Thermodynamics of continua: the challenge of universality

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1. What is non-equilibrium thermodynamics?
2. law or Law?
3. Research methodology

Non-equilibrium thermodynamics:

Construction of constitutive functions

- classical irreversible: fluxes and forces, $q^i = -\lambda \partial^i T$

Construction of equation of motion for internal variables

- rational, extended, etc... $\tau \dot{q}^i + q^i = -\lambda \partial^i T$

General, therefore it is <u>universal</u>.

Non-dissipative is a well-defined limit: *ideal* substances.

Constructive: not a checking method of empirical formulas.

Independent of mechanics (among others).

Other general principles: important role (e.g. objectivity).

Failure or challenge?

We need a proper understanding and formulation.

How far can it be generalized?

What is the role of the second law?

How is it related to the entropy principle?

Are there benchmark problems (areas)?

- Weakly nonlocal: phase field, Ginzburg-Landau, etc. → panel F.
- Relativistic.

How to verify?

Uniform:
Hierarchical and multiscale

Nature

Uniform:
Hierarchical and multiscale

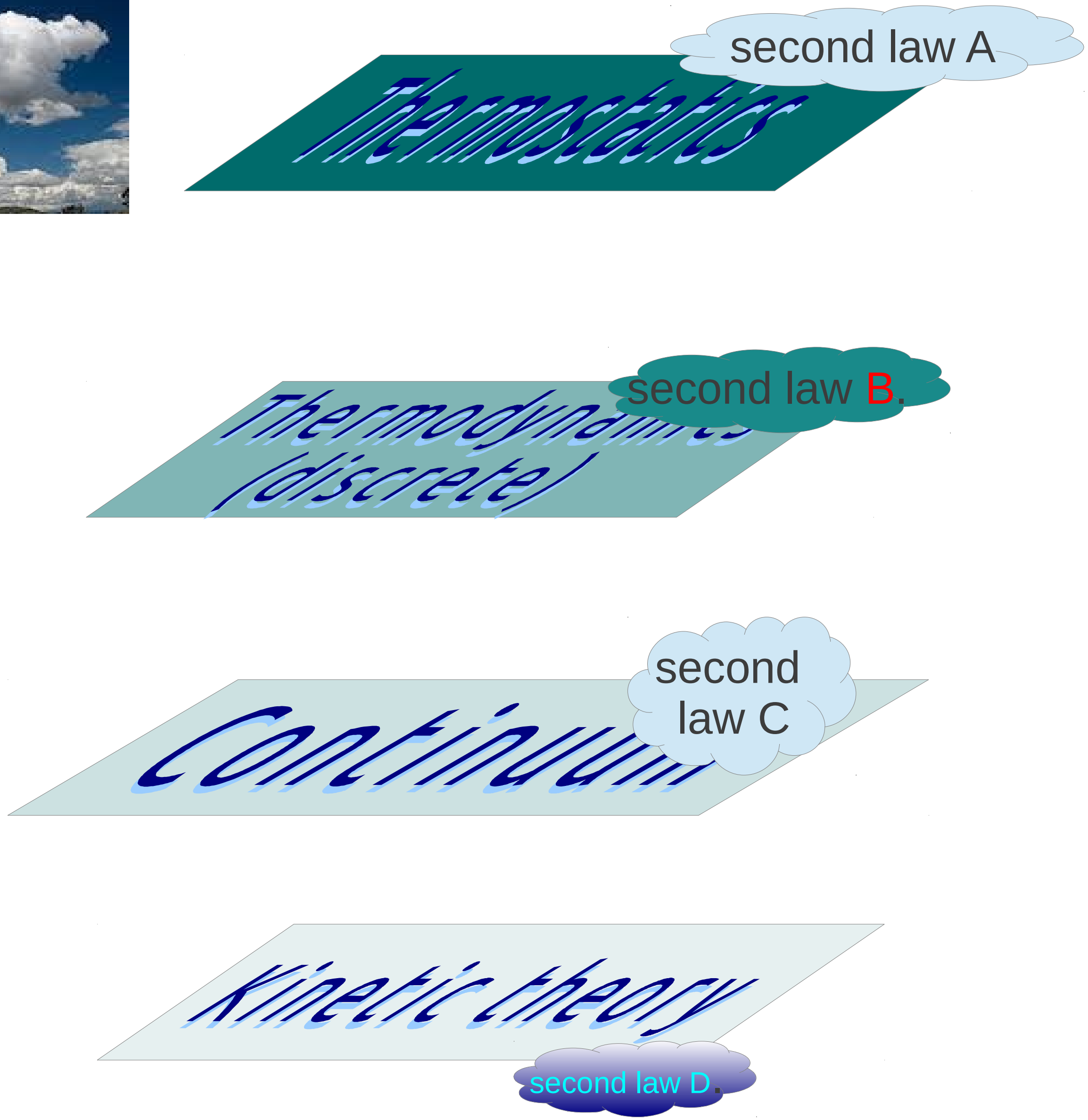
Thermostatics

Thermodynamics (discrete)

