MESOSCOPIC DYNAMICS OF MICROCRACKS

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Abstract. The mesoscopic concept is applied to the description of microcracks. The balance equations of the cracked continuum result in the mesoscopic directional balances of mass, momentum, angular momentum and energy. Averaging over the length of the cracks the corresponding orientational balances are given. A further averaging process leads to the macroscopic balance equations of the microcracked continua. Dynamic equations for the fabric tensors of different order are derived using a multipole moment expansion of the orientational crack distribution function. The simple example of the Griffith cracks is treated. The role of physical assumptions on the microcrack representations and the different macroscopic internal variable representations of microcracks is discussed.

1. Introduction

To find suitable and applicable models for microstructured mechanical materials is a challenge of the contemporary physics, especially of continuum mechanics and statistical physics. An important particular (and relatively simple) example in this respect is to describe the mechanical properties of the microcrack systems in elastic materials. The two basic model levels are the continuum, where macroscopic variables are introduced to characterize the microcrack system and the statistical, where the properties and interactions of single microcracks or the embedding material are considered.

On the macroscopic, continuum level the suitable theories belong to *continuum damage mechanics*. In this phenomenological continuum theory thermodynamic internal variables of different tensorial order are used to calculate the influence of cracks (and other damage) on elastic properties of the material and to predict failure. It is important to remark here that the different continuum theories are far from being capable to propose a single model for *all* important phenomena connected to cracking (multiaxial loading conditions, material stability, dynamics, etc..). The competing theories are using different macroscopic mechanical and thermodynamical concepts. The most important discussed aspects are the nature (e.g. tensorial order, physical meaning) of the proposed macrosopic internal variables and the laws governing their time development, i. e. the corresponding macroscopic dynamical laws. We do not want to analyse the situation on the macroscopic level, but want to emphasize here that the lacking understanding in this level, that is a macroscopic phenomenological model for the experimental observations (e.g. in the frame of irreversible thermodynamics) means a serious disadvantage in the statistical physical modeling.

On the other hand microcracking is an important problem in statistical physics and is treated with two different approaches. *Micromechanics* is building from

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detailed properties of single cracks and extends the results with the help of statistical methods [1]. The 'microscopic laws' for a crack embedded in an ideal elastic continuum are well treated and known [**?**, **?**] and are difficult enough (long range, tensorial, anisotropic interactions with singularities) to mean a challenge for the basic principles of statistical physics on an equilibrium and nonequilibrium level, too. The second large group of statistical models is based on *lattice* calculations and simulations. These models introduce simple interactions between lattice elements (e.g. springs) and want to grasp some general qualitative properties of the phenomena with statistical methods. Some recent numerical and analitical investigations are suggesting the validity of mean field behaviour in precence of quenched disorder in isothermal systems (that we can expect in ordinary experimental situations) arguing that the failure due to microcracking can be treated as a first order phase transition and the whole process as spinodal nucleation [**?**]. At the first investigations spinodal nucleation was discussed as a thermally activated process, where the quenched disorder is irrelevant [**?**, **?**]. Some recent treatments are claiming that more realistic to consider a situation where a system is effectively at zero temperature and only the quenched disorder is relevant [**?**, **?**]. All these investigations are concentrating on the avalanche like behaviour of microfracturing and calculate the scaling properties. However, the observed mean field behaviour in numerical simulations of lattice models supports the view that phenomenological internal variable models can be capable to characterize the material especially when we are far from the quasistatic regime.

In this paper we propose an idea to bridge the microscopic-statistical approach and the macroscopic-phenomenological one. We introduce a new level of modeling, that is called mesoscopic because we go under the continuum level, the statistical distribution function of the microcracks is introduced. Hovewer, instead of a detailed microscopic modeling general ideas are used to get the governing equations of the different distribution functions. The suggested method can be used to derive different macroscopic internal variable models that are compatible with the statistical description and to incorporate micromechanical information of single microcracks, therefore to connect the statistical and phenomenological approaches.

Taking into account directional data distributions (e. g. normal vectors of planar microcracks) Kanatani [2] treated different possible statistical descriptions of directional data and found a coordinate independent description in the form of generalized Fourier series which is a coordinate independent form of the 'multipole moment expansion' known from classical electrodynamics [3]. Kanatani called the corresponding moments of the directional data distribution as 'fabric tensors'. So we can receive a statistically founded classification of the macroscopic internal variables without any information on the possible dynamic properties. After that different macroscopic thermodynamical methods, that are independent of the previous investigations, are used to get dynamic equations on the macroscopic variables.

However, in case of an other important family of microstructured continuum, for liquid crystals, the same moment series expansion is successfully applied to get not only the possible macroscopic thermodynamic variables but to get some general information on their dynamic equations too [4, 5, 6, 7, 8, 9, 10, 11]. Here balance equations are applied for the microstructured continuum and using them we can get some information on the mesoscopic dynamics and we can derive dynamic equations for the macroscopic variables, too. These macroscopic variables are the same moments of the orientation of liquid crystal molecules that are used

for microcracked continuum, however the nomination is different, they are called 'alignment tensors'.

In this paper we apply the mesoscopic theory to get the dynamic equations of the mesoscopic variables, to introduce macroscopic variables and the general form of their dynamic equations. These considerations give a good possibility to compare the macroscopic consequences of the mesoscopic approach with other macroscopic theories for example with rational thermodynamic theories of microstructured continuum, where the derivation of the 'micromomentum balances' is based on a strange application of material frame indifference [12, 13, 14]. On the other hand we can introduce specific 'single crack' properties (that is impossible in case of liquid crystals) to solve the dynamic equation for the distribution function and get information far from the quasistatic range that can be comparable with the micromechanical and lattice models.

2. Basic fields and functions

In liquid crystals molecules of restricted symmetry constitutes the material continuum. In nematic liquid crystals the molecules are rod like, therefore we introduce a quantity that characterizes the orientation of the molecules. There are two basic possibilities: we can give it as an additional vectorial field variable, where this (unit)vector is called the macroscopic "director". In this way we can arrive at the Ericksen-Leslie-Parodi theory of nematics. The other possibility is to introduce the director as a mesoscopic variable. In this case the additional orientational information (a unit vector) plays a similar role as the time and the space are, becomes the variable of the field quantities. In this way we arrive to the mesoscopic theory of liquid crystals, where all the field quantities are defined on this extended 'nematic space' $S^2 \times \mathbb{E} \times \mathbb{I}$, where S^2 denotes the unit sphere, $\mathbb{E}$ and $\mathbb{I}$ represents the space and the time. Sometimes can be useful to introduce more difficult microscopic quantities to characterize the continuum. If the internal structure that we are modeling is more complicated then the characteristic mesoscopic variables can be more complicated, too. For example in case of biaxial nematics the molecules have two axes and the resulted symmetry is best described by quaternyos.

