 2G\mathbf{E} + 2\eta\,\dot{\mathbf{E}},$$
$$\sigma_o = 3K\varepsilon_o.$$

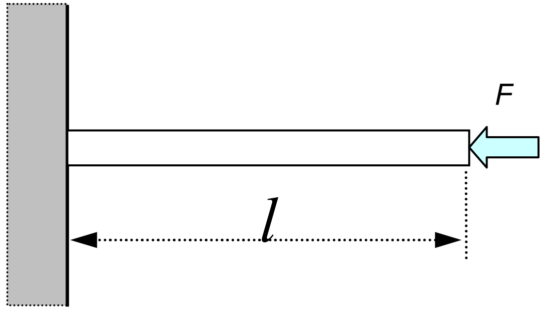
Three stages:

primary: equilibrium is linear elastic

Opening: <u>new initial condition</u>

secondary: processes after the opening

A) Released rod – primary stage



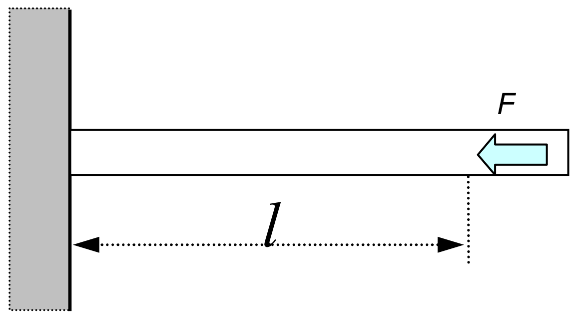
$$\boxed{\begin{aligned}\mathbf{T}_A &= 2G\mathbf{E}_A,\\ \sigma_{oA} &= 3K\varepsilon_{oA}.\end{aligned}}$$

$p=F/A$

$$\mathbf{D}_A = \mathbf{E}_A + \varepsilon_{oA}\mathbf{I} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} = \frac{p}{3}\begin{pmatrix} \frac{1}{3K}+\frac{1}{G} & 0 & 0 \\ 0 & \frac{1}{3K}-\frac{1}{2G} & 0 \\ 0 & 0 & \frac{1}{3K}-\frac{1}{2G} \end{pmatrix}$$

Elastic equilibrium

B) Released rod – opening



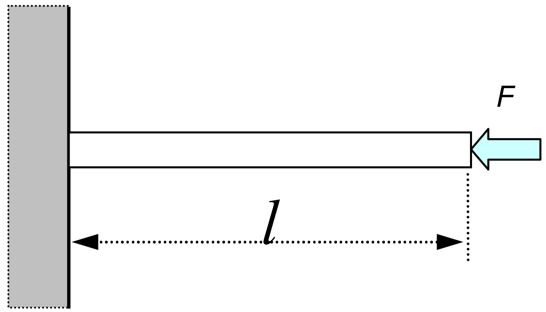
$$\boxed{\tau\,\dot{\mathbf{T}}_B = 2\eta\,\dot{\mathbf{E}}_B}$$