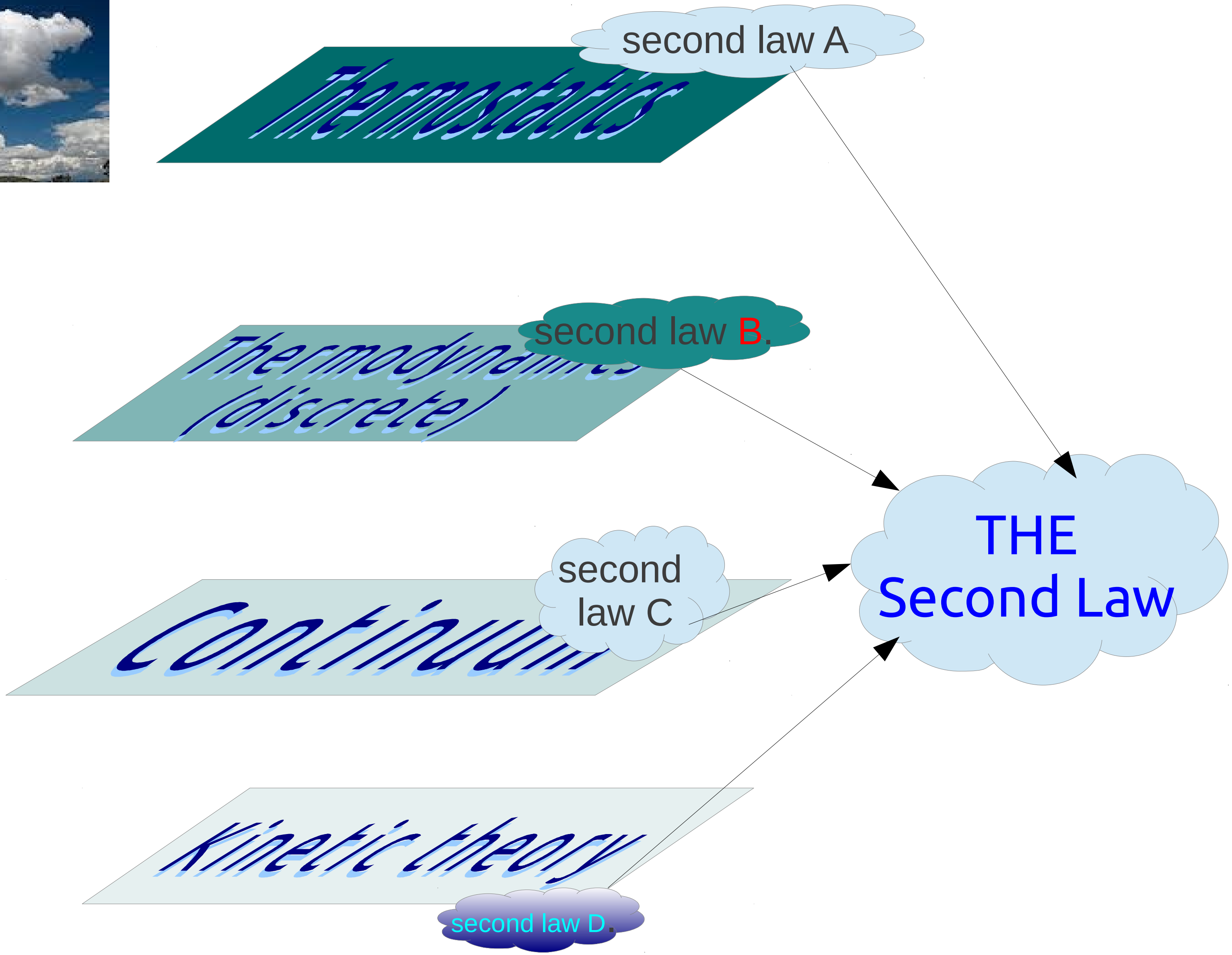
Continuum

Kinetic theory

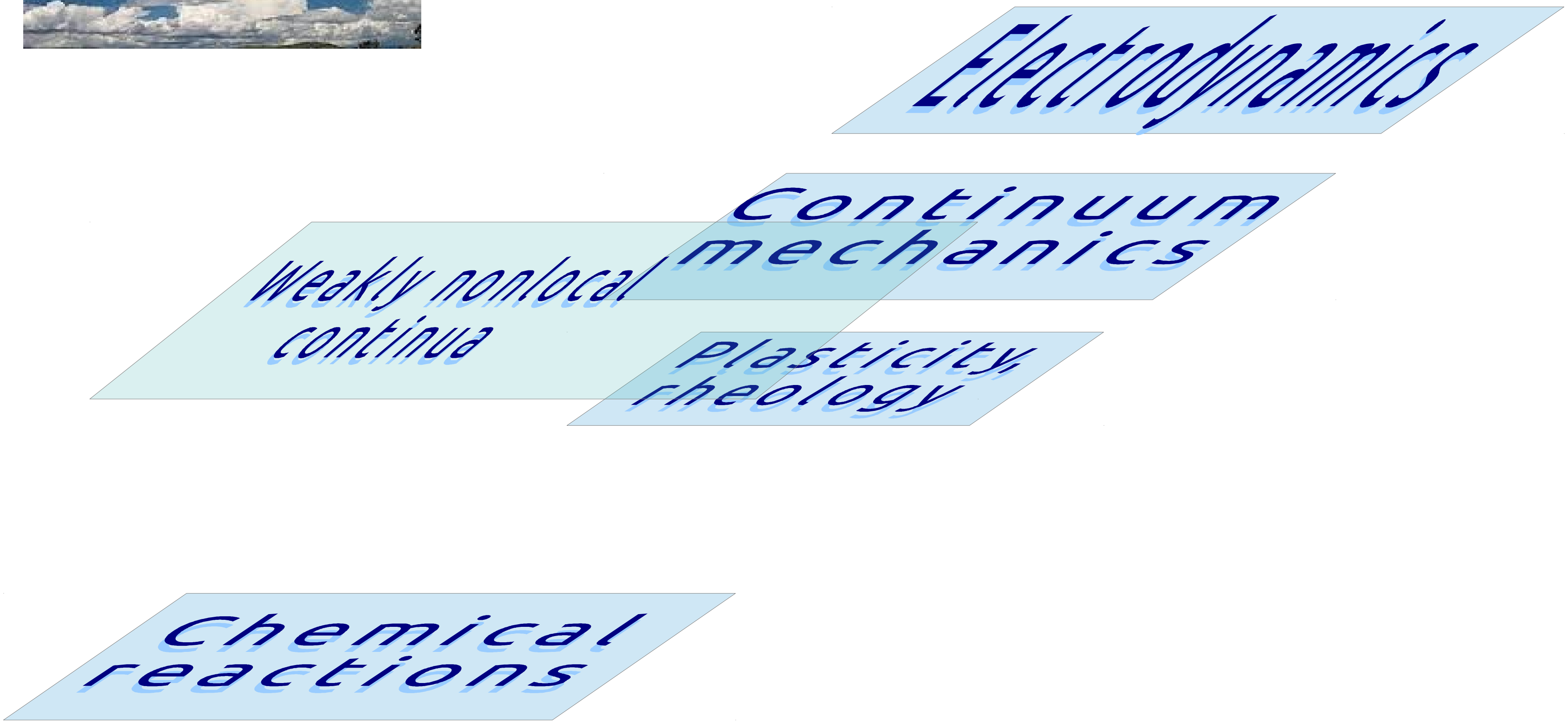