What can we say about the damaged materials? Here the damage can have a more difficult microscopic structure than in liquid crystals, but we can restrict ourselves to the simple and frequently investigated case of planar microcracks. Now the damage consist of little planar surface elements embedded in an elastic or elastoplastic (or any kind of) background material. In this case a crack can be represented by its surface vector. If the cracks are fixed in the material, that is they do not move independently on material elements, then we can apply the mesoscopic concept to describe the microstructure. Therefore a characteristic material element of the microcracked continuum is a crack together with the containing base material.

Let us observe the difference between liquid crystals and microcracks a bit more closely. With a mesoscopic theory we pierce into the representative volume element of the continuum description and instead of the homogenization procedure (from where we would arrive at the continuum theory) we suppose that the macroscopic fields themselves depend on the microstructure and therefore we consider the statistical distribution of the orientations. In liquid crystals the shape of the molecules represents the microstructure and therefore the representative volume elements of the orientation and the other fields (especially mass) can be the same.

However, in case of microcracks the status of the microstructural information can be different, because they can be considered as if to be embedded into an elastic (or visco-elastic or any but continuous) base material. In this case we are using different representative volume elements for the meso-micro transition and for the macro-micro transition.

In this paper we give a mesoscopic model of a continuum that contains several randomly distributed microcracks. The microcracks are supposed to be two dimensional and flat, every microcrack is characterized by its surface 'vector' $\mathrm{l} \in \mathbb{E} \wedge \mathbb{E}$ and spacetime position $r \in M$. Therefore the domain of all field quantities of the mesoscopic theory is interpreted on a subset of this *direction space*. For example the *directional density* $\tilde{\rho}$ of the continuum is given as follows

$$\tilde{\rho} : \mathbb{E} \wedge \mathbb{E} \times M \longrightarrow \mathbb{R}^+, \quad (\mathrm{l}, r) \mapsto \tilde{\rho}(\mathrm{l}, r),$$

where $\mathbb{E}$ is a three dimensional Euclidean vector space and M is the spacetime (a structured four dimensional affine space). If we are in a non-relativistic spacetime and do not insist on a frame independent description we can introduce an inertial observer (as usual) [15]. As a final simplification we will use polar vectors instead of axial ones to represent the surfaces of the cracks introducing the usual form and symmetry requirement for the density function:

$$\rho : \mathbb{E}_l \times \mathbb{E} \times \mathbb{I} \longrightarrow \mathbb{R}^+, \quad (\mathbf{l}, \mathbf{x}, t) \mapsto \rho(\mathbf{l}, \mathbf{x}, t),$$

$$\rho(\mathbf{l}, \mathbf{x}, t) = \rho(-\mathbf{l}, \mathbf{x}, t).$$

The direction, position and time of microcracks are denoted by $(\mathbf{l}, \mathbf{x}, t) \in \mathbb{E}_l \times \mathbb{E} \times \mathbb{I}$. The corresponding mesoscopic space $\mathbb{E}_l \times \mathbb{E} \times \mathbb{I}$, where $\mathbb{E}_l$ and $\mathbb{E}$ are three dimensional Euclidean spaces and $\mathbb{I}$ is a one dimensional oriented vector space, will be called as *direction space*. In the following we suppose that the directional number density of the cracks has finite support, that is we consider a finite piece of material where the maximum length of the cracks is limited by the size of the sample (for example). Let us denote this maximal length by l_{max}.

A further important quantity can be introduced if we decompose the direction $\mathbf{l}$ into a length $l \in \mathbb{R}^+$ and an orientation $\mathbf{n} \in S^2$ as $\mathbf{l} = l\mathbf{n}$, where $\mathbf{n}$ is a unit vector ($\mathbf{n}^2 = 1$). Now the *orientational density* of the cracks is defined by the following integral:

$$\hat{\rho}(\mathbf{n}, \mathbf{x}, t) = \int_0^{l_{max}} \rho(l\mathbf{n}, \mathbf{x}, t) l^2 dl, \tag{1}$$

We will call the $S^2 \times \mathbb{E} \times \mathbb{I}$ mesoscopic space of the orientation, position and time $(\mathbf{n}, \mathbf{x}, t)$ of the microcracks as *orientation space*. For the further calculations is very important to keep in our mind the basic applicability criteria of the mesoscopic concept: the cracks are fixed in the base continuum. We render a fixed amount base material to every microcrack. In this case, and only in this case the density of the media will characterize the density of the number of cracks, too. Taking into account this remarks we can write that

$$\bar{\rho}(\mathbf{x}, t) := \langle \rho(\mathbf{l}, \mathbf{x}, t) \rangle := \frac{1}{2} \int_{\mathbb{R}^3} \rho(\mathbf{l}, \mathbf{x}, t) \mathrm{d}V_l = \frac{1}{2} \int_{S^2} \hat{\rho}(\mathbf{n}, \mathbf{x}, t) \mathrm{d}\mathbf{n}$$

is the macroscopic density of the microcracks at spacetime point $(\mathbf{x}, t)$. Here $\mathrm{d}V_l$ denotes the Lebesque measure of the microcrack part of the direction space, $\mathrm{d}\mathbf{n}$ is

the corresponding surface measure of S^2 in the orientation space. Furthermore

$$\begin{aligned} M(t) &= \int_{\mathbb{R}^3} \langle \rho(\mathbf{l},\mathbf{x},t)\rangle \mathrm{d}V = \int_{\mathbb{R}^3} \bar{\rho}(\mathbf{x},t)\mathrm{d}V = \frac{1}{2}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \rho(\mathbf{l},\mathbf{x},t)\mathrm{d}V_l\mathrm{d}V \\ &= \frac{1}{2}\int_{\mathbb{R}^3}\int_{S^2} \hat{\rho}(\mathbf{n},\mathbf{x},t)\mathrm{d}\mathbf{n}\mathrm{d}V \end{aligned}$$

is the total mass of the sample continuum. The symmetric polar vector representation ($\rho(\mathbf{l},\mathbf{x},t) = \rho(-\mathbf{l},\mathbf{x},t)$ and $\hat{\rho}(\mathbf{n},\mathbf{x},t) = \hat{\rho}(-\mathbf{n},\mathbf{x},t)$) made be necessary to get the half of the last two integrals.

It is useful to normalize the densities introducing the following *directional probability distribution*

$$f(\mathbf{l},\mathbf{x},t) := \frac{\rho(\mathbf{l},\mathbf{x},t)}{\bar{\rho}(\mathbf{x},t)}, \tag{2}$$

orientational probability distribution

$$\hat{f}(\mathbf{n},\mathbf{x},t) := \frac{\hat{\rho}(\mathbf{n},\mathbf{x},t)}{\bar{\rho}(\mathbf{x},t)}, \tag{3}$$

and *length probability distribution* function

$$f_l(\mathbf{l},\mathbf{x},t) := \frac{\rho(\mathbf{l},\mathbf{x},t)}{\hat{\rho}(\mathbf{n},\mathbf{x},t)} = \frac{f(\mathbf{l},\mathbf{x},t)}{\hat{f}(\mathbf{n},\mathbf{x},t)}. \tag{4}$$

At the end of this section let us remark, that a mesoscopic theory formally resemblances to a mixture theory that uses the continuous directional or orientational 'index' $\mathbf{l}$ or $\mathbf{n}$ for the 'components' instead of a discrete one. This analogy can be a help in the interpretation of the directional and orientational 'component' equations.