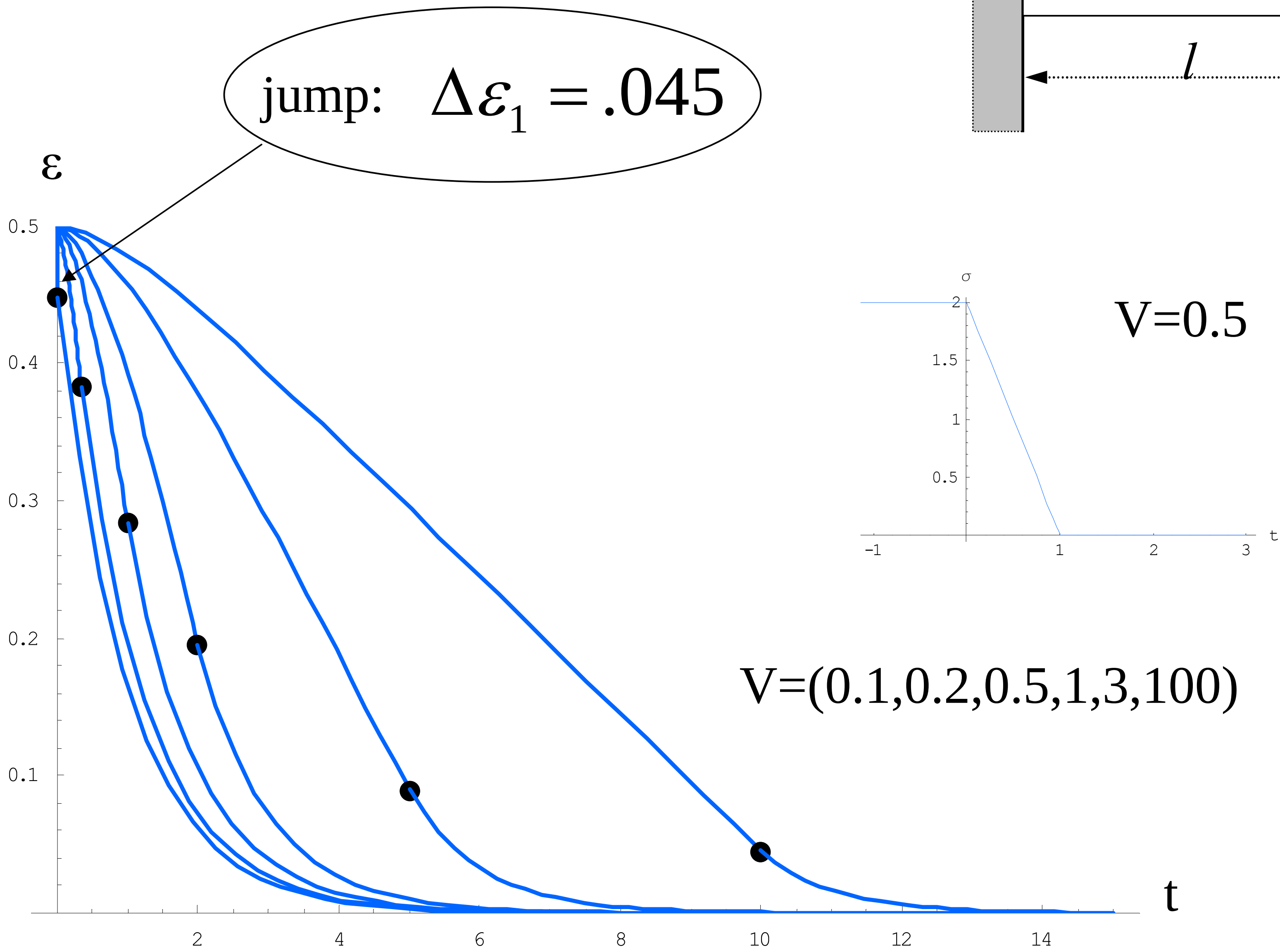
Fast, "quasielastic"

$p=F/A$

Finally:

$$\mathbf{D}_B = \frac{\mathbf{T}_A}{2G}\left(1-\frac{\tau G}{\eta}\right) = \frac{p}{6G}\left(1-\frac{\tau G}{\eta}\right)\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

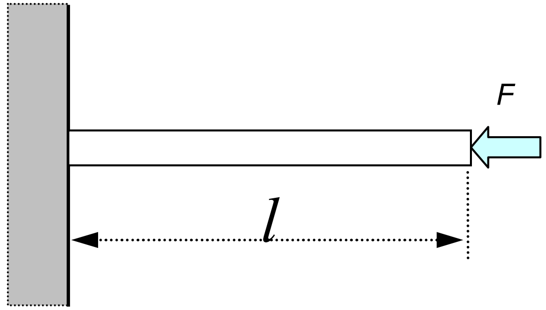




V=0.5

V=(0.1,0.2,0.5,1,3,100)