Uniform:
Hierarchical and multiscale



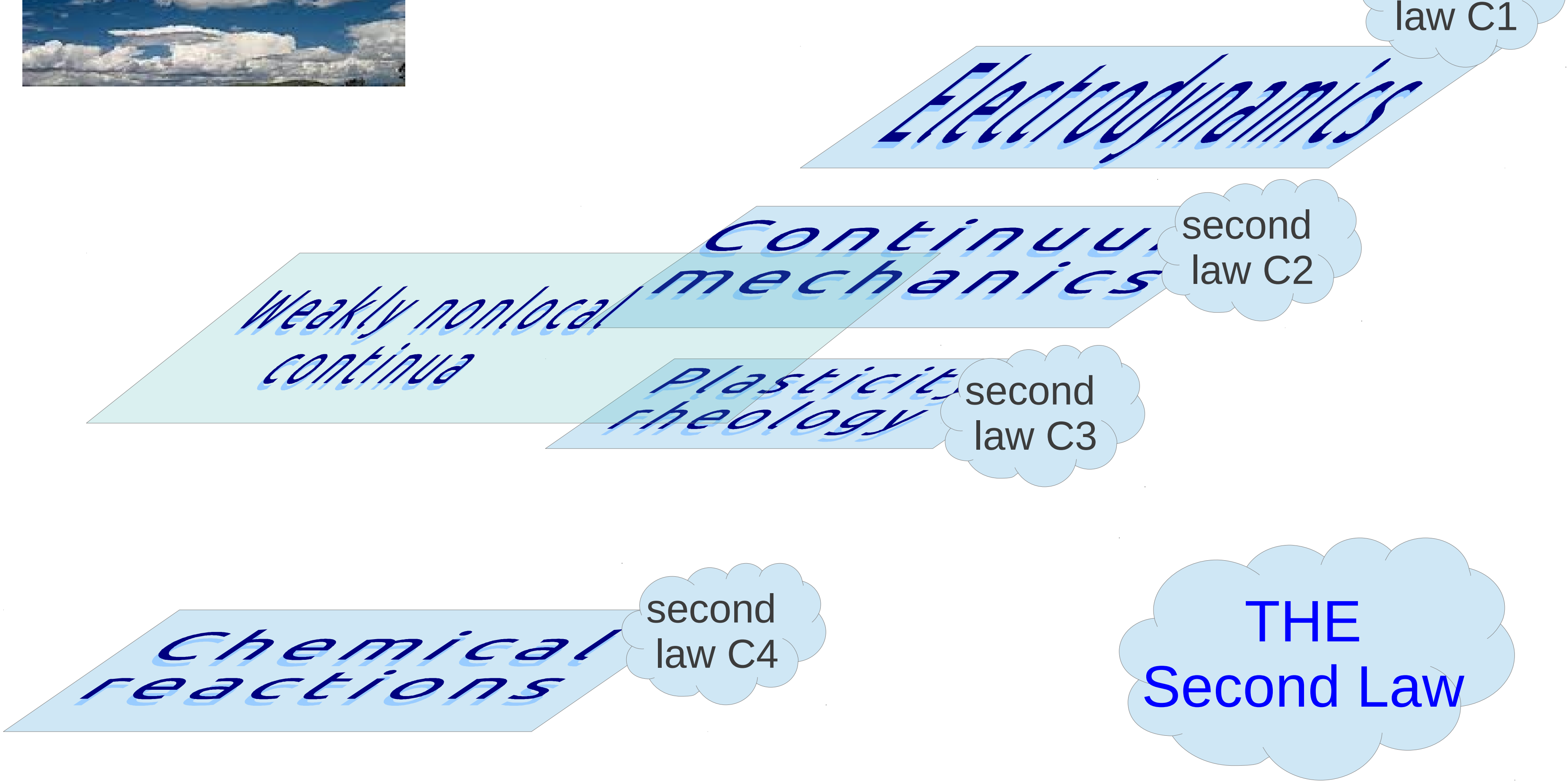
Uniform:
Hierarchical and multiscale



Overdisciplinary



Overdisciplinary



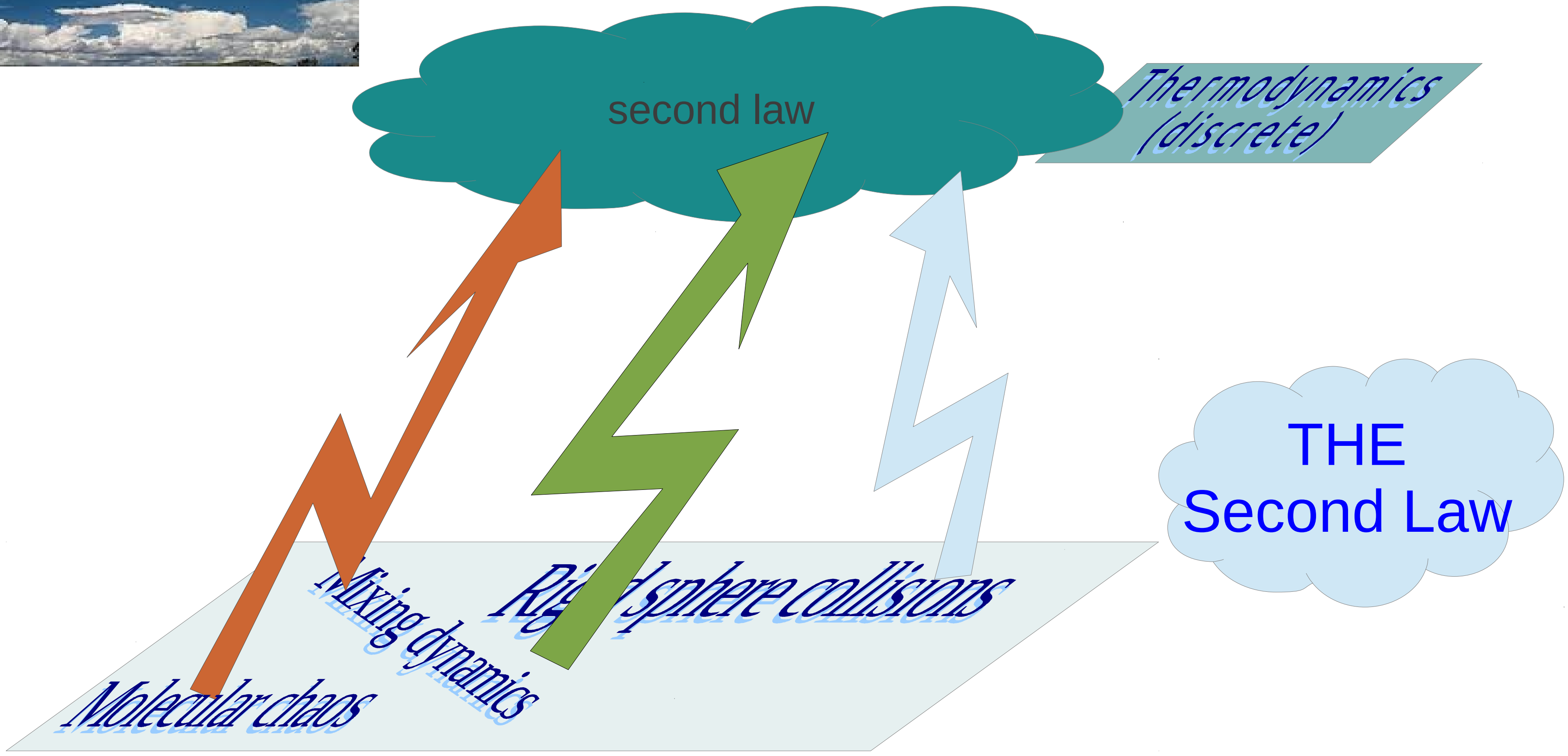