3. Mesoscopic kinematics

The following formulas make possible to give substantial balances in the mesoscopic continuum in addition to the local ones, so we can grasp the meaning of the corresponding mesoscopic balances more easily. Let us consider a piece of continuum material. Now we refer the material elements with their position X at some initial instant t_0, as usual. Let us denote x the position of the appropriate material element at the instant t. We give the position of the material element X at the time t with the map:

$$\mathbf{x} : \mathbb{E}_0 \times \mathbb{I} \rightarrowtail \mathbb{E}, (X,t) \mapsto \mathbf{x}(X,t).$$

Here we denoted the three dimensional Euclidean vector space of positions by $\mathbb{E}$ and the structural space of material points by $\mathbb{E}_0$. Similarly we can give the material element at the position $\mathbf{x}$ and instant t with the map:

$$\mathbf{X} : \mathbb{E} \times \mathbb{I} \rightarrowtail \mathbb{E}_0, (x,t) \mapsto \mathbf{X}(x,t).$$

The two maps are one to one and they are each others inverses at the same instant $\mathbf{x}(\mathbf{X}(x,t),t) = x$ and $\mathbf{X}(\mathbf{x}(X,t),t) = X$.

The mesoscopic structure is characterized by the variable $l \in \mathbb{E}_l$. In case of nematic liquid crystals this is the unit sphere S^2, for biaxial nematics S^3 and for planar microcracks a subset of $\mathbb{E}$. The microstructure is connected to the material element, therefore we can give its value at the instant t corresponding to the material element X: $\mathbf{l} : \mathbb{E}_0 \times \mathbb{I} \rightarrowtail \mathbb{E}_l, (X,t) \mapsto \mathbf{l}(X,t)$.

Now we define the velocities:

$$\mathbf{v} := \frac{\partial \mathbf{x}}{\partial t}(X,t), \quad \text{and} \quad \mathbf{v}_l := \frac{\partial \mathbf{l}}{\partial t}(X,t).$$

If a field quantity depends on the direction l and the position x we can define its material time derivative as follows:

$$\dot{f}(l,x,t) := (\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \mathbf{v}_l \cdot \nabla_l f) \circ (\mathbf{l},\mathbf{x},t)(X,t). \tag{5}$$

Until now we considered a continuum, therefore the position x and the direction l were treated as fields. However, the situation is more difficult, we are under the continuum level. In case of liquid crystals on the microscopic level we have single molecules. For cracks we can suppose that we are in the continuum domain as regards the mass, but the material elements contain single microcracks therefore the direction can be discontinuous from crack to crack at this level. Our task is to get a continuum description and at the same time keeping some information from underneath the usual macroscopic continuum level. Therefore we accomplish a second homogenization, forming a bigger material element from the micro-meso ones and introducing a center of mass X_0 for that macro material element with volume V_m:

$$X_0 = \frac{\int_{V_m} X\rho(X)\mathrm{d}V_m}{\int_{V_m}\rho(X)\mathrm{d}V_m}.$$

Now if we completely replaced the micro material elements with macro ones ($X \to X_0$) we would get the macroscopic director $\mathbf{l}(X_0,t)$. Instead of doing that we would like to keep some microscopic information and therefore we make the $X \to (X_0,l)$ substitution supposing that there is a distribution of the directions inside the macro element. In this way l is not a field quantity any more as we supposed above, but stands on equal footing with X_0 and plays and independent role characterizing the macro continuum element. Therefore the previous functions defined on the "micro-material space" become functions on the mesoscopic space (e.g. the $\mathbf{v},\mathbf{v}_l$ velocities). Moreover, as the continuity of the variables is secured with the homogenization procedure we can introduce the previous (5) derivative as a "material" derivative on the mesoscopic space.

4. Directional balances

After these preparations we are ready to get the mesoscopic balance equations of the directional quantities. All of the following local balances were derived from the proper global balances using a generalized form of the Gauss-Stokes integral theorem (or Reynolds transport theorem equivalently). The difference of the usual spacetime balances and the following generalized balances where the spacetime variables are completed with the direction is that now the 'configuration' space of the continuum is six dimensional. Therefore the velocity space is also six dimensional, we get an additional directional velocity component. Moreover the local balances will have an additional 'current term' with the divergence of the directional part of the total two times three dimensional current densities ($\nabla_l\cdot$).

Using the introduced 'mesoscopic material time derivative' (5) we will give the corresponding substantial balances, too.

First we can get the fundamental balance of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) + \nabla_l \cdot (\rho \mathbf{v}_l) = 0, \tag{6}$$

and

$$\dot{\rho} + \rho(\nabla \cdot \mathbf{v} + \nabla_l \cdot \mathbf{v}_l) = 0, \tag{7}$$

where $\mathbf{v}$ is the directional material velocity, $\mathbf{v}_l$ is the velocity of the change of crack orientation and length. Here, and in this section all quantities are directional, their domain is the subset of the direction space.

The balance of momentum:

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \circ \mathbf{v} - \mathbf{t}^T) + \nabla_l \cdot (\rho \mathbf{v}_l \circ \mathbf{v}) = \rho \mathbf{f}, \tag{8}$$

and

$$\rho \dot{\mathbf{v}} - \nabla \cdot \mathbf{t}^T = \rho \mathbf{f}. \tag{9}$$

Here $\mathbf{t}$ is the stress and $\mathbf{f}$ is the body force density. The superscript T denotes the transpose of the corresponding second order tensor. We applied here that the change of the momentum, even if it depends on the direction, is due only to the body and 'normal' surface forces that is there are no surface forces in the direction component and there is no conductive directional momentum current.