B) Released rod – secondary



$$\boxed{\begin{gathered}\mathbf{T} + \tau\,\dot{\mathbf{T}} = 2G\mathbf{E} + 2\eta\,\dot{\mathbf{E}},\\ \sigma_o = 3K\varepsilon_o.\end{gathered}}$$

$\mathbf{D}(t = 0) = \mathbf{D}_B$ Initial condition

Finally:

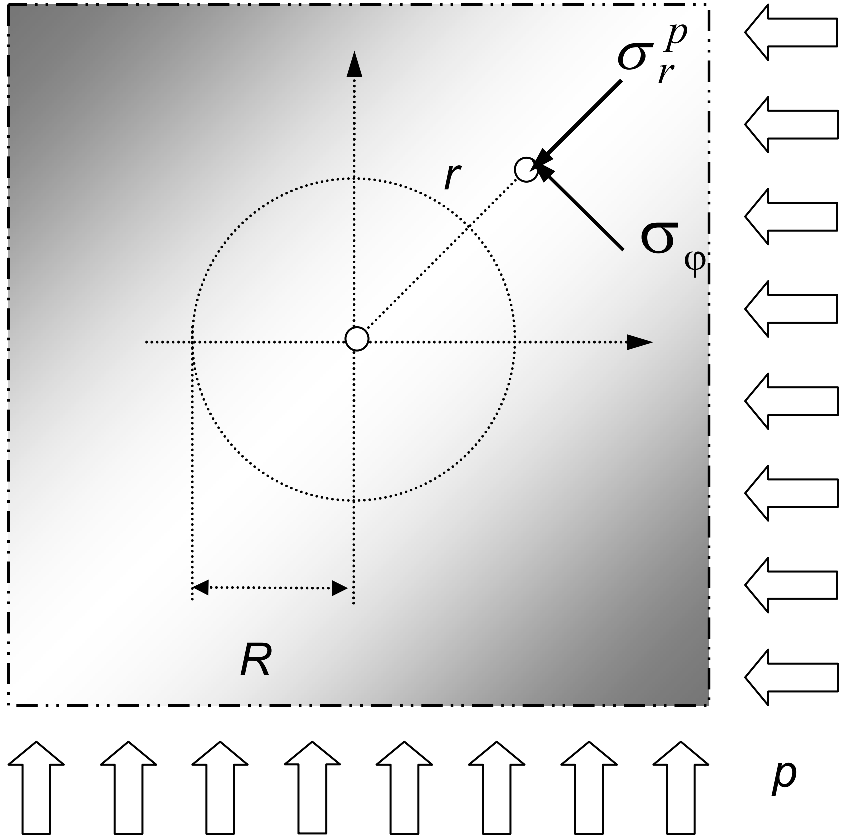
$$\mathbf{D}(t) = \mathbf{D}_B e^{-\frac{G}{\eta}t} = \frac{p}{6G}\left(1 - \frac{\tau G}{\eta}\right) e^{-\frac{G}{\eta}t} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Opening of a circular tunnel

$$\mathbf{T} + \tau\,\dot{\mathbf{T}} = 2G\mathbf{E} + 2\eta\,\dot{\mathbf{E}},$$

$$\sigma_o = 3K\varepsilon_o,$$

$$div(\mathbf{T} + \sigma_o\mathbf{I}) = 0.$$



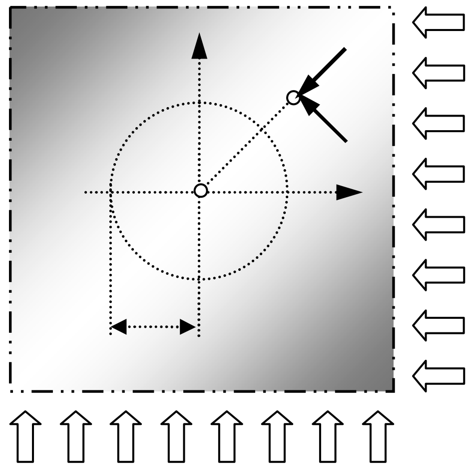
Three stages:

primary: equilibrium is linear elastic

opening: <u>new initial condition</u>

secondary: processes after the opening

Cylindrical symmetry



$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\phi}{r} = 0$$

$$\tau\dot{\sigma}_r + \sigma_r = (K + 4G/3)\varepsilon_r + (K - 2G/3)\varepsilon_\phi + (K\tau + 4\eta/3)\dot{\varepsilon}_r + (K\tau - 2\eta/3)\dot{\varepsilon}_\phi,$$

$$\tau(\dot{\sigma}_r - \dot{\sigma}_\phi) + \sigma_r - \sigma_\phi = 2G(\varepsilon_r - \varepsilon_\phi) + 2\eta(\dot{\varepsilon}_r - \dot{\varepsilon}_\phi).$$

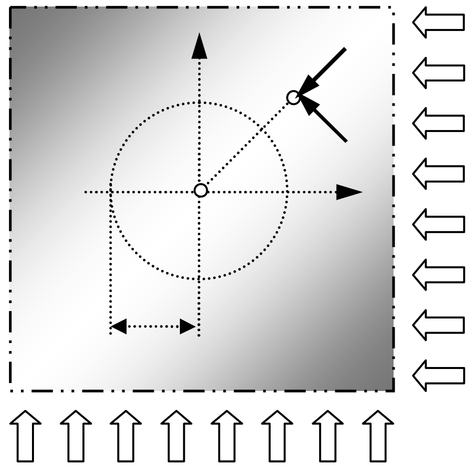
$$\left(1 + \alpha_o \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r}\right) = 0$$

$$u(r,t) = C_1(t)r + \frac{C_2(t)}{r}.$$

Initial conditions:

$$\varepsilon_{rB} = \frac{p}{2}\left(\frac{3}{G+3K} - \frac{\tau}{\eta}\left(\frac{R}{r}\right)^2 \right) = \varepsilon_A - \frac{p\tau}{2\eta}\left(\frac{R}{r}\right)^2,$$

$$\varepsilon_{\phi B} = \frac{p}{2}\left(\frac{3}{G+3K} + \frac{\tau}{\eta}\left(\frac{R}{r}\right)^2 \right) = \varepsilon_A + \frac{p\tau}{2\eta}\left(\frac{R}{r}\right)^2.$$



$$u(r,t) = C_1(t)r + \frac{C_2(t)}{r}.$$

Asymptotic condition:

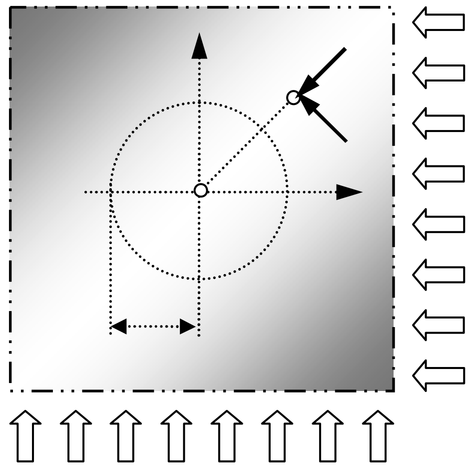
$$u(r, t=\infty) = \varepsilon_A r + \frac{p}{2G}\frac{R^2}{r},$$

$$\varepsilon_r(r, t=\infty) = \varepsilon_A - \frac{p}{2G}\left(\frac{R}{r}\right)^2, \quad \varepsilon_\phi(r, t=\infty) = \varepsilon_A + \frac{p}{2G}\left(\frac{R}{r}\right)^2,$$

$$\sigma_r(r, t=\infty) = p\left(1 - \left(\frac{R}{r}\right)^2\right), \quad \sigma_\phi(r, t=\infty) = p\left(1 + \left(\frac{R}{r}\right)^2\right).$$