second law

or

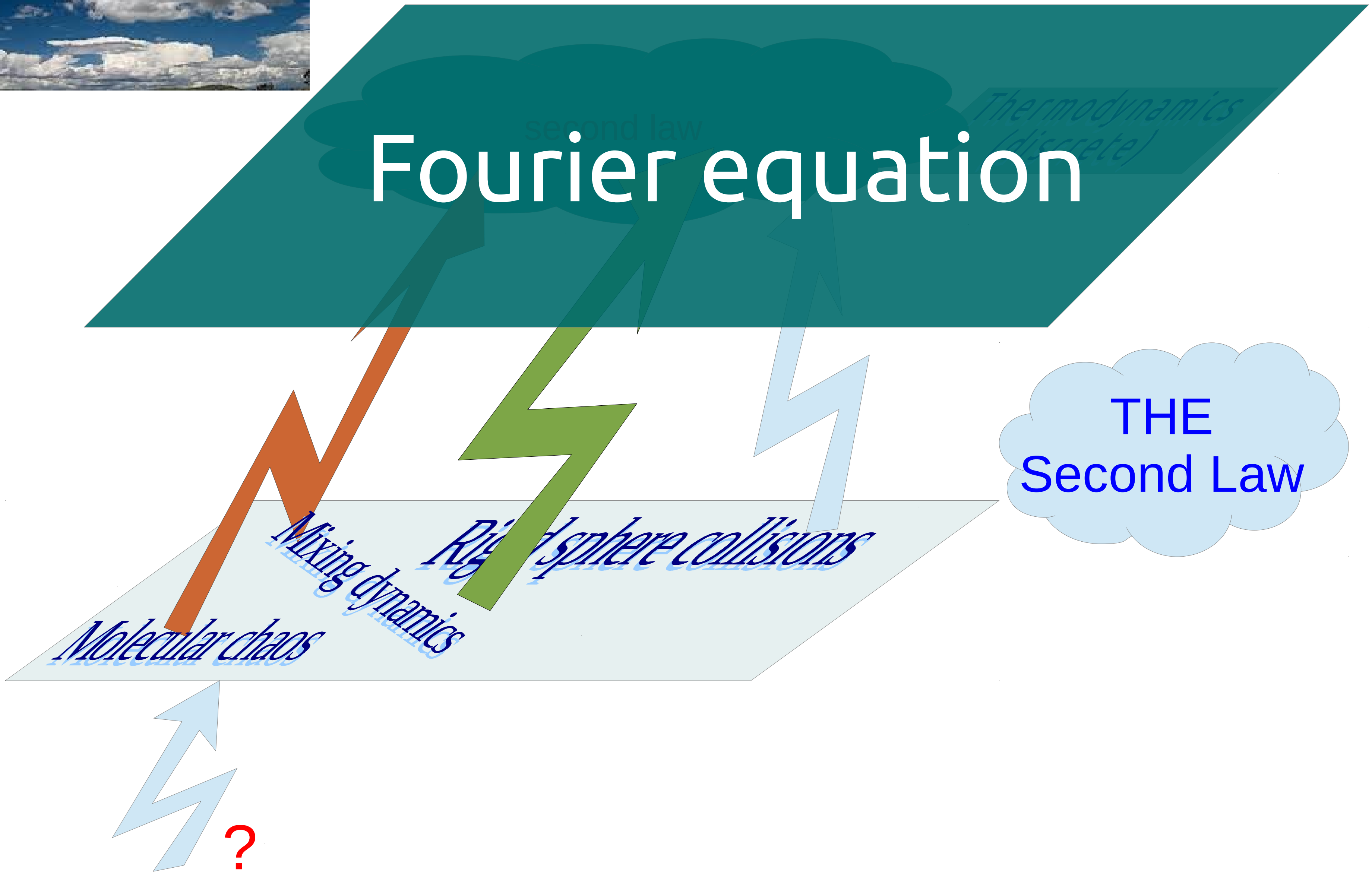
Second Law

of what?