Similarly, for the balance of moment of momentum we will get the form

$$\frac{\partial \rho \mathbf{s}}{\partial t} + \nabla \cdot \left(\rho \mathbf{v} \circ \mathbf{s} - \pi^T\right) + \nabla_l \cdot (\rho \mathbf{v}_l \circ \mathbf{s}) = \mathbf{t}^{as} + \rho \mathbf{g}, \tag{10}$$

where $\mathbf{s}$ is the directional spin density, $\mathbf{t}^{as}$ denotes the antisymmetric part of the stress tensor, π is the couple stress density and $\mathbf{g}$ is the density of couple force. The substantial form is

$$\rho \dot{\mathbf{s}} - \nabla \cdot \pi^T = \mathbf{t}^{as} + \rho \mathbf{g}, \tag{11}$$

At the end we give the directional internal energy density balances. These we got subtracting the balances of the kinetic and rotational energy from the balance of the total energy:

$$\frac{\partial \rho \epsilon}{\partial t} + \nabla \cdot (\rho \mathbf{v} \epsilon + \mathbf{q}) + \nabla_l \cdot (\rho \mathbf{v}_l \epsilon + \mathbf{q}_l) = \nabla \circ \mathbf{v} : \mathbf{t} + \rho \Xi. \tag{12}$$

Here $\mathbf{q}$ and $\mathbf{q}_l$ are the heat current and the directional heat current, respectively (both mesoscopic). Ξ is the internal energy production related directly to the microcrack propagation. The corresponding substantial form:

$$\rho \dot{\epsilon} + \nabla \cdot \mathbf{q} + \nabla_l \cdot \mathbf{q}_l = \nabla \circ \mathbf{v} : \mathbf{t} + \rho \Xi. \tag{13}$$

5. Orientational balances

Traditionally in damage mechanics we are interested only in the orientational part of the data distributions, when the length of the cracks is supposed to be statistically independent on the orientational part of the data distribution, we use averaged, uniform size cracks in the treatment. Therefore here we give the balances of the orientational quantities, too. If we want to get an orientational quantity from a directional one we should average over the microcrack length using the introduced directional distribution function f and length distribution function f_l (see (2) and (4)). To do this we will integrate the directional balances over the microcrack length. Worthy of note here that the time derivation and normal divergence

commutes with the integration over the length, and for an arbitrary directional function g

$$\int_0^{l_{max}} \nabla_l g(\mathbf{l}) l^2 \mathrm{d}l = \nabla_{\mathbf{n}} \int_0^{l_{max}} g(l\mathbf{n}) l^2 \mathrm{d}l, \tag{14}$$

The commutation properties and (14) suppose several identifications and regularity properties. For example calculating the formula (14) the splitting of the directional derivative was accomplished as $\nabla_l = (\mathbf{n}\cdot\nabla_{\mathbf{l}}, (\mathbf{I}-\mathbf{n}\circ\mathbf{n})\nabla_{\mathbf{n}}) = (\frac{\partial}{\partial l}, \nabla_{\mathbf{n}})$, and $(0,\mathbf{a}) = \mathbf{a}$. We will use the hat $\hat{}$ for the orientational quantities (as above) and introduce the notation $\langle\_\rangle_l$ for the length averaging. If the orientational velocities are $\hat{\mathbf{v}} := \langle\mathbf{v}\rangle_l$ and $\hat{\mathbf{v}}_l := \langle\mathbf{v}_l\rangle_l$, then we get the orientational mass balance

$$\frac{\partial\hat{\rho}}{\partial t} + \nabla\cdot(\hat{\rho}\hat{\mathbf{v}}) + \nabla_{\mathbf{n}}\cdot(\hat{\rho}\hat{\mathbf{v}}_l) = 0. \tag{15}$$

In substantial form too:

$$\dot{\hat{\rho}} + \hat{\rho}\nabla\cdot\hat{\mathbf{v}} + \hat{\rho}\nabla_{\mathbf{n}}\cdot\hat{\mathbf{v}}_l = 0. \tag{16}$$

The balance of momentum in local form can be given as:

$$\frac{\partial\hat{\rho}\hat{\mathbf{v}}}{\partial t} + \nabla\cdot(\hat{\rho}\hat{\mathbf{v}}\circ\hat{\mathbf{v}} - \hat{\mathbf{t}}^T) + \nabla_l\cdot(\hat{\rho}\hat{\mathbf{v}}_l\circ\hat{\mathbf{v}} - \hat{T}^T) = \hat{\rho}\hat{\mathbf{f}}. \tag{17}$$

Here we introduced the orientational stress $\hat{\mathbf{t}}$ and orientational *microstress* $\hat{\mathbf{T}}$ as follows:

$$\begin{aligned}
\hat{\mathbf{t}} &= \hat{\rho}\hat{\mathbf{v}}\circ\hat{\mathbf{v}} - \int_0^{l_{max}} (\rho\mathbf{v}\circ\mathbf{v} - \mathbf{t}) l^2\mathrm{d}l = \hat{\rho}(\hat{\mathbf{v}}\circ\hat{\mathbf{v}} - \langle\mathbf{v}\circ\mathbf{v}\rangle_l) + \int_0^{l_{max}} \mathbf{t} l^2\mathrm{d}l,\\
\hat{\mathbf{T}} &= \hat{\rho}\hat{\mathbf{v}}_l\circ\hat{\mathbf{v}} - \int_0^{l_{max}} \rho\mathbf{v}_l\circ\mathbf{v} l^2\mathrm{d}l = \hat{\rho}(\hat{\mathbf{v}}_l\circ\hat{\mathbf{v}} - \langle\mathbf{v}_l\circ\mathbf{v}\rangle_l).
\end{aligned}$$

We can give the substantial form of the orientational momentum balance, too

$$\hat{\rho}\dot{\hat{\mathbf{v}}} - (\nabla\cdot\hat{\mathbf{t}}^T + \nabla_l\cdot\hat{T}^T) = \hat{\rho}\hat{\mathbf{f}}. \tag{18}$$

Remarkable is the appearance of microstress, a conductive orientational momentum current in the orientational momentum balance.

The local balance of moment of momentum:

$$\frac{\partial\hat{\rho}\hat{\mathbf{s}}}{\partial t} + \nabla\cdot(\hat{\rho}\hat{\mathbf{v}}\hat{\mathbf{s}} - \hat{\pi}^T) + \nabla_{\mathbf{n}}\cdot(\hat{\rho}\hat{\mathbf{v}}_l\hat{\mathbf{s}} - \hat{\Pi}^T) = \hat{\mathbf{t}}^{as} + \hat{\rho}\hat{\mathbf{g}} \tag{19}$$

Here $\hat{\mathbf{s}} = \langle\mathbf{s}\rangle_l$ is the orientational spin, $\hat{\mathbf{g}} = \langle\mathbf{g}\rangle_l$ is the orientational couple force vector. However, we should be careful because for example $\hat{\mathbf{t}}^{as} \neq \langle\mathbf{t}^{as}\rangle_l$ but we should consider the previous definition. Moreover the couple stress $\hat{\pi}$ and the new *orientational couple stress* $\hat{\mathbf{T}}$ are defined as

$$\begin{aligned}
\hat{\pi} &= \hat{\rho}(\hat{\mathbf{v}}\circ\hat{\mathbf{s}} - \langle\mathbf{v}\circ\mathbf{s}\rangle_l) + \int_0^{l_{max}} \pi l^2\mathrm{d}l,\\
\hat{\Pi} &= \hat{\rho}(\hat{\mathbf{v}}_l\circ\hat{\mathbf{s}} - \langle\mathbf{v}_l\circ\mathbf{s}\rangle_l).
\end{aligned}$$