Time dependent radial displacement:

$$u(r,t)=\varepsilon_A r+\frac{p}{2G}(1-e^{-\frac{t}{\alpha}})\frac{R^2}{r}+\frac{p\tau}{2\eta}e^{-\frac{t}{\beta}}\frac{R^2}{r}\ .$$



$$u(r,t)=C_1(t)r+\frac{C_2(t)}{r}.$$

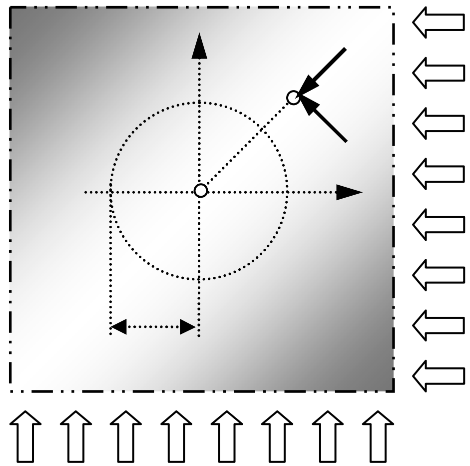
Time dependent deformation fields:

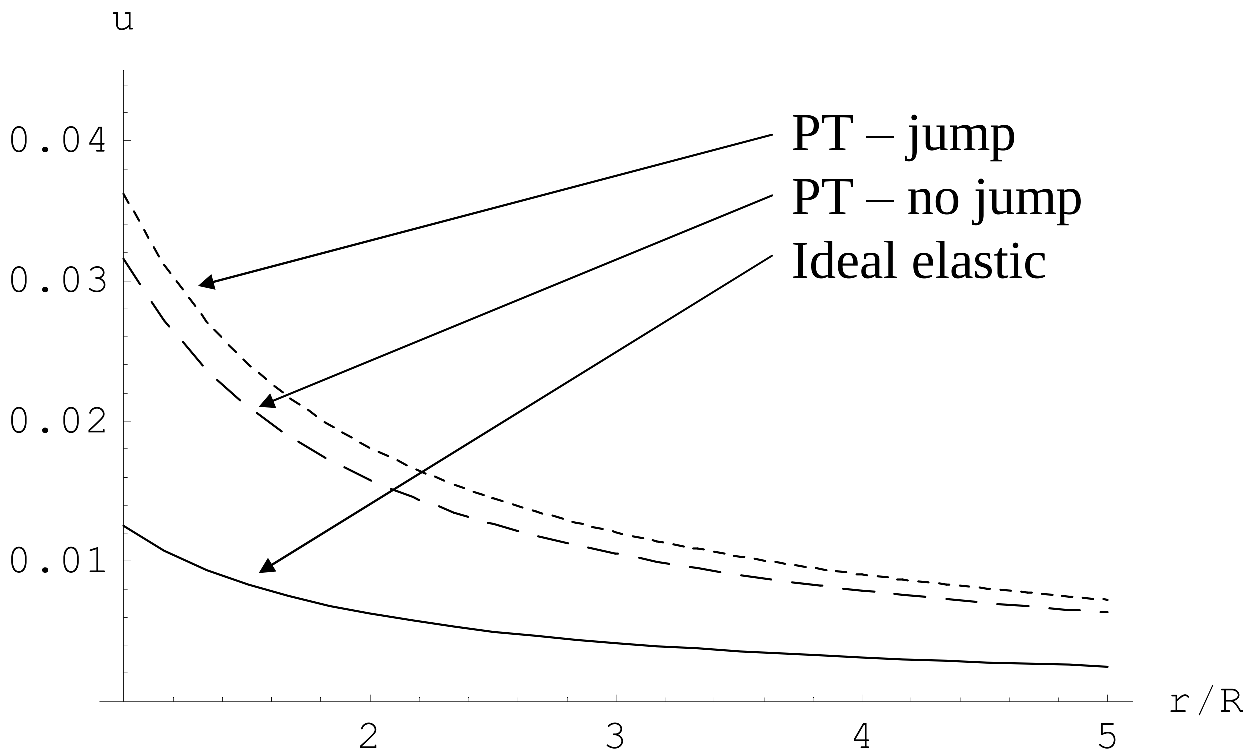
$$\varepsilon_r(r,t)=\varepsilon_A-\frac{p}{2G}(1-e^{-\frac{t}{\alpha}})\left(\frac{R}{r}\right)^2-\frac{p\tau}{2\eta}e^{-\frac{t}{\beta}}\left(\frac{R}{r}\right)^2,$$