Mechanism independent

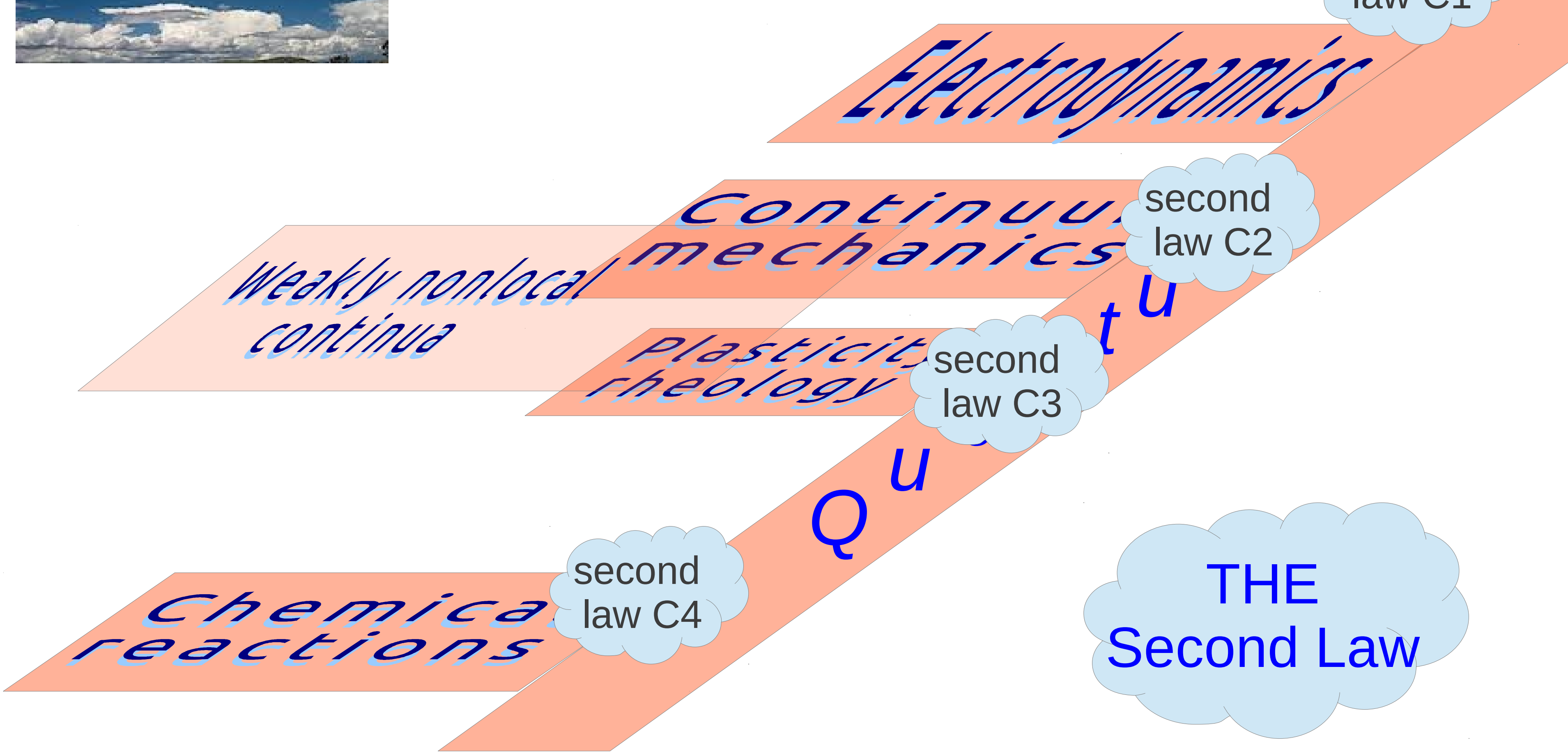


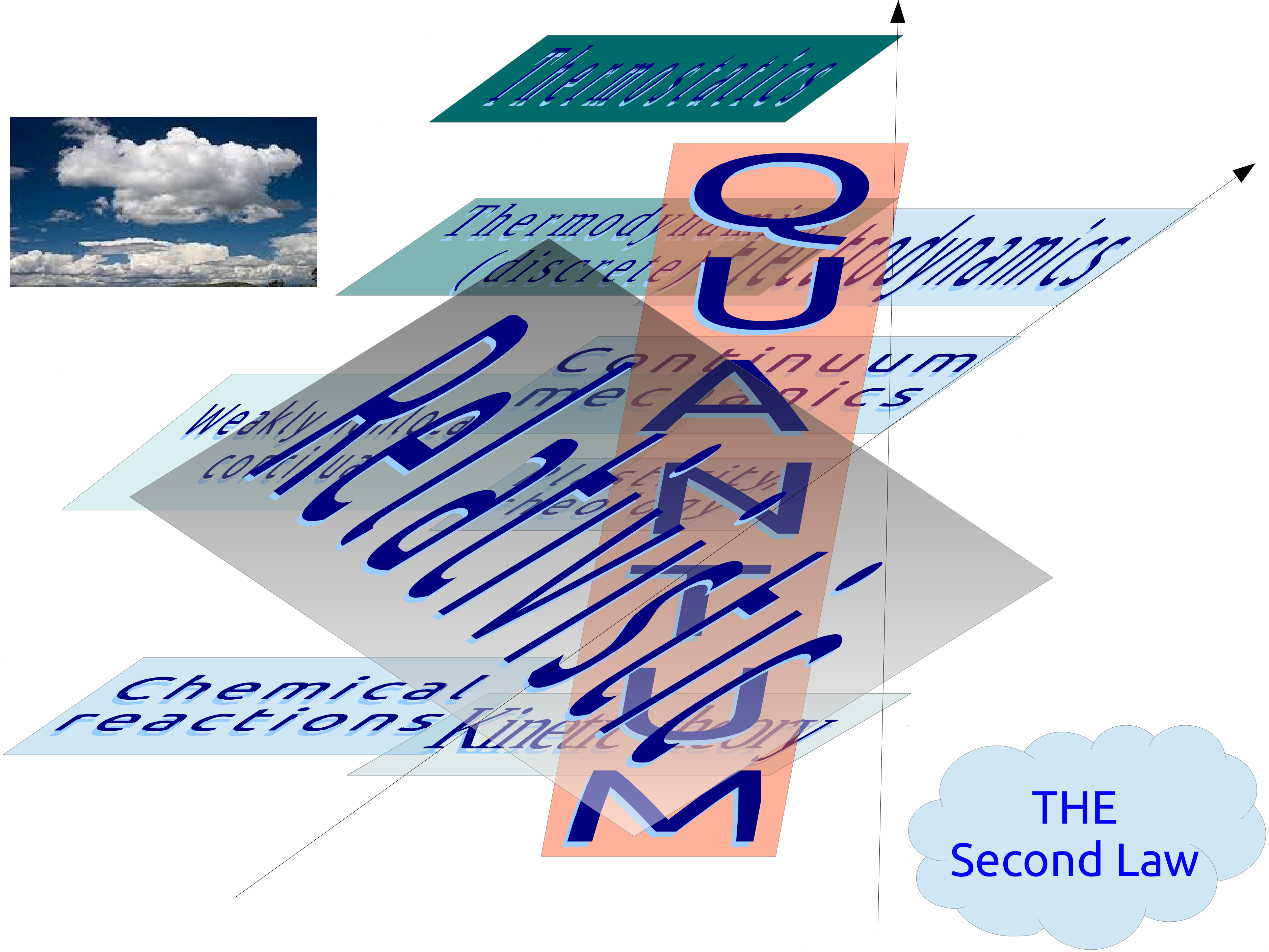
Mechanism independent



More levels behind

More levels behind




Thermostatics
Thermodynamics (discrete)
Continuum mechanics
Relativistic
QUANTUM
Chemical reactions
Kinetic theory
THE
Second Law


Thermostatics
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Universal?

THE
Second Law

A simple and sound formulation.

Thermodynamic equilibrium is stable.

This is a <u>discipline related</u> exact statement.
Dynamic stability, open systems, total entropy, Lyapunov function(al).

Parts of the second law:

- There is entropy: statics.
- It is concave: statics.
- Entropy is not decreasing: dynamics.

? → Stability

Eckart theory: relativistic Navier-Stokes-Fourier

Entropy balance – conditional inequality:

$$\partial_a S^a = \dot{s}(e,n) + s\,\partial_a u^a + \partial_a J^a \geq 0$$

$$ds + \alpha\, dn = \beta\, de$$

$$J^\mu = \beta q^\mu - \alpha j^\mu$$

$$u^\mu \partial_\nu T^{\mu\nu} = \dot{e} + e\ \partial_\mu u^\mu + \partial_\mu q^\mu + u_\mu \dot{q}^\mu + u_\mu \partial_\nu P^{\mu\nu} = 0$$

$$\partial_\mu N^\mu = \dot{n} + n\ \partial_\mu u^\mu + \partial_\mu j^\mu$$

$$\sigma = -j^\mu \partial_\mu \alpha + \beta \Pi^{\mu\nu} \partial_\nu u_\mu + q^\mu \left(\partial_\mu \beta - \underbrace{\boxed{\beta\, \dot{u}_\mu}}_{\text{Eckart term}}\right) \geq 0$$