It is easy to see, that $\hat{\pi}$ is orthogonal to $\mathbf{n}$. The corresponding substantial equation:

$$\hat{\rho}\dot{\hat{\mathbf{s}}} - (\nabla\cdot\hat{\pi}^T + \nabla_{\mathbf{n}}\cdot\hat{\Pi}^T) = \hat{\mathbf{t}}^{as} + \hat{\rho}\hat{\mathbf{g}} \tag{20}$$

The orientational balance of the internal energy is very similar to the directional one, but here again the conductive currents and the source term, the expression of

the energy production is not simply the average of the corresponding directional quantities. The single averaged orientational term is the internal energy itself ($\hat{\epsilon} = \langle\epsilon\rangle_l$).

$$\frac{\partial\hat{\rho}\hat{\epsilon}}{\partial t} + \nabla\cdot(\hat{\rho}\hat{\mathbf{v}}\hat{\epsilon} + \hat{\mathbf{q}}) + \nabla_{\mathbf{n}}\cdot(\hat{\rho}\hat{\mathbf{v}}_l\hat{\epsilon} + \hat{\mathbf{q}}_l) = \nabla\circ\hat{\mathbf{v}} : \hat{\mathbf{t}} + \hat{\rho}\hat{\Xi}. \tag{21}$$

The definition of the heat currents and the source term are as follows:

$$\begin{aligned}
\hat{\mathbf{q}} &= \hat{\rho}(\langle\mathbf{v}\epsilon\rangle_l - \hat{\mathbf{v}}\hat{\epsilon}) + \int_0^{l_{max}} \mathbf{q}l^2\mathrm{d}l,\\
\hat{\mathbf{q}}_l &= \hat{\rho}(\langle\mathbf{v}_l\epsilon\rangle_l - \hat{\mathbf{v}}_l\hat{\epsilon})\\
\hat{\Xi} &= \hat{\rho}\langle\Xi\rangle_l + \int_0^{l_{max}} \nabla\circ\mathbf{v} : \mathbf{t}l^2\mathrm{d}l - \nabla\circ\hat{\mathbf{v}} : \hat{\mathbf{t}}.
\end{aligned}$$

We can easily get the substantial form:

$$\hat{\rho}\dot{\hat{\epsilon}} + \nabla\cdot\hat{\mathbf{q}} + \nabla_{\mathbf{n}}\cdot\hat{\mathbf{q}}_l = \nabla\circ\hat{\mathbf{v}} : \hat{\mathbf{t}} + \hat{\rho}\hat{\Xi}. \tag{22}$$

6. Macroscopic balances

In the calculation of the macroscopic balances we can use the directional or the orientational balances, too. Maybe the first way is the more convenient. We will denote the macroscopic quantities with the bar $\bar{\ }$ and the averaged directional quantities that are calculated with the help of the directional distribution function (2) with brackets $\langle\_\rangle$. The corresponding macroscopic equations are calculated with the integration of the directional balances over $\mathbf{l}$. This integration commutes with the time and space derivatives and eliminates the divergence of the directional-orientational derivative, because g has a compact support

$$\int_{V_l} \nabla_l\cdot g\mathrm{d}V_l = 0.$$

If the macroscopic (barycentric) velocity $\bar{\mathbf{v}} = \langle\mathbf{v}\rangle$ then with the previously introduced macroscopic density $\bar{\rho}$, the balance of mass can be written as

$$\frac{\partial\bar{\rho}}{\partial t} + \nabla\cdot(\bar{\rho}\bar{\mathbf{v}}) = 0. \tag{23}$$

The balance of momentum:

$$\frac{\partial\bar{\rho}\bar{\mathbf{v}}}{\partial t} + \nabla\cdot(\bar{\rho}\bar{\mathbf{v}}\circ\bar{\mathbf{v}} - \bar{\mathbf{t}}^T) = \bar{\rho}\bar{\mathbf{f}}, \tag{24}$$

where the macroscopic force density $\bar{\mathbf{f}} = \langle\mathbf{f}\rangle$. Again the macroscopic stress is not a simple average, it can be calculated as

$$\bar{\mathbf{t}} = \int_{V_l} \mathbf{t}\mathrm{d}V_l + \rho(\bar{\mathbf{v}}\circ\bar{\mathbf{v}} - \langle\mathbf{v}\circ\mathbf{v}\rangle).$$

The balance of moment of momentum and the balance of internal energy can be calculated similarly.

$$\frac{\partial\bar{\rho}\bar{\mathbf{s}}}{\partial t} + \nabla\cdot(\bar{\rho}\bar{\mathbf{v}}\circ\bar{\mathbf{s}} - \bar{\pi}) = \bar{\mathbf{t}}^{as} + \bar{\rho}\bar{\mathbf{g}}, \tag{25}$$

where the macroscopic couple force density $\bar{\mathbf{g}} = \langle\mathbf{g}\rangle$. The macroscopic couple stress also includes a contribution due to deviations of $\mathbf{v}$ and $\mathbf{s}$ from the average, it can be calculated as

$$\bar{\pi} = \int_{V_l} \pi\mathrm{d}V_l + \rho(\bar{\mathbf{v}}\circ\bar{\mathbf{s}} - \langle\mathbf{v}\circ\mathbf{s}\rangle).$$

The balance of the internal energy becomes:

$$\frac{\partial \bar{\rho}\bar{\epsilon}}{\partial t} + \nabla \cdot (\bar{\rho}\bar{\mathbf{v}}\bar{\epsilon} - \bar{\mathbf{q}}) = \nabla \circ \bar{\mathbf{v}} : \bar{\mathbf{t}} + \bar{\rho}\bar{\Xi}, \tag{26}$$

where

$$\begin{aligned} \bar{\mathbf{q}} &= \bar{\rho}(\langle \mathbf{v}\epsilon \rangle - \bar{\mathbf{v}}\bar{\epsilon}) + \int_{V_l} \mathbf{q} \mathrm{d}V_l, \\ \bar{\rho}\bar{\Xi} &= \bar{\rho}\langle \Xi \rangle + \int_{V_l} \nabla \circ \mathbf{v} : \mathbf{t} \mathrm{d}V_l - \nabla \circ \bar{\mathbf{v}} : \bar{\mathbf{t}}. \end{aligned}$$

Let us remark that on the directional and orientational level it was unreasonable to suppose a positive entropy production but it has sense on a macroscopic level and we can exploit it.

7. Crack propagation

For the most frequently used materials in damage mechanics the balance equations given above are too general. Therefore we are introducing some simplifying assumptions:

1. The base material does not have an internal spin, that is a crack does not rotate independently from the base material.
2. There are no couple forces ($\mathbf{g} = 0$) and couple stresses ($\Pi = 0$).
3. There are no external body forces ($\mathbf{f} = 0$),
4. The material is in mechanical equilibrium ($\dot{\mathbf{v}} = 0$).
5. The velocity does not depend on the crack size and orientation, that means it is equal to the barycentric velocity ($\mathbf{v}(\mathbf{l}, x, t) = \bar{\mathbf{v}}(x, t)$).