$$\varepsilon_\phi(r,t)=\varepsilon_A+\frac{p}{2G}(1-e^{-\frac{t}{\alpha}})\left(\frac{R}{r}\right)^2+\frac{p\tau}{2\eta}e^{-\frac{t}{\beta}}\left(\frac{R}{r}\right)^2.$$

Radial displacement :

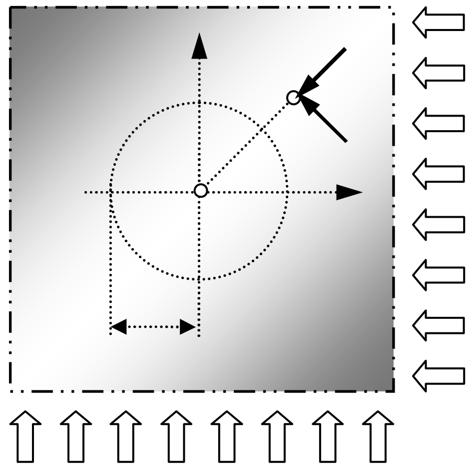
$$u(r,t) = \varepsilon_A r + \frac{p}{2G}(1 - e^{-\frac{t}{\alpha}})\frac{R^2}{r} + \frac{p\tau}{2\eta}e^{-\frac{t}{\beta}}\frac{R^2}{r} \; .$$





G=10000MPa,
K=30000MPa,
p=1,
R=1m,
η=400000 MPah
t=10h,
α=9h
β =11h

Time dependent stress field:

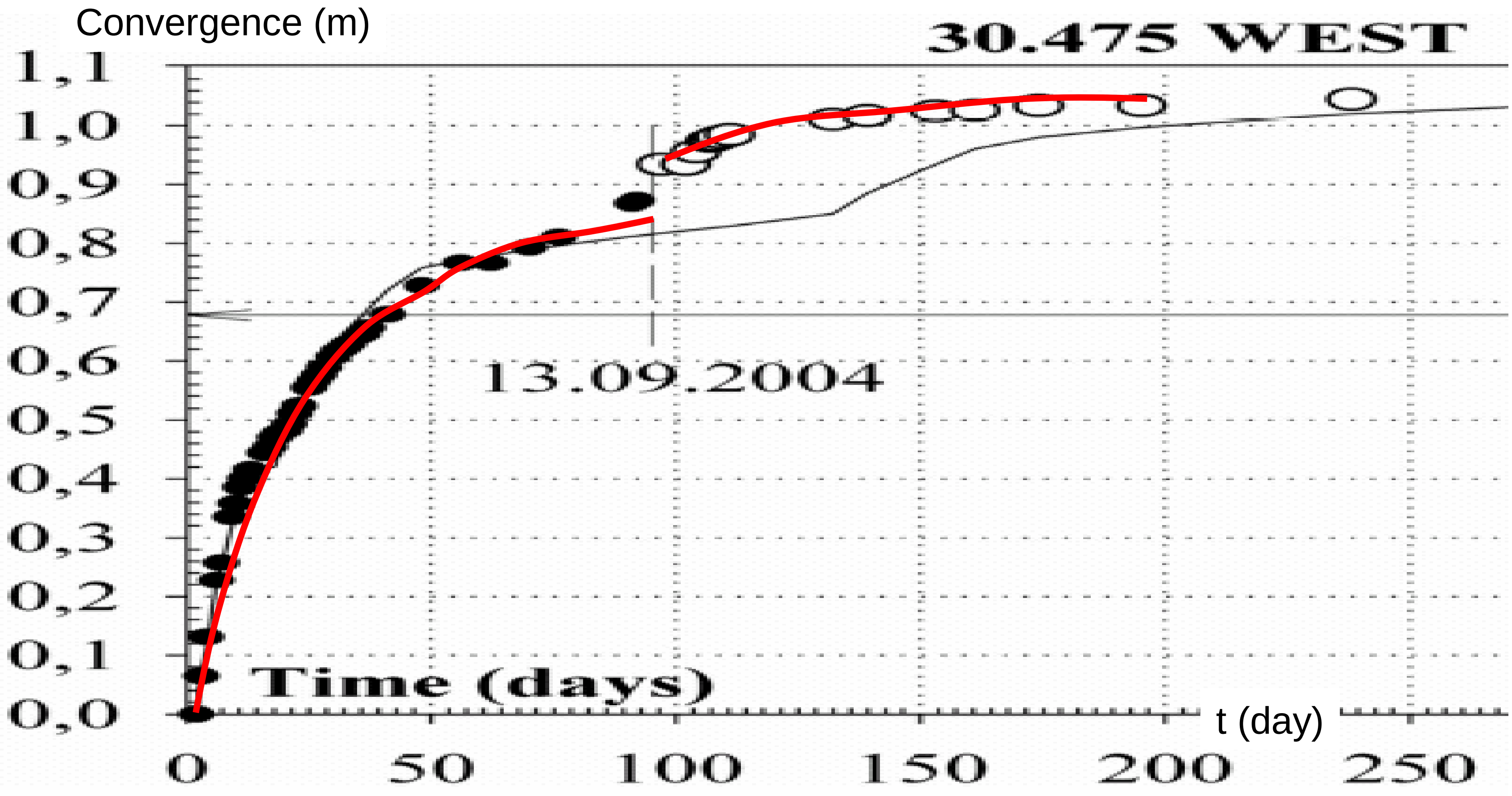


$$\sigma_r(r,t) = p\left(1-\left(\frac{R}{r}\right)^2\right) + \sigma_t(r,t),$$

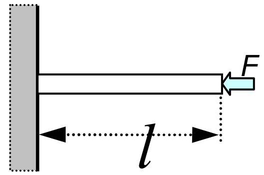
$$\sigma_\varphi(r,t) = p\left(1+\left(\frac{R}{r}\right)^2\right) - \sigma_t(r,t),$$

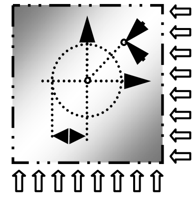
$$\sigma_r(r,t) = p\left(\frac{R}{r}\right)^2\left(\frac{\tau(G\beta-\eta)}{\eta(\tau-\beta)}e^{-\frac{t}{\beta}} - \frac{(G\alpha-\eta)}{G(\tau-\alpha)}e^{-\frac{t}{\alpha}} + (G\tau-\eta)\left(\frac{1}{G(\tau-\alpha)} - \frac{\beta}{\eta(\tau-\beta)}\right)e^{-\frac{t}{\tau}}\right).$$

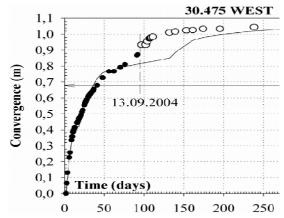
Remark to the presentation of
F. Sandrone, J.P. Dudt, V. Laiouse and F. Dexcoeudres



Conclusions







– Poynting-Thomson is
  not just one of them
– Separate timescales: initial jump
– Analytical solution can be generalized:
  elliptical, lining, etc.
– Measurements of the material parameters
– Numerics
  mesh: gradient generalization (thermodynamics!)
– Practice:
  stops and starts in tunnel excavations
  swelling?

Thank you for your attention!