Unstable (Hiscock-Lindblom 1980-85).
Momentum balance must also be a constraint.

Universality with surprises

Quantum continuum?

Schrödinger-Madelung fluid. Weakly nonlocal in density:

$$\dot{\rho}+\rho\nabla\cdot v=0$$

$$\rho\dot{v}-\nabla\cdot P(\rho,\nabla\rho,\nabla^2\rho)=0$$

$$P(\rho,\nabla\rho,\nabla^2\rho)=\frac{\hbar^2}{8m^2}\left(\Delta\rho I+\nabla^2\rho-\frac{2\nabla\rho\circ\nabla\rho}{\rho}\right)$$

Let us wonder about this fact.

1) No interpretation.
2) Quantum field theories (R. Jackiw).

Summary

Entropy principle for continuum physics:

- Entropy inequality is conditional.

  Balances are constraints. Coleman-Noll or Liu procedures.

- The entropy current density is constitutive.

  The entropy four-vector is a constitutive quantity. Objectivity!

- Momentum is a state variable.

  Total energy, internal energy.

- Space and time derivatives of the basic state may appear in the constitutive state space.

  Evolution and dynamics. → Ruggeri/Trovato/Hütter

- Stability is a benchmark.

  Linear stability is necessary.

Outlook

Let us recognize the interconnections.

Schools are isolation, isolation is ignorance.

Subfields are restrictive.

Thermodynamics is autonomous.

(Panel discussions are great.)


Thank you for
your attention!