Because of the first condition we do not need the balance of the internal energy the third condition simplifies the spin balance to a symmetric stress. For this symmetric stress the balance of the momentum together with the fourth and fifth condition results in an equation of the mechanical equilibrium

$$\nabla \cdot \mathbf{t} = 0. \tag{27}$$

Very similarly we can get in the orientational space:

$$\nabla \cdot \hat{\mathbf{t}} = 0 \tag{28}$$

Moreover, the balance of mass simplifies considerably because of the last condition:

$$\frac{\partial f}{\partial t} + \bar{\mathbf{v}} \cdot \nabla f + \nabla_l \cdot (f \mathbf{v}_l) = 0, \tag{29}$$

where $f = \rho/\bar{\rho}$ is the directional probability density as it was given in (2) and $\bar{\mathbf{v}}$ is the macroscopic velocity. The substantial form is remarkable simple:

$$\dot{f} + f \nabla_l \cdot \mathbf{v}_l = 0.$$

Integrating over the crack length we get the following orientational balance:

$$\frac{\partial \hat{f}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \hat{f} + \nabla_{\mathbf{n}} \cdot \hat{f} \hat{\mathbf{v}}_l = 0, \tag{30}$$

where the orientational crack velocity $\hat{\mathbf{v}}_l$ was introduced. Let us observe, that (29) is formally the same as (30) but that the functions are different (directional-orientational).

Therefore our simplified final system of equations is (27) and (29) at the direction space, or (28) and (30) is at the orientation space. Now we can consider several possibilities to have a closed, soluble system. We can try to close the system on the mesoscopic or on the macroscopic level.

- If we consider some specific information on the crack propagation and calculate the crack growth speed $\mathbf{v}_l$. This is promising because this velocity is connected to the micromaterial element and therefore we need to investigate a single crack to calculate it. In this way, introducing the corresponding state space and considering some constitutive assumptions on the mesoscopic stress, there is a good hope to close the system at the mesoscopic level. The problematic point can be the constitutive assumption for $\mathbf{t}$. On the mesoscopic level, without an inequality from the second law for the mesoscopic functions, the constitutive theory is more approximative.
- The other possibility can be to calculate the macroscopic balances from the mesoscopic ones. In this case the orientational balances are the more promising, because here the moment series expansion gives a familiar and understood process (see e.g. [5]). We can try similar series expansions at the directional space too, but the most straightforward choices will mix the length and orientational information, therefore the meaning of the macroscopic quantities is not evident.

7.1. **Moment series expansion and order parameters.** First we will investigate the consequences of moment series expansion of the distribution function $\hat{f}$ and the equation (30). We can introduce the following alignment-fabric tensors

$$\mathbf{a}^{(k)}(\mathbf{x},t) := \int_{S^2} \hat{f}(\mathbf{n},\mathbf{x},t)\ \overline{\mathbf{n}\circ\ldots\circ\mathbf{n}}\ \mathrm{d}\mathbf{n}, \tag{31}$$

where $\overline{\ldots}$ denotes the symmetric irreducible part of a tensor [16]. Remarkable, that only the even order tensors appear in the series because the microcracks are represented by axial vectors. These damage parameters are macroscopic quantities and they called 'fabric tensors of the second kind' in damage mechanics (see Kanatani [2] or Krajcinovic [1]) and were introduced on a purely statistical ground, without a mesoscopic foundation.

Now let us turn our attention to the series expansion of the equation (30). We can get the following system of equations for the k-th moment [**?**]:

$$\begin{aligned}
&\frac{da^{(k)}_{n_1\dots n_k}}{dt} - k\ \overline{(\underline{\omega}\times a^{(k)})}_{\ n_1\dots n_k} = \frac{2k+1}{k!4\pi}\sum_{l\,even}(2l-1)!! \\
&\left(\left(\oint_{S^2}\delta\hat{v}^p\ \overline{n_{m_1}\dots n_{m_l}}\ \overline{n_{n_1}\dots n_{n_k}}\ d^2n\ a^{(l)}_{m_1\dots m_l}\right)_{,p} + \right. \\
+\quad &\left(\oint_{S^2}\delta\hat{v}^p\ \overline{n_{m_1}\dots n_{m_l}}\ \overline{n_{n_1}\dots n_{n_k}}\ d^2n(\ln\rho)_{,p} + \right. \\
+\quad &\left.\oint_{S^2}\overline{n_{m_1}\dots n_{m_l}}\ \overline{n_{n_1}\dots n_{n_k}}\ (\nabla_n\times\delta\underline{\hat{\omega}})\cdot d\mathbf{n}\right)a^{(l)}_{m_1\dots m_l} - \\
-\quad &l\oint_{S^2}\overline{n_{m_1}\dots n_{m_l}}\ \overline{n_{n_1}\dots n_{n_k}}\ \left(\delta\underline{\hat{\omega}}\times a^{(l)}\right)_{m_1\dots m_l}\cdot d\mathbf{n} + \\
+\quad &\oint_{S^2}\delta\hat{v}^p\ \overline{n_{n_1}\dots n_{n_k}}\ d^2n(\ln\rho)_{,p} + \left(\oint_{S^2}\delta\hat{v}^p\ \overline{n_{n_1}\dots n_{n_k}}\ d^2n\right)_{,p} + \\
+\quad &\left.\oint_{S^2}\overline{n_{n_1}\dots n_{n_k}}\ (\nabla_n\times\delta\underline{\hat{\omega}})\cdot d\underline{n}\right),
\end{aligned}$$

where $\underline{\omega}=\frac{1}{2}\nabla\times\mathbf{v}$, $\delta\underline{\hat{\omega}}=\underline{\hat{\omega}}-\underline{\bar{\omega}}$, $\delta\hat{\mathbf{v}}=\hat{\mathbf{v}}-\bar{\mathbf{v}}$ and we denoted the components of the velocity with the index p to avoid misunderstanding. In this way we have got a whole set of possible macroscopic damage parameters together with a general form of their dynamic equation. Let us investigate more closely the dynamic equation of the second order tensor term in the expansion. It seems to be useful to put down the definition and the dynamic equation for that term separately as follows

$$\mathbf{a}(x,t) := \int_{S^2}\hat{f}\ \overline{\mathbf{n}\circ\mathbf{n}}\ \mathrm{d}\mathbf{n}, \tag{32}$$

and we get the following dynamic equation

$$\frac{\partial\mathbf{a}}{\partial t} + \bar{\mathbf{v}}\cdot\nabla\mathbf{a} + \int_{S^2}\overline{\mathbf{n}\circ\mathbf{n}}\ \nabla_{\mathbf{n}}\cdot(\hat{f}\hat{\mathbf{v}}_l) = 0. \tag{33}$$

Or equivalently

$$\frac{\partial\mathbf{a}}{\partial t} + \bar{\mathbf{v}}\cdot\nabla\mathbf{a} + 2\langle\,\overline{\mathbf{v}_l\circ\mathbf{n}}\,\rangle = 0. \tag{34}$$

Without calculating the last term we can see that $\mathbf{a}$ is a normal internal variable in the sense that a local first order differential equation describes its change.