Thermodynamics

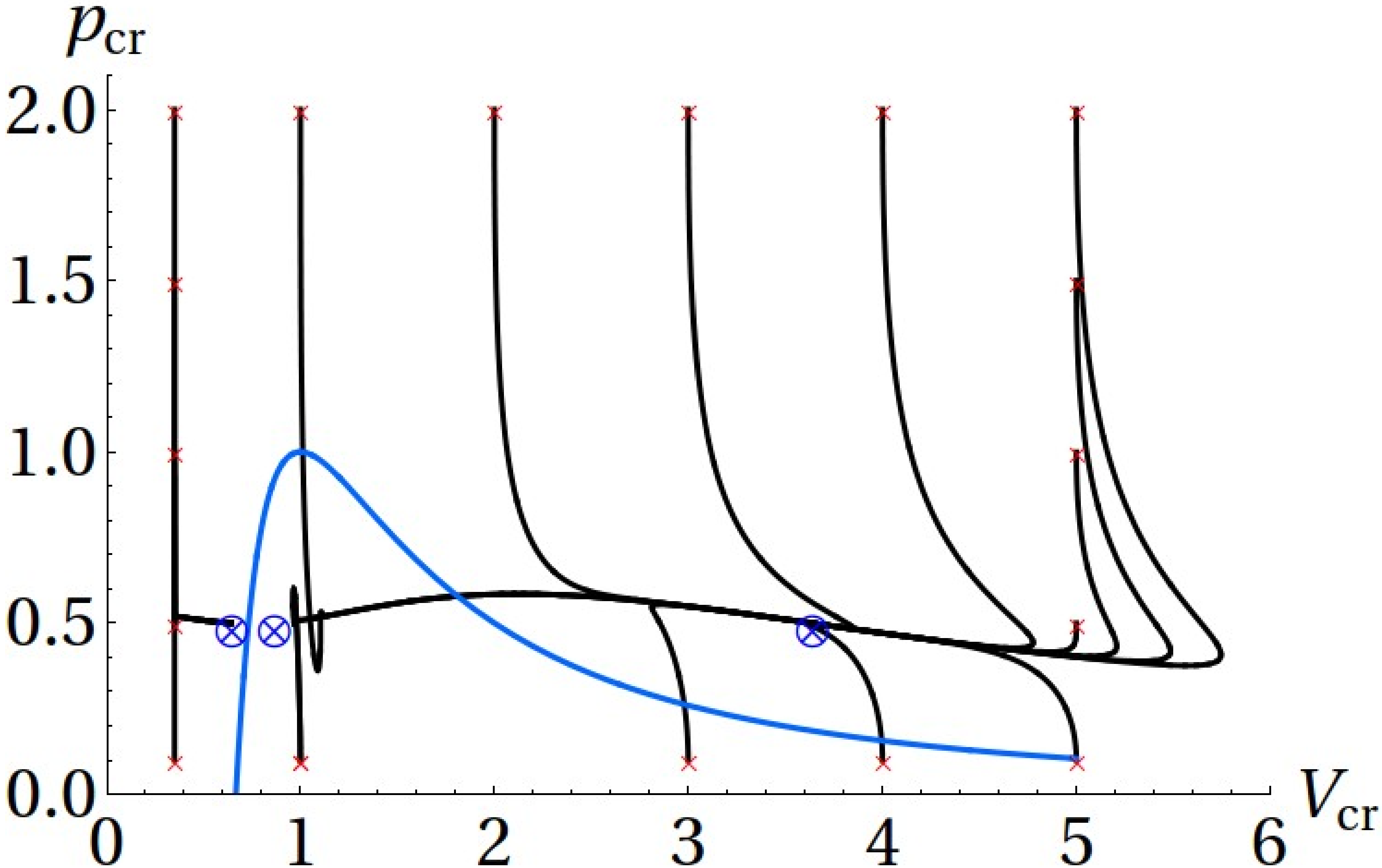
is

~~amazing~~.

universal?

Verification?

1) Ordinary thermodynamics –
Time dependent, internal variables or else. Processes of a Van der Waals gas.



Verification? → *Challenge!*

Relativistic thermostatics?

Momentum is a state variable.

Quantum continuum?

Only density? What kind of weak nonlocality is quantum like?

Violation of the second law of violation of the Second Law?

Perpetuum mobiles of different kind...

Stability of gradient equations?

Failure of rational mechanics...

Stability of dissipative relativistic fluids.

Is that universal??

Heat conduction:

$$\rho\dot{e} + \partial^i q^i = 0$$

$$\boxed{q^i = -\lambda\partial^i T,}$$ Fourier (1822)

$$\tau\dot{q}^i + q^i = -\lambda\partial^i T,$$ Cattaneo (1948),

(Vernotte (1958), Maxwell ~1880)

$$\tau\dot{q}^i + q^i = -\lambda\partial^i T + a_1\partial^{ij} q^j + a_2\partial^{jj} q^i,$$ Guyer and Krumhansl (1966)

$$\tau\dot{q}^i + q^i = -\lambda\partial^i T + l\partial^i \dot{T},$$ Jeffreys type

(Joseph and Preziosi, 1989))

$$\dot{q}^i = -\lambda\partial^i T + +a_2\partial^{jj} q^i.$$ Green-Naghdi type (1991)

$$\left(\rho c\ddot{\alpha} = k_1\partial^{ii}\alpha + k_2\partial^{ii}\dot{\alpha}, \qquad T = \dot{\alpha}\right)$$

there are more…

$$c_1 \dot{T}_1 - \partial^i \left(\lambda \partial^i T_1 \right) = -\beta \left(T_1 - T_2 \right)$$

$$c_2 \dot{T}_2 = -\beta \left(T_2 - T_1 \right)$$

$$\frac{c_1 c_2}{c_1 + c_2} \frac{1}{\beta} \ddot{T}_1 + \dot{T}_1 = \frac{\lambda}{c_1 + c_2} \partial^{ii} T_1 + \frac{\lambda}{\beta} \frac{c_2}{c_1 + c_2} \partial^{ii} \dot{T}_1$$

$$\boxed{\tau \ddot{T}_1 + \dot{T}_1 = \hat{\lambda} \partial^{ii} T_1 + a \partial^{ii} \dot{T}_1}$$

Guyer-Krumhansl, Jeffreys type