It is worth to investigate separately the uniaxial case, when the alignment tensors can be expressed in terms of order parameters $S^{(k)}$ and a unit vector $\mathbf{d}$ in the following way:

$$\mathbf{a}^{(k)} = S^{(k)}\ \overline{\mathbf{d}\circ\ldots\circ\mathbf{d}} \qquad (k=2,4,\ldots)$$

where the value of the order parameters $S^{(k)}$ is one in case of total alignment (the microcracks stand parallely) and zero for randomly oriented cracks. In case of a second order alignment tensor ($\mathbf{a}^{(2)} = S^{(2)}\ \overline{\mathbf{d}\circ\mathbf{d}}$) the following dynamic equations can be written for the order parameter:

$$\frac{\mathrm{d}S}{\mathrm{d}t} + e\mathbf{d}\cdot\langle\,\overline{\mathbf{v_l}\circ\mathbf{n}}\,\rangle\cdot\mathbf{d} = 0. \tag{35}$$

Moreover for the vector $\mathbf{d}$ we can get:

$$S\frac{\mathrm{d}\mathbf{d}}{\mathrm{d}t} = 2(\mathbf{d}\circ\mathbf{d}-\delta)\mathbf{d} : \langle \overline{\mathbf{v_l}\circ\mathbf{n}} \rangle$$

Let us remark that are cases when a truncation of the series leads to paradoxes. If the microcrack distribution is uniaxial than the best fitting alignment tensor can result in negative crack densities, the so called 'anticracks' regions in the approximated data distribution [17]. This unexpected property can be removed if we use a director (vectorial) internal variable representation instead of the even order traceless tensors. The single vectorial approximation results in a macroscopic 'director' theory.

7.2. **Solution for the distribution function: Griffith cracks.** In this section the equation of motion for the mesoscopic distribution function is specialized considering a specific single crack model. In this case we can start from the directional level and we can calculate the crack size distribution function. The following additional assumptions are introduced:

- The crack surface area can increase, but cannot decrease.
- The crack velocity is independent of other cracks in the vicinity, i.e. cracks do not interact.
- crack inertia is neglected in the expression for the crack velocity, i.e. it is assumed that the crack stops enlarging instantanuously when the external load stops changing.
- All the idealizations assumed by Griffith [**?**] (e.g. two dimension, ideal elliptic cracks, etc..) are supposed here.

Let us observe that these seemeingly restrictive conditions are in some respect more general than the restrictions used explicitly or implicitly in models of micromechanical origin [**?**, 1, **?**]. For example we did not assume special crack orientations or definite interactions between the microcracks.

From the mesoscopic balance of mass we have derived the following differential equation for the directional distribution function

$$\frac{\partial f}{\partial t} + \bar{\mathbf{v}}\cdot\nabla f + \nabla_l\cdot(f\mathbf{v}_l) = 0. \tag{36}$$

In the following we use spherical coordinates. In spherical coordinates the mesoscopic velocity $\mathbf{v}_l$ is decomposed into the length change velocity v_c and the orientation change velocity ω, which is zero in our model:

$$\mathbf{v}_l(\mathbf{l},x,t) = (v_c(\mathbf{l},x,t),\omega(\mathbf{l},x,t)). \tag{37}$$

From the model of Griffith [**?**] it follows for the crack length change velocity:

$$v_c = -\frac{2}{m}l^{3/2}\dot{R}, \tag{38}$$

where R is the stress at the location of the crack, and m is a material depenent constant. In the following we consider the case where slowly changing external loads P and Q are applied to the sample as shown on figure 1.

The R used by Griffith has been given by Inglis [**?**] in terms of the parameters α_0, β, and θ, where θ is the angle between the crack orientation and the z-axis, α_0 is the ratio of diameter to thickness of the crack (ratio of the large to small axis of the ellipse describing the crack in the model of Griffith). α_0 is very large and is assumed to be constant in time according to our preliminary asumptions (because

otherwise the crack length and orientation would not be the only crack variables). β is the parameter on the ellipse. According to the previous assumption ($\alpha_0 \gg 1$) the stress is maximal for $\beta = \pi$, i.e. on the tip of the crack (the real maximum is very close to that, at least for Griffith cracks). Therefore we will set here $\beta = \pi$. θ is constant in time because the crack cannot change its orientation. With these assumptions we obtain from the paper by Griffith [**?**] the following expression for the stress change rate:

$$\begin{aligned} \dot{R} &= c_1(\dot{P}+\dot{Q}) + c_2(\dot{P}-\dot{Q}) \\ c_1 &= \frac{(e^{2\alpha_0}-1)cos(2\Theta)}{ch(2\alpha_0)-1} \\ c_2 &= \frac{sh(2\alpha_0)}{ch(2\alpha_0)-cos(2\Theta)} \end{aligned} \tag{39}$$

The coefficients c_1 and c_2 depend on crack orientation, but in our model not on position and time. If we avarage over different crack orientations, the result will depend on the above introduced order parameters.

Now the mesoscopic velocity $\mathbf{v}_l$ derived from the expressions (39) and (38) is introduced into the differential equation for the distribution f. In spherical coordinates we have:

$$\begin{aligned} \nabla_l \cdot (\mathbf{v}_l f_l) &= \frac{1}{l^2}\frac{\partial}{\partial l}\left(l^2 v_c f\right) = \\ &= \frac{1}{l^2}\frac{\partial}{\partial l}\left(-\frac{2}{m}l^{7/2}\left(c_1(\dot{P}+\dot{Q}) + c_2(\dot{P}-\dot{Q})\right) f\right), \end{aligned} \tag{40}$$

which results in the equation for the distribution function:

$$\frac{df}{dt} = -\nabla_l \cdot (\mathbf{v}_l f) = \frac{1}{l^2}\frac{\partial}{\partial l}\left(\frac{2}{m}l^{7/2}\left(c_1(\dot{P}+\dot{Q}) + c_2(\dot{P}-\dot{Q})\right) f\right). \tag{41}$$

Separation of the variables gives the solution of the differential equation.

Moreover, we can go further introducing the moments of the distribution function as macroscopic variables. From (41) we can derive evolution equations for the particular moments, too.

In case of Griffith cracks we can introduce the orientational order parameters and the length order parameters as macroscopic quantities describing the mesoscopic distribution. However, there are several other possibilities. The question arises which macroscopic parameter is relevant for the mechanical properties of the material. Here we mention an example for a macroscopic parameter that is different to the moments. In the simple variation of the one dimensional "loose bundle parallel bar" model of Krajcinovic [1] the material is assumed to consist of elastic parallel bars of fixed diameter l_0. When the projection of the crack length perpendicular to the bar axis is greater than l_0, the bar is broken and does not support stresses any more. The damage parameter D is introduced as the ratio of broken bars over the whole number of bars. Translating this definition to the mesoscopic theory with Griffith cracks we can define:

$$D(\mathbf{x},t) = \int_0^{l_0} f(l,\mathbf{x},t)l^2 dl \tag{42}$$

as a new macroscopic parameter. The mesoscopic theory provides tools to deal (dynamics, relation to the moment series expansion, etc...) with this damage variable, too.

8. Discussion

In this paper we investigated the applicability of the mesoscopic concept for microcracks. The physical conditions show that if we consider planar microcracks that are fixed in the surrounding media (no diffusion, unlike the dislocations) then the formalism and the results developed for liquid crystals are applicable and can give some fundamental informations on the possible macroscopic internal variables and on their dynamics, too.

For example according to the present investigations the moment series expansion of the orientational distribution function does not close the long discussion on the nature of the tensorial order of the internal variables in continuum damage mechanics. First of all the introduction of an orientational distribution function is only a convenient simplification of the situation and there can be cases when the length and the orientation of the cracks are statistically dependent. On the other hand the dynamics of the microcrack distribution depends on the mesoscopic space.

Sometimes a vectorial representation is simpler and fits better than a tensorial one (uniaxial case). This can be interpreted as special case of uniaxiality in the fabric tensor description. The situation is best seen from the point of view of liquid crystal theories, where both kind of descriptions are present. Similar symmetry requirements as in the case of microcracks (head-tail symmetry) results in only even order terms in the alignment tensor series expansion, but the vectorial director theory of Ericksen-Leslie-Parodi-Verhás is well usable (and a little bit simpler) in a lot of systems.

In continuum damage mechanics we can find examples of very different damage descriptors (Scalars: [18, 19], vectors: [20, 21, 22, 23], second order tensors: [1], higher order tensors: [24]). From a mesoscopic point of view the relation between the macroscopic theories with internal variables of different tensorial order is clear [6, 7, 9]. Furthermore the mesoscopic theory gave a particular form of the possible dynamic equations on the mesoscopic and on the macroscopic level, too. Without calculating a particular source term we can see that it is a first order equation in the time and space derivatives. Using further specific assumptions on the dynamics of the extension of single microcracks one can get a closed system of equations for the dynamics of the moments of the microcrack distribution and for the distribution function itself.

It can be important to see, that our results do not correspond to some other microstructural continuum theories [12, 13, 25], where a second order equation is supposed for the dynamics of the microstructure.

Let us give a closer look to this proposition. According to the suggestion of Capriz we include a general kinetic energy term into the energy balance and after some calculations based on the principle of material frame indifference you can conclude on the micromomentum balance [13]:

$$\rho\left(\left(\frac{\partial\dot{\kappa}}{\partial\dot{\nu}}\right)-\frac{\partial\kappa}{\partial\nu}\right)-\rho\beta+\chi=0 \tag{43}$$

where ν is the parameter of the microstructure (e. g. microcrack length), β, χ can be interpreted as 'microforces' and 'microstresses' and they must be given constitutively. The first term contains $\kappa(\nu,\dot{\nu})$, the "micro-kinetic energy". Easy to prove that this term cannot result in a first order equation for ν.

On the other hand we can give some remarks on the statistical approaches, too. The mesoscopic theory in some sense supports the validity of the mean field descriptions in case of simple crack orientation distributions when the first terms of the momentum series expansion can represent the length distribution functions. For example this can be expected when uniaxial loading conditions were applied to an initially undamaged material as it is expected in lattice models where the mean field scaling was observed [**?**]. Hovewer, the reason of long standing metastable states should be explained on the phenomenological level, too.

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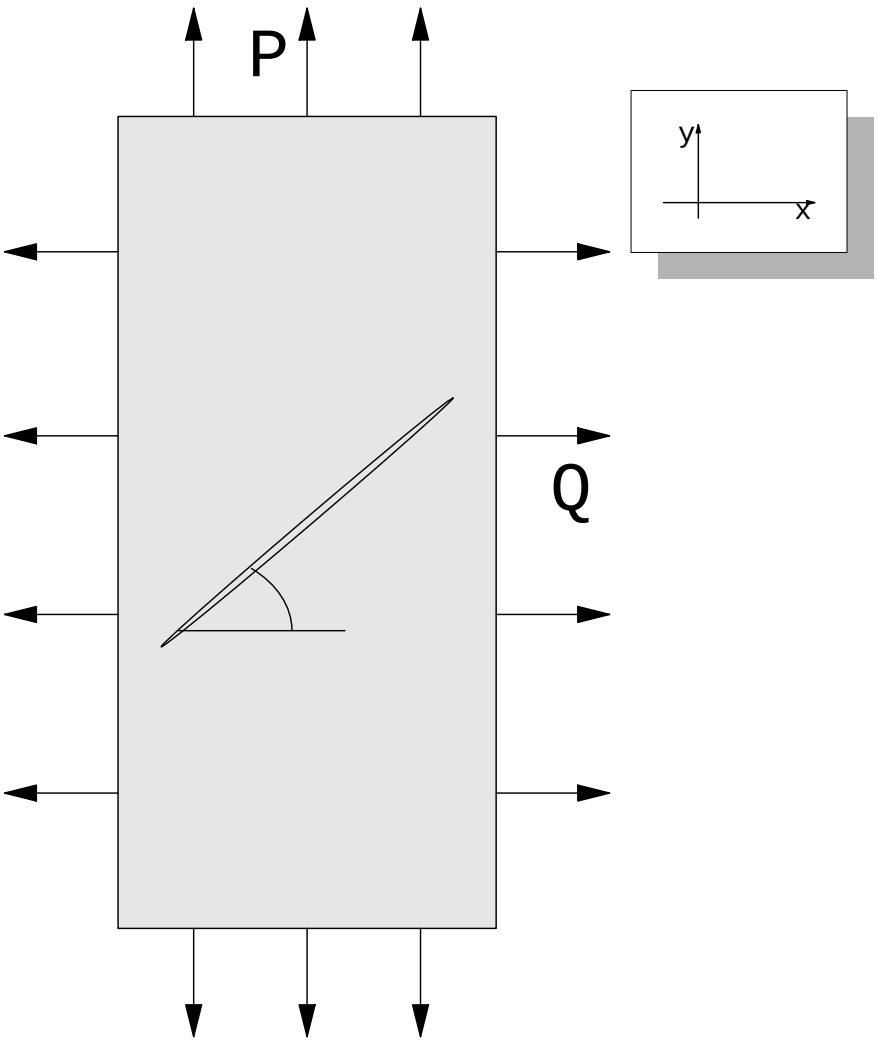

FIGURE 1. The loading sample with a crack, according to Griffith

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